श्री BalaJi

ADVANCED PROBLEMS IN

Algebra

for **JEE**

Main & Advanced

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Solutions

1 CHAPTER

QUADRATIC EQUATIONS

SECTION-1

SINGLE CHOICE QUESTIONS

1. If the equation $\sin x + \cos x = y^2 - y + a$ has no solution in x and y, then

(A) $a < -2$
(B) $a \in (-1, 1)$
(C) $a < \frac{1}{4} - \sqrt{2}$
(D) $a > \sqrt{2} + \frac{1}{4}$

2. The set of values of 'a' for which the equation $\cos^4 x - \sin^4 x + \cos 2x + a^2 + a = 0$ will have atleast one real solution is

(A) $[-2, 1]$
(B) $[-1, 2]$
(C) $[-1, 1]$
(D) $[1, 2]$

3. The solution set of the inequality $\sqrt{x} - 3 \leq \frac{2}{\sqrt{x} - 2}$ is

(A) $[0, 1] \cup [4, 16]$
(B) $[0, 1] \cup (4, 16]$
(C) $(0, 1) \cup (4, 16)$
(D) $(0, 1] \cup (4, 16]$

4. Let $a, b, c \in R$, $a \neq 0$ such that a and $4a + 3b + 2c$ have the same sign. Then the equation $ax^2 + bx + c = 0$ must have

(A) both roots in (1, 2)
(B) no roots in (1, 2)
(C) not both roots in (1, 2)
(D) exactly one root in (1, 2)

5. Let $x = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}, y = \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$, then the value of $x^4 + y^4 + (x + y)^4$ is equal to

(A) 527
(B) 1254
(C) 976
(D) 1152

6. Let ‘p’ is a root of the equation $x^2 - x - 3 = 0$. Then the value of $\dfrac{p^3+1}{p^5-p^4-p^3+p^2}$ is equal to

(A) $\frac{4}{3}$
(B) $\frac{4}{9}$
(C) $\frac{2}{9}$
(D) $\frac{2}{3}$

7. Given that the equation $(x - 19)(x - 97) = p$ has real roots α and β. Then the minimum real root of the equation $(x - \alpha)(x - \beta) = -p$ is

(A) 13
(B) 16
(C) 18
(D) 19

8. Let a, b, c are real numbers with $a^2 + b^2 + c^2 > 0$. Then the equation $x^2+(a + b + c)x+(a^2 + b^2 + c^2) = 0$ has

(A) 2 positive real roots
(B) 2 negative real roots
(C) 2 real roots with opposite sign
(D) no real roots

9. Let a, b are two different positive integers and the two quadratic equations $(a - 1)x^2 - (a^2 + 2)x + (a^2 + 2a) = 0$ and $(b - 1)x^2 - (b^2 + 2)x + (b^2 + 2b) = 0$ have one common root. Then the value of $\dfrac{a^b+b^a}{a^{-b}+b^{-a}}$ is equal to

(A) 256
(B) 64
(C) 16
(D) 72

10. If p, q_1 and q_2 are real numbers with $p = q_1 + q_2 + 1$, then which of the following must be correct about the equations $E_1 : x^2 + x + q_1 = 0$ and $E_2 : x^2 + px + q_2 = 0$,

(A) Nothing can be said about roots of the two equations.
(B) atleast one of the equation has distinct real roots.
(C) atleast one must have imaginary roots.
(D) atleast one must have real roots of oposite sign.

11. Given that the solution set of the quadratic inequality $ax^2 + bx + c > 0$ is (2, 3). Then the solution set of the inequality $cx^2 + bx + a < 0$ will be

(A) $\left(\frac{1}{3}, \frac{1}{2}\right)$
(B) $(-\infty, 2) \cup (3, \infty)$
(C) $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$
(D) Nothing can be said

12. Complete set of real values of k for which the inequality $kx^2 - kx - 1 < 0$ holds for any real x, satisfy

(A) $k \in (-4, 0)$
(B) $k \in (-4, 0]$
(C) $k \in [-4, 0)$
(D) $k \in [-4, 0]$

13. The poduct of all values of x satisfying the equation

$$\frac{1}{x^2+2x} + \frac{1}{x^2+6x+8} + \frac{1}{x^2+10x+24} = \frac{1}{5} - \frac{1}{x^2+14x+48}$$ is

(A) –80
(B) 40
(C) –10
(D) –20

14. Let x_1, x_2, x_3 be the roots of the equation $x^3 + 3x + 5 = 0$. Then the value of expression $\left(x_1 + \frac{1}{x_1}\right)\left(x_2 + \frac{1}{x_2}\right)\left(x_3 + \frac{1}{x_3}\right)$ is equal to

(A) $-\frac{29}{5}$
(B) –5
(C) $-\frac{24}{5}$
(D) –1

15. The value of maximum real root minus the minimum real root of the equation $(x^2 - 5)^4 + (x^2 - 7)^4 = 16$ is

(A) $\sqrt{5} + \sqrt{7}$
(B) $2\sqrt{5}$
(C) $\sqrt{28}$
(D) $4\sqrt{2}$

16. Three real numbers x, y, z are such that $x^2 + 6y = -17$, $y^2 + 4z = 1$ and $z^2 + 2x = 2$. Then the value of $x^3 + y^3 + z^3$ is equal to

(A) 30 (B) –24

(C) –36 (D) –28

17. The degree of the polynomial p(x) which satisfies the condition

$(x)p(x - 1) = (x - 15)\, p(x)$ is :

(A) 4 (B) 15

(C) 7 (D) can not be determined

18. The equations $x^2 - 4x + k = 0$ and $x^2 + kx - 4 = 0$, where k is a real number, have exactly one common root. Then the number of values of k is

(A) 0 (B) 1

(C) 2 (D) 3

19. Suppose 1, 2, 3 are the roots of the equation $x^4 + ax^2 + bx = c$. Then the value of c is

(A) 25 (B) 8

(C) 24 (D) 36

20. If a, b are the roots of the quadratic equation $x^2 + \lambda x - \frac{1}{2\lambda^2} = 0$, where λ is a real parameter. Then the minimum value of $a^4 + b^4$ is

(A) $2\sqrt{2}$ (B) $2(\sqrt{2} - 1)$

(C) $\sqrt{2}$ (D) $2 + \sqrt{2}$

21. If a, b, c are complex numbers and $a + b + c = ab + bc + ca = abc = 1$, then find $a + b^4 + c^4$

(A) 1 (B) 0

(C) 3 (D) 8

22. Let $p(x) = x^6 + ax^5 + bx^4 + x^3 + bx^2 + ax + 1$. Given that 1 is a root of $p(x) = 0$ and -1 is not. What is the maximum number of distinct real roots that p could have

(A) 6 (B) 5

(C) 4 (D) 3

23. If the roots of quadratic equation $ax^2 + bx + c = 0$ are $\frac{m}{m-1}$ and $\frac{m+1}{m}$, then $(a + b + c)^2$ is equal to

(A) $b^2 - ac$ (B) $4(b^2 - 4ac)$

(C) $b^2 - 4ac$ (D) $\frac{b^2 + c^2}{a^2}$

24. The set of all real values of 'a' for which both the roots of the equation $x^2 - 1 = 0$ lie between the roots of the equation $x^2 + (3a - a^2)x - 3a^3 = 0$ is equal to

(A) $(-\infty, -1)$ (B) $\left(-1, -\frac{1}{3}\right) \cup (1, \infty)$

(C) $(1, \infty)$ (D) $\left(-\frac{1}{3}, 1\right)$

25. The complete set of real values of 'a' for which the smaller root of the equation $x^2 + 2ax - 3 = 0$ lies in the interval $(-1, 1)$ is

(A) $(1, \infty)$ (B) $(0, 1)$

(C) $\left(-\frac{1}{4}, \infty\right)$ (D) $a \in (2, \infty)$

26. If the ratio of the roots of equation $ax^2 + 2bx + c = 0$ is same as the ratio of the roots of equation $px^2 + 2qx + r = 0$, where a, b, c, p, q, r are non zero real numbers. Then the value of $\left(\frac{b^2}{q^2}\right)\left(\frac{p}{a}\right)\left(\frac{r}{c}\right)$ is equal to :

(A) 4 (B) 2

(C) 1 (D) -1

27. The complete set of real values of 'a' for which the inequality $ax^2 - (3+2a)x + 6 > 0$, $a \neq 0$ holds for exactly three negative integral values of x is

(A) $\left[-\frac{5}{4}, -\frac{3}{4}\right)$ (B) $\left[-2, -\frac{5}{4}\right]$

(C) $(-1, 0)$ (D) $\left(-1, -\frac{3}{4}\right]$

28. If the equation $ax^2 + bx + c = x$, a, b, c $\in$ R and $a \neq 0$, has no real roots, then the equation $a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c = x$ will have

(A) 2 distinct real roots (B) no real roots

(C) 2 equal real roots (D) nothing can be said

29. The equation $x^2 - 4ax + 1 = 0$ has real roots gien by α and β, where a is real. Then the complete set of values of a for which $\alpha \geq a$ and $\beta \geq 0$ is

(A) $[1, \infty)$ (B) $[2, \infty)$

(C) $(-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$ (D) $[\frac{1}{2}, \infty)$

30. If the quadratic equation $ax^2 - bx + 7 = 0$ does not have two distinct real roots, then the minimum value of a + b is equal to :

(A) –8 (B) –7

(C) –6 (D) –5

31. If α and β are roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $ax^2 - bx(x+1) + c(x+1)^2 = 0$ are

(A) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ (B) $\frac{-\alpha}{\alpha+1}, \frac{-\beta}{\beta+1}$

(C) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ (D) $\frac{\alpha}{\alpha-1}, \frac{\beta}{\beta-1}$

32. If the equation $(6a + 3b + 4c)x^2 + (11a + 8b + 7c)x + (3c + 5a + 5b) = 0$ has equal real roots, where a, b, c are positive real numbers. Then a,b, c are in

(A) A.P.
(B) G.P.
(C) H.P.
(D) nothing can be said

33. Let a, b, are real numbers such that $a + b = 5$. Then the equation $x^2 - ax - b = 0$ must have for all real values of a

(A) equal real roots
(B) imaginary roots
(C) distinct real roots
(D) nothing can be said

34. If both roots of the equation $x^2 - 2(a - 1)x + 2a + 1 = 0$ are positive , then

(A) $a < 2$
(B) $a \geq 4$
(C) $1 \leq a \leq 4$
(D) $1 < a < 2$

35. The complete set of real values of p for whcih both roots of the equation $x^2 + 2(p - 3)x + 9 = 0$ lie in $(-6, 1)$ is

(A) $\left[6, \frac{27}{4}\right)$
(B) $\left(\frac{27}{4}, 9\right)$
(C) $[6, 9)$
(D) $(-2, 0] \cup (2, 9)$

36. the complete set of real values of 'a' for which the equation $9^x - (4a)3^x + 4 - a^2 = 0$ has an unique root in the interval $(0, 1)$ is

(A) $(-12, 0)$
(B) $(0, 11)$
(C) $(-11, -1) \cup \left\{\frac{2}{\sqrt{5}}\right\}$
(D) $(-13, -5) \cup \left\{\frac{2}{\sqrt{5}}\right\}$

37. Let a, b, c be real numbers, $a \neq 0$, if α is a root $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

(A) $\gamma < \alpha$
(B) $\gamma > \beta$
(C) $\alpha < \gamma < \beta$
(D) $\gamma = \frac{\alpha + \beta}{2}$

38. If the maximum and minimum values of $y = \frac{x^2 - 3x + c}{x^2 + 3x + c}$ are 7 and $\frac{1}{7}$ respectively, $c \in R$. Then the value of c is equal to

(A) 2 (B) 4

(C) 3 (D) 5

39. The complete set of real values of 'a' for which there are distinct real numbers x, y satisfying the equations $x = a - y^2$ and $y = a - x^2$ is

(A) $\left[\frac{3}{4}, \infty\right)$ (B) $\left[\frac{5}{4}, \infty\right)$

(C) $\left[\frac{7}{4}, \infty\right)$ (D) $(0, \infty)$

40. If a, b, c, p, q, r are non-zero real numbers, such that $a < b < c$ and $f(x) = (x - a)(x - b)(x - c) - p^2(x - a) - q^2(x - b) - r^2(x - c)$, then $f(x) = 0$ must have

(A) exactly 1 real root (B) exactly 3 distinct real roots

(C) 2 equal and 1 distinct real roots (D) nothing can be said

41. If $ax^2 + bx + 8 = 0$, $a, b \in R$, $a \neq 0$ has no distinct real roots, then the least value of $4a + b$ is

(A) –4 (B) –3

(C) –2 (D) –1

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. Let the inequalities $y(-1) > -4$, $y(1) < 0$ and $y(3) > 5$ are known to hold for $y = ax^2 + bx + c$, then

(A) $b > 3$
(B) $b < 2$
(C) $a < 0$
(D) $a > \frac{1}{8}$

2. Let $ax^2 + bx + c$ is integer for all integral values of x, then which of the following must be true ?

(A) c is integer
(B) $b = \frac{n}{2}, n \in I$
(C) $a = \frac{n}{2}, n \in I$
(D) a, b both are integers

3. If the value of quadratic trinomial $ax^2 - bx + c$ is an integer for $x = 0$, $x = 1$ and $x = 2$, then the valueof the given trinomial is an integer for

(A) $x = 2017$
(B) $x = 2018$
(C) $x = -4$
(D) $x = -2017$

4. How many roots does the equation $x^4 = 5x + 2a$ possess depending on 'a' ?

(A) if $a < -\frac{15}{16}\sqrt[3]{10}$
(B) 2 if $a > -\frac{15}{16}\sqrt[3]{10}$
(C) 3 if $a = -\frac{15}{16}\sqrt[3]{10}$
(D) 1 if $a = -\frac{15}{16}\sqrt[3]{10}$

5. How many roots does the equation $x^3 + ax + 2 = 0$ possess depending on 'a'?

(A) 1 if $a \in (-\infty, -3)$
(B) 3 if $a \in (-\infty, -3)$
(C) 2 if $a = -3$
(D) 1 if $a \in (-3, \infty)$

6. How many solutions does the system of equations $|x| + |y| = 1$, $x^2 + y^2 = a^2$ possess depending on 'a'?

(A) if $|a| < \frac{1}{\sqrt{2}}$

(B) 0 if $|a| > 1$

(C) 4 if $|a| = \frac{1}{\sqrt{2}}, 1$

(D) 8 if $\frac{1}{\sqrt{2}} < |a| < 1$

7. The values of 'a' for which the curves $y = 1 + \frac{x^2}{a^3}$ and $y = 4\sqrt{x}$ possess only one point in common is/are

(A) $a \in \left(0, \frac{1}{3}\right)$

(B) $a \in (-\infty, 0)$

(C) $a = \frac{1}{3}$

(D) $a \in \left(\frac{1}{3}, \infty\right)$

8. The greatest value of the function $f(x) = \frac{1}{2bx^2 - x^4 - 3b^2}$ on the interval $[-2, 1]$ depending on the parameter 'b' is/are

(A) $-\frac{1}{3b^2}$ if $b \in [0, 2]$

(B) $\frac{1}{4b - 4 - 3b^2}$ if $b \in [0, 4]$

(C) $\frac{1}{8b - 16 - 3b^2}$ if $b \le 2$

(D) $-\frac{1}{3b^2}$ if $b \ge 2$

9. The greatest value of the function $f(x) = x^4 - 6bx^2 + b^2$ on the interval $[-2, 1]$ depending on the parameter b is

(A) b^2 if $b \ge \frac{2}{3}$

(B) $16 - 24b + b^2$ if $b \le \frac{2}{3}$

(C) $4 - 12b + b^2$ if $0 \le b \le \frac{4}{3}$

(D) $16 - 24b + b^2$ if $b \ge \frac{2}{3}$

10. Let $f(x) = ax^2 + bx + c$, $a \ne 0$, a, b, c are integers. Let $f(1) = 0$, $50 < f(7) < 60$ and $70 < f(8) < 80$, then

(A) $f(10) = 135$

(B) $f(3) = 4$

(C) $f(5) = 20$

(D) $f(-2) = 21$

11. Let x satisfy the equation $\frac{\sqrt{a}+\sqrt{x-b}}{\sqrt{b}+\sqrt{x-a}}=\sqrt{\frac{a}{b}}$, a, b > 0, then

(A) $x \in [a, \infty)$ if a = b

(B) Number of values of x is exactly one if a > b

(C) Number of values of x is zero if zero if a ≠ b

(D) Number of values of x is exactly one if a < b

12. Given that a, b, c are positive distinct real numbers such that quadratic expressions $ax^2 + bx + c$, $bx^2 + cx + a$ and $cx^2 + ax + b$ are always non-negative. Then the expression $\frac{a^2+b^2+c^2}{ab+bc+ca}$ can never lie in

(A) $(-\infty, 2]$
(B) $(-\infty, 1]$
(C) $(2, 4)$
(D) $[4, \infty)$

13. Given that the equation $mx^2 - 2(m + 2)x + m + 5 = 0$, $m \in R$ has no real root. Then the equation $(m - 6)x^2 - 2(m + 2)x + (m + 5) = 0$ can have

(A) Two equal real roots
(B) Two distinct real roots
(C) No real roots
(D) Exactly one real root

14. The possible values of 'b', $b \in R$ for which the equations $2017x^2 + bx + 7102 = 0$ and $7102x^2 + bx + 2017 = 0$ have a common root is/are

(A) –9119
(B) –10879
(C) 9119
(D) 10879

15. The equation $(x - a)(x - a - b) = 1$, where a, b are positive constants has

(A) One root less than a and the other is greater than a.

(B) One root less than a + b and the other is greater than a + b.

(C) One root less than a and the other is greater than a + b.

(D) Roots lying between a and a + b.

16. Let p, q are integers and the two roots of the equation $x^2 - \left(\frac{p^2+11}{9}\right)x + \frac{15}{4}(p+q) + 16 = 0$ are p and Then

(A) $p = 4$ (B) $p = 13$

(C) $q = 13$ (D) $q = 7$

17. Let $M = 3x^2 - 8xy + 9y^2 - 4x + 6y + 13$, where $x, y \in R$, then

(A) M can not be equal to zero (B) M must be negative

(C) $M > 2$ (D) M must be positive

18. The equation $8x^4 - 16x^3 + 16x^2 - 8x + a = 0$, $a \in R$ has

(A) Atleast two real roots $\forall\ a \in R$.

(B) At least two imaginary roots $\forall\ a \in R$.

(C) The sum of all non-real roots equal to 2, if $a > \frac{3}{2}$.

(D) The sum of all non-real roots equal to 1, if $a \le \frac{3}{2}$.

19. If a, b, c are the roots of the cubic equation $x^3 - 3ax^2 + 3bx - c^3 = 0$, then which of the following may be possible ?

(A) $a = 3$ (B) $b = 6$

(C) $c = 0$ (D) $a = b = c$

20. Consider the equation $\frac{4x^2 + x + 4}{x^2 + 1} + \frac{x^2 + 1}{x^2 + x + 1} = \frac{31}{6}$, has

(A) The number of distinct real roots equal to 4.

(B) The number of distinct real roots equal to 3.

(C) The sum of all distinct real roots of the equation is –2.

(D) The sum of all distinct real roots of the equation is –1.

21. Given that the equation $\frac{x}{x+1}+\frac{x+1}{x}=\frac{4x+a}{x(x+1)}$, a ∈ R, has only one real root, then

(A) Number of values of a is 3.
(B) Number of values of a is 1.
(C) Sum of all values of 'a' is $\frac{1}{2}$.
(D) Sum of all values of 'a' is $\frac{13}{2}$.

22. Let $f(x) = x^3 - 3x + b$ and $g(x) = x^2 + bx - 3$, where b is a real number. If the equations f(x) = 0 and g(x) = 0 have a common root, then

(A) Number of possible values of b is 3.
(B) Number of possible value of b is 2.
(C) Sum of all possible values of b is 0.
(D) Sum of all possible values of b is 2.

23. Consider the equation $3x^7 - x^4 - 30x^5 + 10x^2 + 3x^3 - 1 = 0$ then

(A) The minimum real root of equation is $(-\sqrt{3}-\sqrt{2})$.
(B) The minimum real root of equation is $(-\sqrt{3}+\sqrt{2})$.
(C) The maximum real root of equation is $(\sqrt{3}+\sqrt{2})$.
(D) The number of positive roots of equation is 3.

24. The equation $x^4 - 9x^3 + 2(10 - a)x^2 + 9ax + a^2 = 0$ for x where 'a' is real parameter has

(A) 4 distinct real roots if $a > -4$, $a \neq 0$.
(B) 2 distinct real roots if $-\frac{25}{4} < a < -4$.
(C) No real roots if $a < -\frac{25}{4}$.
(D) 3 distinct real roots for 2 values of a.

25. The number of real roots of the equation $\sqrt{|1-x|} = kx$, where k is a parameter is

(A) 1, if $k > \frac{1}{2}$
(B) 2, if $k = \frac{1}{2}$
(C) 3, if $0 < k < \frac{1}{2}$
(D) 1, if $k \leq 0$

26. The complete set of values of real parameter 'm' for which the equation $m\sin^2x + (m-1)\sin x + (m-2) = 0$ has

(A) No real roots is $(-\infty, 1) \cup \left(1+\frac{2}{\sqrt{3}}, \infty\right)$

(B) 5 roots in $[0, 2\pi]$ is $\{2\}$

(C) 4 roots in $[0, 2\pi]$ is $(1, 2) \cup \left(2, \frac{\sqrt{3}+2}{\sqrt{3}}\right)$

(D) 2 roots in $[0, 2\pi]$ is $\left\{-1, 1, \frac{\sqrt{3}+2}{\sqrt{3}}\right\}$

27. The set of values of real number 'a' for which the equation $a^3 + a^2|a + x| + |a^2x + 1| = 1$ has not less than 4 different solutions which are integers can be

(A) $\left(\frac{1}{2}, 1\right)$ (B) $(-3, -1)$

(C) $(-\infty, -3]$ (D) $\left[-\frac{1}{\sqrt{3}}, \frac{1}{2}\right]$

28. The possible value(s) of 'p' for which the equations $ax^2 - px + ab = 0$ and $x^2 - ax - bx + ab = 0$ may have a common root, given that a, b are non zero real numbers is/are :

(A) $b^2 + a$ (B) $a + ab$

(C) $a^2 + b$ (D) $b + ab$

29. If $\sin 10^\circ$ is a root of the equation $4ax^3 - 3ax + b = 0$, where a, b are real parameters, $a \neq 0$, then the remaining two roots are

(A) $\sin 130^\circ$ (B) $\sin 40^\circ$

(C) $\sin 250^\circ$ (D) $\sin 200^\circ$

30. Suppose a and b are two positive real numbers such that the roots of the cubic equation $x^3 - ax + b = 0$ are all real. If α is a root of this cubic with minimum absolute value, then

(A) $\alpha < \frac{b}{a}$ (B) $\alpha > \frac{b}{a}$

(C) $\alpha \leq \frac{3b}{2a}$ (D) $\alpha > \frac{3b}{2a}$

31. If the equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$, $a, b, c \in R$ and $ac \neq 0$ have a common non real root, then

(A) $a = -c$ (B) $a = c$

(C) $|b| > 2|a|$ (D) $|b| < 2|a|$

32. If the equation $ax^2 + bx + c = 0$, where $a, b, c \in R$ and $a > 0$ has two real roots α and β such that $\alpha < -2$ and $\beta > 2$, then

(A) $c < 0$ (B) $a - |b| + c < 0$

(C) $4a + 2|b| + c < 0$ (D) $6a + |b| + c < 0$

33. Let the expression $\dfrac{(ax-b)(dx-c)}{(bx-a)(cx-d)}$ takes all real values when x is real, a, b, c, d are all distinct real parameters. Then which of the following is/are possible

(A) $a^2 > b^2$ and $c^2 < d^2$ (B) $a^2 > b^2$ and $c^2 > d^2$

(C) $a^2 < b^2$ and $c^2 > d^2$ (D) $a^2 < b^2$ and $c^2 < d^2$

34. The possible set of real values of 'a' for which, for all x not exceeding unity in absolute value, the inequality $\dfrac{ax - a(1-a)}{a^2 - ax - 1} > 0$ is valid

(A) $(1, \infty)$ (B) $\left(-\infty, \dfrac{-1+\sqrt{5}}{2}\right)$

(C) $\left(-\infty, \dfrac{-1-\sqrt{5}}{2}\right)$ (D) $(2, \infty)$

35. Given that system of inequalities $x^2 + 4x + 3 \leq \alpha$ and $x^2 - 2x \leq 3 - 6\alpha$ have a unique solution, then

(A) Sum of all possible values of α is equal to 0.

(B) Number of all possible values of α is 2.

(C) Sum of all possible values of α is equal to –1.

(D) Number of all possible values of α is 3.

36. If $ax^2 + bx + c = 0$, $a \neq 0$ be an equation with integral coefficients and $D > 0$ be its discriminant. Then the equation $b^2x^2 - Dx - 4ac = 0$ must have

(A) Two integral roots (B) Two irrational roots

(C) Two rational roots (D) At least one integral root

37. If all the roots of the equation $x^4 - 12x^3 + ax^2 + bx + 81 = 0$ where $a, b \in R$ are positive, then

(A) $b + 2a = 0$ (B) $b + a = -54$

(C) $a = 54$ (D) $a = 27$

38. If each pair of equations $x^2 + ax + 2 = 0$, $x^2 + bx + 6 = 0$ and $x^2 + cx + 3 = 0$ has a common root, then $a + b + c$ can be equal to

(A) -3 (B) 3

(C) -12 (D) 12

39. Let p and q be real numbers such that the parabola $y = x^2 - 2px + q$ has no common point with the x-axis. Let there exist points A and B on the parabola such that AB is parallel to the x-axis and $\angle AOB = 90^\circ$ ('O' is origin), then possible values of q is/are

(A) -1 (B) $\frac{3}{17}$

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

40. Let the equation $x^n + px^2 + qx + r = 0$, where $n \geq 6$, $r \neq 0$ has roots $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ and $S_k = \sum_{i=1}^{n} \acute{a}_i^k$, where k is an natural number, then

(A) $S_n + pS_2 + qS_1 + nr = 0$

(B) Roots of the equation can not all be real

(C) $S_n = -nr$

(D) $S_n = r$

41. The three roots of equation $x^4 - px^3 + qx^2 - rx + s = 0$, where $p, q, r, s \in R$ and $s < 0$, are tanA, tanB and tanC where A, B, C are angles of a triangle. Then the fourth root of the equation can be equal to :

(A) $\frac{p + \sqrt{p^2 - 4s}}{2}$ (B) $\frac{p - \sqrt{p^2 - 4s}}{2}$

(C) $\frac{p + r}{1 - q + s}$ (D) $\frac{p - r}{1 - q + s}$

SECTION-3

COMPREHENSION BASED QUESTIONS

COMPREHENSION: (1 TO 3)

Given that $a > 0$, $|ax^2 + bx + c| \leq 1$, if $-1 \leq x \leq 1$, $a, b, c \in R$ and $ax + b$ has its maximum value 2, when $-1 \leq x \leq 1$. Then :

1. a =

(A) 3 (B) 1 (C) 2 (D) 4

2. b =

(A) –1 (B) 2 (C) 1 (D) 0

3. c =

(A) –1 (B) 0 (C) 1 (D) 2

COMPREHENSION: (4 TO 6)

Consider a function $f : R \to R$, $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are real such that max $|f(x)| \leq 2$ for all $x \in [-2, 2]$.

4. f(x) must b

(A) odd function (B) even function
(C) neither odd nor even function (D) nothing can be said

5. b =

(A) 1 (B) 0 (C) –2 (D) –3

6. f(1) is equal to

(A) –2 (B) –1 (C) 1 (D) 2

COMPREHENSION: (7 TO 8)

Let $f(x) = x^3 + ax^2 + bx + c$ and $g(x) = x^3 + bx^2 + cx + a$, where a, b, c are integers with $c \neq 0$. Let $f(1) = 0$ and the roots of g(x) are squares of the roots of f(x). Then

7. $a^3 + b^2 + c =$

(A) -1 (B) 1

(C) -3 (D) 3

8. $ab + bc + ca =$

(A) -1 (B) 3

(C) 1 (D) -3

COMPREHENSION: (9 TO 11)

Consider the equation $x^4 - (k - 1)x^2 + (2 - k) = 0$. The complete set of possible values of real k for which the equation has

9. Four distinct real roots is

(A) $(-\infty, 2)$ (B) $(2\sqrt{2} - 1, 2)$

(C) $(\sqrt{2} - 1, 2\sqrt{2} - 1)$ (D) $(2, \infty)$

10. 3 distinct real roots is

(A) $\{2\}$ (B) $\{\sqrt{2} - 1, 2\}$

(C) $\{\sqrt{5} - 1\}$ (D) $\{2\sqrt{2}, \sqrt{3} - \sqrt{2}\}$

11. 2 distinct real roots is

(A) $(0, 2)$ (B) $(-\infty, 2\sqrt{2} - 1)$

(C) $(2, \infty)$ (D) $\left\{2\sqrt{2} - 1\right\} \cup (2, \infty)$

COMPREHENSION: (12 TO 13)

Consider the inequality $9 - x^2 > |x - a|$, where a is a real number. Then the complete set of values of 'a' for which the given inequality has

12. Atleast one negative solution is

(A) $(-8, 8)$
(B) $(-9, 9)$
(C) $(-3, 8)$
(D) $\left(-\frac{37}{4}, 9\right)$

13. Only positive solutions is

(A) $\left(-9, \frac{41}{4}\right)$
(B) $\left(9, \frac{41}{4}\right)$
(C) $\left(9, \frac{37}{4}\right)$
(D) $(8, 9)$

COMPREHENSION: (14 TO 15)

Consider a function, $f(\theta) = \tan^2\theta + (a + 1)\tan\theta - (a - 3)$, $\theta \in R$ where a is a real parameter. The complete set of values of 'a' for which

14. $f(\theta) > 0\ \forall\ \theta \in \left(0, \frac{\pi}{2}\right)$ is

(A) $[-3, 3]$
(B) $\left(-3-\sqrt{5}, 0\right)$
(C) $\left[3+2\sqrt{5}, \infty\right)$
(D) $\left(-3-2\sqrt{5}, 3\right]$

15. $f(\theta) < 0\ \forall\ \theta \in \left(0, \frac{\pi}{2}\right)$ is

(A) $(0, \infty)$
(B) $(-\infty, -3)$
(C) $(3 + 2\sqrt{5}, \infty)$
(D) Null set

COMPREHENSION: (16 TO 18)

If rational number $\frac{p}{q}$, $q \neq 0$ and p, q are relatively prime integers is a root of polynomial function, then p is a divisor of constant term and q that of leading coefficient. Therefore the possible rational zeroes are formed by listing the factors of constant term over the factors of leading coefficient.

16. The sum of all possible rational x-intercepts of the curve $y = 6x^4 - 13x^3 - 35x^2 - x + 3$ is

(A) $-\frac{1}{3}$ (B) -1 (C) $-\frac{11}{6}$ (D) $-\frac{13}{6}$

17. The number of possible rational roots of the equation $x^4 + (2p_1+1)x^3 + (2p_2+1)x^2 + (2p_3+1)x + (2p_4+1) = 0$, p_1, p_2, p_3, p_4 are integers is

(A) 0 (B) 2

(C) 4 (D) nothing can be said

18. The number of possible integral roots of the equations $ax^2 + bx + c = 0$, where a, b, c are prime numbers greater than 2 is

(A) 0 (B) 1

(C) 2 (D) nothing can be said

COMPREHENSION: (19 TO 21)

Consider the equation $x^4 + (\alpha - 1)x^3 + x^2 + (\alpha - 1)x + 1 = 0$, where 'α' is a real parameter. Then

19. The given equation has 2 positive and 2 negative roots if

(A) $\alpha \leq -\frac{1}{2}$ (B) $\alpha \geq \frac{5}{2}$ (C) $-\frac{1}{2} \leq \alpha \leq \frac{5}{2}$ (D) None of these

20. The given equation has 2 distinct negative roots if

(A) $\alpha < -\frac{1}{2}$ (B) $\alpha > \frac{5}{2}$ (C) $-\frac{1}{2} < \alpha < \frac{5}{2}$ (D) $-\frac{5}{2} < \alpha < \frac{1}{2}$

21. The given equation has no real roots if

(A) $\alpha < -\frac{1}{2}$ (B) $\alpha > \frac{5}{2}$ (C) $-\frac{1}{2} < \alpha < \frac{5}{2}$ (D) $-\frac{5}{2} < \alpha < \frac{1}{2}$

COMPREHENSION: (22 TO 23)

Given that the two quadratic equations $E_1 : x^2 - ax + b = 0$ and $E_2 : x^2 - px + q = 0$, where a, b, p, q are real parameters have a common root.

22. If the other roots of equations are reciprocal to each other, then $(q-b)^2$ is equal to

(A) $(p-a)^2$ (B) $b(p-a)^2$

(C) $q(p-a)^2$ (D) $bq(p-a)^2$

23. If the equation E_2 has equal roots, then b + q is equal to

(A) ap (B) $\frac{ap}{2}$

(C) 2ap (D) 4ap

COMPREHENSION: (24 TO 26)

One root of the equation $x^4 - 5x^3 + ax^2 + bx + c = 0$ is $3 + \sqrt{\ }$. If all the roots of the equation are real given that a, b, c are rational parameters, then

24. The greatest value of 'a' is equal to

(A) $\frac{7}{4}$ (B) $\frac{5}{4}$ (C) 2 (D) $\frac{9}{4}$

25. The least value of 'b' is equal to

(A) $\frac{13}{2}$ (B) 6 (C) $\frac{9}{2}$ (D) $\frac{11}{2}$

26. The greatest value of 'c' is equal to

(A) $\frac{3}{4}$ (B) $\frac{5}{4}$ (C) $\frac{7}{4}$ (D) $\frac{9}{4}$

COMPREHENSION (27 TO 29) :

Let x_1, x_2, x_3, x_4 be the roots of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$. If $x_1 + x_2 = x_3 + x_4$ and a, b, c, d $\in$ R, then

27. If a = 2, then b – c is equal to

(A) 0 (B) 1 (C) 2 (D) 4

28. If b < 0, then how many different real values of 'a', we may have?

(A) 0 (B) 1 (C) 2 (D) 3

29. If b + c = 1 and a $\neq$ –2, then for real values of 'a', the complete set of real values of 'c' is

(A) $(-\infty, 1]$ (B) $[1, \infty)$ (C) $\left(-\infty, \frac{1}{4}\right]$ (D) $\left(-\infty, \frac{5}{4}\right)$

COMPREHENSION (30 TO 31)

Let $f(x) = x^4 + ax^3 + bx^2 + ax + 1$ be a polynomial where a, b are real numbers, then

30. If f(x) = 0 has two different pairs of equal roots, then the least value of a + b is

(A) 0 (B) 1 (C) 2 (D) 4

31. If all roots of f(x) = 0 are imaginary and b = –1, then the complete set of values of 'a' is

(A) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (B) $(-1, 1)$ (C) $\left(-\frac{5}{2}, \frac{5}{2}\right)$ (D) $(-3, 3)$

COMPREHENSION (32 TO 33) :

Let m be a real number, such that the roots x_1 and x_2 of the equation $x^2 + (m-4)x + m^2 - 3m + 3 = 0$ are real numbers, then

32. The value of m for which $x_1^2 + x_2^2 = 6$ is equal to

(A) $-1-\sqrt{5}$
(B) $\sqrt{5}+1$
(C) $\sqrt{5}-1$
(D) $1-\sqrt{5}$

33. The maximum value of $\frac{mx_1^2}{1-x_1} + \frac{mx_2^2}{1-x_2} + 8$ is equal to

(A) $\frac{11}{3}$
(B) 1
(C) $\frac{49}{9}$
(D) $\frac{121}{9}$

COMPREHENSION (34 TO 35) :

Consider functions $f(x) = \frac{x^2+4x+3}{x^2+7x+14}$ and $g(x) = \frac{x^2-5x+10}{x^2+5x+20}$, for all real values of x, then

34. The greatest value of f(x) is equal to

(A) 3
(B) 4
(C) 1
(D) 2

35. The greatest value of function $(g(x))^{f(x)}$ is equal to

(A) 9
(B) 4
(C) 8
(D) 16

SECTION-4

MATCH THE COLUMN

1.

	Column-I		**Column-II**
(A)	If a, b, c are length of sides of a triangle, then the roots of the equation $a^2x^2 + (b^2 + a^2 - c^2)\,x + b^2 = 0$ are	(P)	of opposite signs
(B)	If a, b, c are unequal positive numbers and b is A.M. of a and c, then the roots of the equation $ax^2 + 2bx + c = 0$ are	(Q)	both positive
(C)	If $a \in R$, then the roots of the equation $x^2 - (a + 1)\,x - a^2 - 4 = 0$ are	(R)	both negative
(D)	If a, b, c are unequal positive numbers and b is H.M. of a and c, then the roots of the equation $ax^2 + 2bx + c = 0$ are	(S)	real and distinct
		(T)	imaginary

2. Let α, β, γ be three numbers such that $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$ and $\alpha^3 + \beta^3 + \gamma^3 = 11$, then

	Column-I		**Column-II**
(A)	$\alpha^4 + \beta^4 + \gamma^4$ is equal to	(P)	13
(B)	$\alpha^5 + \beta^5 + \gamma^5$ is equal to	(Q)	26
(C)	$(\alpha^2 - 4)(\beta^2 - 4)(\gamma^2 - 4)$ is equal to	(R)	57
(D)	$\alpha^6 + \beta^6 + \gamma^6$ is equal to	(S)	119
		(T)	129

3.

	Column-I		Column-II
(A)	The possible integral values of λ for which $(\lambda - 2)x^2 + 8x + (\lambda + 4) > 0 \ \forall \ x \in R$ is/are	(P)	2
(B)	The equation $x^2 + 2(a^2 + 1)x + (a^2 - 14a + 48) = 0$ possesses roots of opposite signs, then the value of 'a' can be	(Q)	3
(C)	If the equation $ax^2 + 2bx + 4c = 16$ has no real roots and $a + c > b + 4$, then the integral value of 'c' can be equal to	(R)	4
(D)	the possible values of x satisfying the equation $\|x^2 - x - 6\| = x + 2$ is/are	(S)	5
		(T)	7

4.

	Column-I		Column-II
(A)	The possible value(s) of 'a' for which the largest value of $\sin^2 x - 2a \sin x + a + 3$ is 7 is/are	(P)	–3
(B)	The possible value(s) of 'a' for which the largest value of $x^4 - ax^2 + 2a - 1$ for $x \in [-1, 2]$ is 9 is/are	(Q)	1
(C)	The possible value(s) of 'a' for which the equation $\tan^4 x - 3\tan^2 x + (a - 1) = 0$ has 4 roots in $(0, \pi)$ is/are	(R)	3
(D)	The possible value(s) of 'a' for which the smallest value of $x^4 - ax^2 + 2a - 1$ for $x \in [-1, 2]$ is – 7 is/are	(S)	5
		(T)	11

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. The number of real values of x satisfying the equation $\sqrt{1-\sqrt{x^4-x^2}} = x - 1$ is

2. Let ABCD be a rectangle and let E and F be points on CD and BC respectively such that area (ΔADE) = 16, area (ΔCEF) = 9 and area (ΔABF) = 25. Then the area (ΔAEF) is k, find $\frac{k}{5}$.

3. The number of integral values of 'a' such that the equation $x^3 - 3x + a = 0$ has three integer roots is

4. Let the equation $x^2 - (2a + b)x + (2a^2 + b^2 - b + \frac{1}{2}) = 0$ where $a, b \in R$ has two real roots. Then the value of 6a + 2b is equal to

5. Given that the quadratic equation $ax^2 + bx + c = 0$ has no real roots, but Mr. A got two roots 2 and 4 since he wrote down a wrong value of 'a'. Mr. B also got two roots –1 and 4 because he wrote the sign of a term wrongly. Then the value of $\frac{2b+3c}{a}$ is equal to

6. Given that m is a real number not less than –1, such that the equation $x^2 + 2(m - 2)x + m^2 - 3m + 3 = 0$ has two distinct real roots x_1 and x_2. Find the maximum value of $\frac{1}{2}\left(\frac{mx_1^2}{1-x_1}+\frac{mx_2^2}{1-x_2}\right)$.

7. Given that the quadratic equation $x^2 - px + q = 0$, where $p, q \in R$ has two real roots α and β. If the equation having roots α^3, β^3 is also $x^2 - px + q = 0$, then find the number of possible pairs (p, q).

8. Let α, β are the roots of the equation $x^2 + x - 3 = 0$. Then the value of $\alpha^3 - 4\beta^2 + 19$ is equal to

9. If a, b are two real numbers satisfying the relations $19a^2 + 99a + 1 = 0$ and $b^2 + 99b + 19 = 0$ and $ab \neq 1$. Then the value of $\left|\frac{ab + 4a + 1}{b}\right|$ is equal to

10. Given that a, b, c are the lengths of three sides of ΔABC, $a > b > c$, $2b = a + c$ and b is a positive integer. If $a^2 + b^2 + c^2 = 84$, find the value of b.

11. The number of ordered pairs (p, q), $p, q \in N$ such that the equation $x^2 - pqx + p + q = 0$ has two integer roots.

12. Let p be an integer such that both roots of the equation $5x^2 - 5px + (66p - 1) = 0$ are positive integers. Then the value of $\left[\frac{p}{10}\right]$ is equal to ([.] denotes greatest integer function)

13. Given that the integers a, b satisfy the equation

$$\left(\frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}}-\frac{\frac{1}{b}}{\frac{1}{a}+\frac{1}{b}}\right)\left(\frac{1}{a}-\frac{1}{b}\right)\times\frac{1}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}=\frac{2}{3}$$

. Then the value of a + b is equal to

14. Find the number of integral values of parameter 'a' so that the inequality $\frac{1}{8}(2a - a^2) \leq x^2 - 3x + 2 \leq 3 - a^2$ holds for any real x in the interval [0, 2].

15. Given that a, b are integers and the two real roots α, β of the equation $3x^2 + 3(a + b)x + 4ab = 0$ satisfy the relation $\alpha(\alpha + 1) + \beta(\beta + 1) = (\alpha + 1)(\beta + 1)$. Then the number of ordered pairs (a, b) is equal to

16. Let a, b, c are real numbers and satisfy $a = 8 - b$ and $c^2 = ab - 16$, then $\frac{a}{b}$ is equal to

17. The number of triplets (a, b, c), where a, b, c are rational numbers such that the equation $x^3 + ax^2 + bx + c = 0$ has roots a, b, c.

18. If α is a real root of equation $x^5 - x^3 + x - 2 = 0$, then $[\alpha^6]$ is equal to ([.] denotes greatest integer function)

19. The number of ordered pairs of integers (x, y) satisfying $x + y = x^2 - xy + y^2$ is

20. How many integer pairs (x, y) satisfy $x^2 + 4y^2 - 2xy - 2x - 4y - 8 = 0$?

21. Given that α, β, γ are all real roots of the equation $x^3 - 2007x + 2002 = 0$, then the value of $\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}+\frac{\gamma-1}{\gamma+1}$ is equal to

22. The number of integral values of 'a' for which the equation $x^3(x + 1) = (x + a)(2x + a)$ has four distinct real solutions is

23. Given that real numbers a, b satisfy $a^3 - 6a^2 + 15a - 21 = 0$ and $b^3 - 6b^2 + 15b - 7 = 0$ respectively, then find the value of a + b.

24. Natural numbers k, ℓ, p and q are such that if a and b are roots of $x^2 - kx + \ell = 0$ then $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of $x^2 - px + q = 0$. What is the sum of all possible values of q ?

25. Given that $a + b + c = 3$, $a^2 + b^2 + c^2 = 5$ and $a^3 + b^3 + c^3 = 7$, then the value of $a^4 + b^4 + c^4$ is equal to

26. The equation $x^4 - (r + 1)x^2 + r = 0$ has 4 distinct real solutions which form an arithmetic progression. The number of such real values of r is

27. The smallest value that $x^2 - 7x + 6$ takes in the set $\{x \in R | x^2 - x - 30 \geq 0\}$ is

28. Given that $x^2 + y^2 = 8x + 6y + 11$, where x and y are integers . What is the smallest possible value of $|4x-2y|$.

29. The number of different integer triplets (x, y, z) satisfying the equations $x + y^2 - z = 124$ and $x^2 + y - z = 100$ is

30. The polynomial equation $x^4 - 2x^2 + ax + b = 0$ has four distinct real roots. The maximum possible integral value of its root is

31. Find the least possible integral value of $\sqrt{8a+8b+c}$ for which both the roots of equation $ax^2 - bx + c = 0$ are distinct and lie in interval (0, 1), is, $(a, b, c \in N)$.

32. For a natural number b, let N(b) denotes the number of natural numbers a for which the equation $x^2 + ax + b = 0$ has integer roots. Let x is the sum of digits of the smallest value of b for which N(b) = 20. Then $\frac{n+1}{2}$ is equal to

33. Let α, β be the roots of equation $x^2 - 2px + p^2 - 2p - 1 = 0$. If p_1, p_2 be two distinct real values of 'p' for which $\frac{1}{2}\left(\frac{(\alpha-\beta)^2-2}{(\alpha+\beta)^2+2}\right)$ is an integer. Find $\frac{1}{10}\left(\frac{1}{p_1^2}+\frac{1}{p_2^2}\right)$.

34. The number of real number pairs (x, y) which will satisfy the equation $x^2 - xy + y^2 = 4(x + y - 4)$ is

35. The equation $x^4 - 8x^3 + 24x^2 - 32x - 14 = 0$ has two real roots x_1, x_2 and two imaginary roots x_3, x_4, then the value of $x_1x_2 + x_3x_4$ is equal to

36. If α and β, $\alpha \neq \beta$ are the roots of the equation $x^2 - p(x + 1) - c = 0$, then find the value of $\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+c}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+c}$.

37. The least integral value of 'a' for which the inequality $\frac{x^2+a^2}{a(4+x)} \geq 1$, holds true for all real values of x in the interval (–1, 1) is

38. The number of integral values of 'a' for which the inequality $\log_x(x^2 - 4x + a) > 0$ holds for all real values of x in the interval (0, 1).

39. The smallest positive value of 'a' for which every solution of inequality $\log_{\frac{1}{2}} x^2 \geq \log_{\frac{1}{2}}(x+2)$ is a solution of inequality $49x^2 - 4a^4 \leq 0$ is

40. Let S be the sum of all distinct real values of 'm' for which the equation $mx^3 - 9x^2 + 12x - 5 = 0$ has two equal real roots. Then [S] = ([.] denotes greatest integer function).

41. The number of integral values of 'a' for which any real value of x that satisfies the inequality $ax^2 + (1 - a^2)x - a > 0$ does not exceed two in absolute value.

42. Let P(x) be a polynomial with real coefficients and leading coefficient unity satisfy the identity $(x - 8) P(2x) = 8(x - 1) P(x)$. Then $[\sqrt{P(10)}]$ is equal to ([.] denotes greatest integer function).

43. Let P(x) = 0 be a fifth degree polynomial equation with integer coefficients that has a integral root 'α'. If P(2) = 13 and P(10) = 5 then $\left[\frac{á}{2}\right]$ is equal to ([.] denotes greatest integer function).

44. The number of integral values of 'a' for which the equation cos2x + asinx = 2a – 7 possesses solution is

45. The number of ordered pairs (x, y), x, y $\in$ R satisfying the equation

$4^{|x2-8x+12|-\log_4 7} = 7^{2y-1}$ and the inequality $|y - 3| - 3|y| - 2(y + 1)^2 \geq 1$ is

46. The number of integral values of n for which the equation $nx^2 + (n + 1)x + (n + 2) = 0$ has rational roots only is equal to

47. If p_1, p_2 are the roots of the quadratic equation $ax^2 + bx + c = 0$ and q_1, q_2 are the roots of the quadratic equation $cx^2 + bx + a = 0$ such that p_1, q_1, p_2, q_2 is an A.P. of distinct terms, then a + c is equal to (a, b, c $\in$ R).

48. The number of integral values of 'a' for which the equation$(x^2 + x + 2)^2 - (a - 3)(x^2 + x + 2)(x^2 + x + 1) + (a - 4)(x^2 + x + 1)^2 = 0$ has atleast one real root is

49. The smallest integral value of |a| for which the range of function $f(x) = \frac{x^2 + a}{x + 1}$ is R is

50. The number of distinct values of 'c' for which the equation

$x^4 - (c^2-7c+11)\,x^2 + (18-21c + 8c^2-c^3) = 0$ doesn't have 4 distinct roots is

51. For each real number m, the parabolas $y = (m^2 + 4)x^2 + (m - 2)^2x - 4m + 2$ passes through the same point (a, b), then $a^2 + b^2$ is equal to

52. Suppose 1, 2, 3 are the roots of equation $x^4 + ax^2 + bx = c$, then the value of c is equal to

53. Let a and b be real numbers such that $a \neq 0$. Then the maximum number of possible real roots of the equation $ax^4 + bx^3 + x^2 + x + 1 = 0$ is equal to

54. Let x, y, z are all real numbers such that $x + y + z = 0$ and $xy + yz + zx = -3$, then the value of $|x^3y + y^3z + z^3x|$ is equal to

55. Find the number of positive integers 'n' such that $3^{2n} + 3n^2 + 7$ is a perfect square.

Answer Key

Single choice Questions

1. D	**2.** A	**3.** B	**4.** C	**5.** D	**6.** B
7. D	**8.** D	**9.** A	**10.** B	**11.** C	**12.** B
13. D	**14.** A	**15.** C	**16.** C	**17.** B	**18.** B
19. D	**20.** D	**21.** C	**22.** B	**23.** C	**24.** C
25. A	**26.** C	**27.** D	**28.** B	**29.** D	**30.** B
31. B	**32.** A	**33.** C	**34.** B	**35.** A	**36.** D
37. C	**38.** B	**39.** A	**40.** B	**41.** C	

One or More Than One Correct Questions

1. B,D	**2.** A,C	**3.** A,B,C,D	**4.** A,B,D	**5.** B,C,D	**6.** A,B,C,D
7. B,C	**8.** C,D	**9.** A,B	**10.** A,B,C	**11.** A,B,D	**12.** B,D
13. B,D	**14.** A,C	**15.** A,B,C	**16.** B,D	**17.** A,C,D	**18.** B,C,D
19. A,B,C,D	**20.** B,C	**21.** A,D	**22.** A,C	**23.** A,C,D	**24.** A,B,D
25. A,B,C,D	**26.** A,B,C,D	**27.** C,D	**28.** B,C	**29.** A,C	**30.** B,C
31. B,D	**32.** A,B,C	**33.** B,D	**34.** C,D	**35.** B,C	**36.** C,D
37. A,B,C	**38.** C,D	**39.** B,C	**40.** A,B,C	**41.** A,B,D	

Comprehension Based Questions

1. C	**2.** D	**3.** A	**4.** A	**5.** D	**6.** A
7. A	**8.** A	**9.** B	**10.** A	**11.** D	**12.** D
13. C	**14.** D	**15.** D	**16.** C	**17.** A	**18.** A
19. D	**20.** B	**21.** C	**22.** D	**23.** B	**24.** B
25. D	**26.** C	**27.** B	**28.** B	**29.** C	**30.** B
31. A	**32.** C	**33.** D	**34.** D	**35.** A	

Match The Column

1. A – T ; B – R, S; C – P, S ; D – T
2. A – Q ; B – R ; C – P ; D – T
3. A – S, T ; B – T ; C – S, T ; D – P, R
4. A – P, Q ; B – R, S ; C – R ; D – P, T

Subjective Type Questions

1. 1	**2.** 6	**3.** 2	**4.** 5	**5.** 6	**6.** 5
7. 6	**8.** 0	**9.** 5	**10.** 5	**11.** 5	**12.** 7
13. 3	**14.** 1	**15.** 4	**16.** 1	**17.** 3	**18.** 3
19. 6	**20.** 6	**21.** 2	**22.** 0	**23.** 4	**24.** 4
25. 9	**26.** 2	**27.** 0	**28.** 2	**29.** 8	**30.** 1
31. 9	**32.** 8	**33.** 2	**34.** 1	**35.** 8	**36.** 1
37. 5	**38.** 0	**39.** 3	**40.** 4	**41.** 2	**42.** 9
43. 7	**44.** 5	**45.** 2	**46.** 3	**47.** 0	**48.** 1
49. 2	**50.** 4	**51.** 5	**52.** 36	**53.** 2	**54.** 9
55. 1					

Previous Year Questions

SECTION-1

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is **[AIEEE 2002]**

(A) $3x^2 - 19x + 3 = 0$ (B) $3x^2 + 19x - 3 = 0$

(C) $3x^2 - 19x - 3 = 0$ (D) $x^2 - 5x + 3 = 0$

2. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then **[AIEEE 2002]**

(A) $a + b + 4 = 0$ (B) $a + b - 4 = 0$

(C) $a - b - 4 = 0$ (D) $a - b + 4 = 0$

3. If p and q are the roots of the equation $x^2 + px + q = 0$, then **[AIEEE 2002]**

(A) $p = 1, q = -2$ (B) $p = 0, q = 1$

(C) $p = -2, q = 0$ (D) $p = -2, q = 1$

4. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is **[AIEEE 2002]**

(A) less than 1 (B) equal to 1

(C) greater than 1 (D) any real no.

5. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is **[AIEEE 2003]**

(A) $-\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $-\frac{2}{3}$ (D) $\frac{1}{3}$

6. The real positive number x when added to its inverse gives the minimum value of the sum at x equal to **[AIEEE 2003]**

(A) –2 (B) 2

(C) 1 (D) –1

7. If (1 – p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its root are **[AIEEE 2004]**

(A) –1, 2 (B) –1, 1

(C) 0, –1 (D) 0, 1

8. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is **[AIEEE 2004]**

(A) 4 (B) 12

(C) 3 (D) $\frac{49}{4}$

9. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then **[AIEEE 2005]**

(A) a = b + c (B) c = a + b

(C) b = c (D) b = a + c

10. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[AIEEE 2005]**

(A) (5, 6] (B) (6, ∞)

(C) (–∞, 4) (D) [4, 5]

11. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan30º and tan15º, respectively, then the value of 2 + q – p is **[AIEEE 2006]**

(A) 2 (B) 3

(C) 0 (D) 1

12. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval **[AIEEE 2006]**

(A) $-2 < m < 0$
(B) $m > 3$
(C) $-1 < m < 3$
(D) $1 < m < 4$

13. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is **[AIEEE 2006]**

(A) $\frac{1}{4}$
(B) 41
(C) 1
(D) $\frac{17}{7}$

14. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[AIEEE 2007]**

(A) $(3, \infty)$
(B) $(-\infty, -3)$
(C) $(-3, 3)$
(D) $(-3, \infty)$

15. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE 2009]**

(A) 1
(B) 4
(C) 3
(D) 2

16. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is **[AIEEE 2009]**

(A) less than 4ab
(B) greater than –4ab
(C) less than –4ab
(D) greater than 4ab

17. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has **[AIEEE 2012]**

(A) infinite number of real roots
(B) no real roots
(C) exactly one real root
(D) exactly four real roots

18. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is : **[IIT Mains 2013]**

(A) 3 : 1 : 2 (B) 1 : 2 : 3

(C) 3 : 2 : 1 (D) 1 : 3 : 2

19. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] **[IIT Mains 2013]**

(A) does not exist (B) lies between 1 and 2

(C) lies between 2 and 3 (D) lies between – 1 and 0

20. Let α and β be the roots of equation $x^2 – 6x – 2 = 0$. If $a_n = \alpha^n – \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[IIT Mains 2015]**

(A) – 3 (B) 6

(C) –6 (D) 3

21. The sum of all real values of x satisfying the equation

$(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is : **[IIT Mains 2016]**

(A) 3 (B) –4

(C) 6 (D) 5

22. If, for a positive integer, n the quadratic equation,

$x(x + 1) + (x + 1)(x + 2) + ... + (x + \overline{n - 1})\ (x + n) = 10n$

has two consecutive integral solutions, then n is equal to : **[IIT Mains 2017]**

(A) 10 (B) 11

(C) 12 (D) 9

23. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 – x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to : **[IIT Mains 2018]**

(A) 2 (B) –1

(C) 0 (D) 1

SECTION-2

1. Find the values of α & β, $0 < \alpha,\ \beta < \pi/2$, satisfying the following equation,

$\cos\alpha\ \cos\beta\ \cos(\alpha + \beta) = -1/8$. **[REE '99, 6]**

2. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real & less than 3 then **[JEE '99, 2 + 2]**

(A) $a < 2$ (B) $2 \le a \le 3$

(C) $3 < a \le 4$ (D) $a > 4$

3. If α, β are the roots of the equation, $(x - a)(x - b) + c = 0$, find the roots of the equation, $(x - \alpha)(x - \beta) = c$. **[REE 2000 (Mains), 3]**

4.(a) For the equation, $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to:

(A) 1/3 (B) 1

(C) 3 (D) 2/3

(b) If α & β ($\alpha < \beta$), are the roots of the equation, $x^2 + bx + c = 0$, where $c < 0 < b$, then

(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$

(C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

(c) If $b > a$, then the equation, $(x - a)(x - b) - 1 = 0$, has :

[JEE 2000 Screening, 1 + 1 + 1 out of 35]

(A) both roots in [a, b] (B) both roots in $(-\infty, a)$

(C) both roots in $[b, \infty)$ (D) one root in $(-\infty, a)$ & other in $(b, +\infty)$

(d) If α, β are the roots of $ax^2 + bx + c = 0$, $(a \ne 0)$ and $\alpha + \delta$, $\beta + \delta$, are the roots of, $Ax^2 + Bx + C = 0$, $(A \ne 0)$ for some constant δ, then prove that,

[JEE 2000, Mains, 4 out of 100]

$$\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}.$$

5. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β.

[JEE 2001, Mains, 5 out of 100]

6. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is

[JEE 2002 (screening), 3]

(A) $(-\infty, -2) \cup (2, \infty)$
(B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(C) $(-\infty, -1) \cup (1, \infty)$
(D) $(\sqrt{2}, \infty)$

7. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'.

[JEE 2003, Mains-4 out of 60]

8. (a) If one root of the equation $x^2 + px + q = 0$ is the square of the other, then

(A) $p^3 + q^2 - q(3p + 1) = 0$
(B) $p^3 + q^2 + q(1 + 3p) = 0$
(C) $p^3 + q^2 + q(3p - 1) = 0$
(D) $p^3 + q^2 + q(1 - 3p) = 0$

(b) If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then **[JEE 2004 (Screening)]**

(A) $-5 < a < 2$
(B) $a < -5$
(C) $a > 5$
(D) $2 < a < 5$

9. Find the range of values of *t* for which $2 \sin t = \dfrac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

[JEE 2005(Mains), 2]

10. (a) Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then

[JEE 2006, 3]

(A) $\lambda < \dfrac{4}{3}$
(B) $\lambda > \dfrac{5}{3}$
(C) $\lambda \in \left(\dfrac{1}{3}, \dfrac{5}{3}\right)$
(D) $\lambda \in \left(\dfrac{4}{3}, \dfrac{5}{3}\right)$

(b) If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d, then find the value of $a + b + c + d$. (a, b, c and d are distinct numbers) **[JEE 2006, 6]**

11. (a) Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2$, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of 'r' is

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

Match the column :

(b) Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Match the expressions / statements in **Column I** with expressions / statements in **Column II**.

	Column-I		Column-II
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	(P)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, the $f(x)$ satisfies	(Q)	$f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	(R)	$f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	(S)	$f(x) < 1$

[JEE2007,3+6]

12. The smallest value of k, for which both the roots of the equation

$x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

[JEE 2009]

13. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -$ If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[JEE 2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

14.. A value of b for which the equations

$$x^2 + bx - 1 = 0$$

$$x^2 + x + b = 0,$$

have one root in common is **[JEE 2011]**

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$

(C) $i\sqrt{5}$ (D) $\sqrt{2}$

15. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

[IIT Advance - 2015]

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

16. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals **[IIT Advance 2016]**

(A) $2(\sec\theta - \tan\theta)$
(B) $2\sec\theta$
(C) $-2\tan\theta$
(D) 0

COMPREHENSION : (17 TO 18) **[IIT ADVANCE 2017]**

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For n = 0, 1, 2 ......, let $a_n = p\alpha^n + q\beta^n$.

Fact : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b.

17. If $a_4 = 28$, then $p + 2q =$

(A) 14
(B) 7
(C) 12
(D) 21

18. $a_{12} =$

(A) $2a_{11} + a_{10}$
(B) $a_{11} - a_{10}$
(C) $a_{11} + a_{10}$
(D) $a_{11} + 2a_{10}$

19. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}$ a cos x + 2 b sin x = c, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is ______________. **[IIT Advance 2018]**

Answer Key

Section-1

1. A **2.** A **3.** A **4.** A **5.** B **6.** C
7. C **8.** D **9.** B **10.** C **11.** B **12.** C
13. B **14.** C **15.** D **16.** B **17.** B **18.** B
19. A **20.** D **21.** A **22.** B **23.** D

Section-2

1. $\alpha = \beta = \pi/3$, **2.** A **3.** (a, b) **4.** (a) C, (b) B, (c) D
5. $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$ **6.** B **7.** $a > 1$
8. (a) D; (b) A **9.** $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ **10.** (a) A, (b) 1210
11. **(a)** D, **(b)** (A) P, R, S; (B) Q, S; (C) Q, S; (D) P, R, S
12. $k = 2$ **13.** B **14.** B **15.** A, D **16.** C **17.** C
18. C **19.** 0.5

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2 CHAPTER

SEQUENCE AND PROGRESSION

SECTION-1

SINGLE CHOICE QUESTIONS

1. The first and last two terms of an A.P. are a, b, c respectively in order. Then the sum of all the terms is :

(A) $\frac{(2c-a-b)(a+c)}{2(c-b)}$

(B) $\frac{(2c-a-b)(a+c)}{(c-b)}$

(C) $\frac{(4c-2a-2b)(a+c)}{(c-b)}$

(D) $\frac{(2c+a+b)(c-a)}{(c-b)}$

2. If $a^x = b^y = c^z$ and x, y, z are in G.P., where a, b, c, x, y, z are positive real numbers not equal to unity, then :

(A) $\log_a b = \log_c b$

(B) $\log_b a = \log_c b$

(C) $\log_{10} b = \log_{10} a \log_{10} c$

(D) $\log_b a + \log_c b = 0$

3. The sum of three numbers in H.P. is 11 and the sum of their reciprocals is 1, then their product is equal to

(A) 30

(B) 48`

(C) 24

(D) 36

4. If p^{th}, q^{th} and r^{th} terms of an H.P. be respectively x, y, z, then $(p-q)\,xy + (q-r)\,yz + (r-p)\,xz =$

(A) xyz + pqr

(B) pqr

(C) xyz

(D) 0

5. If A_1, A_2 be A.M.'s, G_1, G_2 be G.M.'s and H_1, H_2 be H.M.'s between two numbers, then

(A) $\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2}$

(B) $\frac{G_1G_2}{H_1H_2} = \frac{H_1+H_2}{A_1+A_2}$

(C) $\frac{G_1G_2}{A_1A_2} = \frac{H_1+H_2}{A_1+A_2}$

(D) $\frac{G_1G_2}{A_1A_2} = \frac{A_1+A_2}{H_1+H_2}$

6. If a, b, c are in H.P., then $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ is equal to

(A) $\frac{1}{b^2}-\frac{1}{ab}$ (B) $\frac{3}{b^2}-\frac{2}{ab}$

(C) $\frac{4}{b^2}-\frac{2}{ab}$ (D) $\frac{3}{b^2}-\frac{1}{ab}$

7. If A, G, H be respectively the A.M., G.M. and H.M. between two positive numbers and if xA = yG = zH where x, y, z are non-zero positive quantities, then x, y, z are in

(A) A.P. (B) G.P.

(C) H.P. (D) Nothing can be said

8. If the $(m + 1)^{th}$, $(n + 1)^{th}$ and $(r + 1)^{th}$ terms of an A.P. are in G.P. and m, n, r are in H.P., then the ratio of the common difference to the first term in the A.P. is equal to

(A) $\frac{2}{n}$ (B) $\frac{1}{n}$

(C) $-\frac{1}{n}$ (D) $-\frac{2}{n}$

9. If ℓ, m, n are three numbers in G.P., then the ratio of first term to the common difference of an A.P. whose ℓ^{th}, m^{th} and n^{th} terms are in H.P. is equal to

(A) $\frac{m+1}{2}$ (B) m

(C) m + 1 (D) $\frac{1}{m+1}$

10. If between any two quantities there be inserted two arithmetic means A_1, A_2 ; two geometric means G_1, G_2 ; and two harmonic means H_1, H_2, then $G_1G_2 : A_1 + A_2$ is equal to

(A) $H_1H_2 : H_1 + H_2$ (B) $\frac{1}{H_1}+\frac{1}{H_2}$

(C) $H_1 + H_2$ (D) H_1H_2

11. Let S_n be sum of the first 'n' terms of an A.P. $\{a_n\}$. If $S_6 = S_9$, then the ratio $a_3 : a_5$ is equal to

(A) 9 : 5 (B) 5 : 9

(C) 3 : 5 (D) 5 : 3

12. If in the arithmetic progression $\{a_n\}$, $a_1 > 0$, $3a_8 = 5a_{13}$, then among the following listed partial sums the largest one is

(A) S_{10} (B) S_{11}

(C) S_{20} (D) S_{21}

13. Let α, β be the roots of the equation $ax^2 + 2bx + c = 0$ and γ, δ be the roots of the equation $px^2 + 2qx + r = 0$. If α, β, γ, δ are in G.P., then

(A) $\frac{ac}{b^2} = \frac{pr}{q^2}$ (B) $\frac{ab}{c^2} = \frac{pq}{r^2}$

(C) $\frac{ac}{q^2} = \frac{pr}{b^2}$ (D) $\frac{pq}{c^2} = \frac{ab}{r^2}$

14. $\sum_{r=1}^{99} r!(r^2 + r + 1)$ is equal to

(A) 102! – 100! (B) 100(100!) – 1

(C) 99(100!) – 1 (D) 100(99!) – 1

15. If a, b, c form an A.P. with comon difference d (≠ 0) and x, y, z form a G.P. with common ratio r (≠ 1), then the area of triangle with vertices (a, x), (b, y) and (c, z) is independent of

(A) a (B) d

(C) x (D) r

16. A G.P. consist of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying the odd places. Then the common ratio will be equal to

(A) 3
(B) 4
(C) 5
(D) 6

17. A convex n-sided polygon has a circumcircle and an inscribed circle, its area is B and the area of its circumcircle and inscribed circle are A and C respectively, then

(A) $B = \frac{A+C}{2}$
(B) $B > \frac{A+C}{2}$
(C) $B < \frac{A+C}{2}$
(D) Nothing can be said

18. The sum $\sum_{k=1}^{n} \frac{k^2 - \frac{1}{2}}{k^4 + \frac{1}{4}}$ is equal to

(A) $\frac{2n^2 - 2n + 1}{2n^2 + 2n + 1}$
(B) $\frac{2n^2 - n}{2n^2 + 2n + 1}$
(C) $\frac{n^2}{2n^2 + 2n + 1}$
(D) $\frac{2n^2}{2n^2 + 2n + 1}$

19. The value of $\frac{\left(1^4 + \frac{1}{4}\right)\left(3^4 + \frac{1}{4}\right)\ldots\ldots\left((2n-1)^4 + \frac{1}{4}\right)}{\left(2^4 + \frac{1}{4}\right)\left(4^4 + \frac{1}{4}\right)\ldots\ldots\left((2n)^4 + \frac{1}{4}\right)}$ is equal to

(A) $\frac{1}{4n^2 + 2n + 1}$
(B) $\frac{1}{8n^2 + 4n + 1}$
(C) $\frac{1}{4(2n^2 + n + 1)}$
(D) $\frac{n}{8n^2 - 4n + 1}$

20. The sum $\sum_{k=1}^{n} \frac{1}{(k+1)\sqrt{k} + k\sqrt{k+1}}$ is equal to

(A) $\frac{\sqrt{n+1} - 2}{\sqrt{n+1}}$
(B) $\frac{\sqrt{n+1} - 1}{\sqrt{n+1}}$
(C) $\frac{\sqrt{n+1} + 1}{\sqrt{n+1}}$
(D) $\frac{n+1}{n\sqrt{n+1}}$

21. The sum $\sum_{k=1}^{n} k!(k^2+k+1)$ is equal to

(A) (n + 1) (n + 1)! – 1
(B) nn! – 1
(C) (n + 1)! – 1
(D) n(n + 1)! – 1

22. Let $a_1, a_2, a_3, \ldots\ldots, a_n$ be an arithmetic progression with common difference 'd'.

Then $\sum_{k=1}^{n-1} \frac{1}{a_k a_{k+1}}$ is equal to

(A) $\frac{n}{a_1(a_1+nd)}$
(B) $\frac{(n+1)}{a_1(a_1+nd)}$
(C) $\frac{(n-1)}{a_1(a_1+(n-1)d)}$
(D) $\frac{n}{a_1(a_1+(n-1)d)}$

23. Let S_n, S_{2n}, S_{3n} are respectively the sums of first n, 2n, 3n terms of an arithmetic progression, then $S_{3n} =$

(A) $2(S_{2n} - S_n)$
(B) $\frac{3}{2}(S_{2n} - S_n)$
(C) $3(S_{2n} - S_n)$
(D) $6(S_{2n} - S_n)$

24. The sum $\frac{7}{2\times3}\left(\frac{1}{3}\right)+\frac{9}{3\times4}\left(\frac{1}{3}\right)^2+\frac{11}{4\times5}\left(\frac{1}{3}\right)^3+\frac{13}{5\times6}\left(\frac{1}{3}\right)^4+\ldots.$ upto 10 terms is equal to

(A) $1-\frac{1}{12\times3^{10}}$
(B) $\frac{1}{2}-\frac{1}{12\times3^{10}}$
(C) $\frac{1}{3}-\frac{1}{12\times3^{10}}$
(D) $\frac{1}{2}-\frac{1}{11\times3^{10}}$

25. $\sum_{0\le i<j\le10} \sum (i+j)$ is equal to

(A) 545
(B) 548
(C) 550
(D) 552

26. Let there be a G.P. whose first term is 'a' and common ratio is 'r'. If A and H are the arithmetic mean and the harmonic mean respectively for the first 'n' terms of the G.P. Then AH is equal to

(A) ar^{n-1} (B) a^2r^{n-1}

(C) a^2r^n (D) $ar^{\frac{n-1}{2}}$

27. Let $0 < x < y < 2019$, then the number of ordered pair of integers (x, y) such that A.M. of x and y exceeds their G.M. by 2 is

(A) 42 (B) 43

(C) 44 (D) 45

28. The sum $\frac{19}{1\cdot2\cdot3}\frac{1}{4}+\frac{28}{2\cdot3\cdot4}\frac{1}{8}+\frac{39}{3\cdot4\cdot5}\frac{1}{16}+\frac{52}{4\cdot5\cdot6}\frac{1}{32}+ ....+$ upto infinite terms is equal to

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) 4

29. The sum of infinite series

$1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}+\frac{5^2}{5^4}-\frac{6^2}{5^2}+.........\infty$ is equal to

(A) $\frac{5}{27}$ (B) $\frac{25}{27}$

(C) $\frac{25}{108}$ (D) $\frac{25}{54}$

30. Let x, y are real numbers such that $x, x + 2y, 2x + y$ form an A.P. while the numbers $(y + 1)^2, xy + 5, (x + 1)^2$ form a G.P., then $|x - y|$ is equal to

(A) 0 (B) 1

(C) 2 (D) 4

31. The sum of all values of θ in interval $[0, 4\pi]$ for which $\frac{\sin\theta}{6}$, cosθ, tanθ taken in that order constitute a geometric progression is

(A) $\frac{23\pi}{3}$ (B) 24π

(C) 16π (D) 8π

32. $\frac{n}{1\cdot2\cdot3}+\frac{n-1}{2\cdot3\cdot4}+\frac{n-2}{3\cdot4\cdot5}+$ .... upto n terms is equal to

(A) $\frac{1}{2(n+2)}+\frac{n+1}{4}-\frac{1}{2}$ (B) $\frac{1}{2(n+2)}+\frac{n+1}{4}+\frac{1}{2}$

(C) $\frac{1}{n+2}+\frac{n+1}{4}-\frac{1}{2}$ (D) $\frac{1}{2(n+2)}+\frac{n+1}{2}+\frac{1}{2}$

33. $\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\frac{8}{1+x^8}+$ .... upto 'n' terms is equal to

(A) $\frac{1}{1-x}-\frac{2^{n-1}}{1-x^{2^{n-1}}}$ (B) $\frac{1}{1-x}+\frac{2^n}{1-x^{2^n}}$

(C) $\frac{1}{x+1}-\frac{2^n}{x^{2^n}-1}$ (D) $\frac{1}{x-1}-\frac{2^n}{x^{2^n}-1}$

34. If x, y, z are positive and $x+y+z=1$, then the minimum value of

$\left(\frac{1}{x}-1\right)\left(\frac{1}{y}-1\right)\left(\frac{1}{z}-1\right)$ is

(A) 12 (B) 4

(C) 8 (D) 9

35. $\frac{a_1}{1+a_1}+\frac{a_2}{(1+a_1)(1+a_2)}+\frac{a_3}{(1+a_1)(1+a_2)(1+a_3)}+....+\frac{a_n}{(1+a_1)(1+a_2)...(1+a_n)}$ is equal to

(A) $\frac{1}{1+a_1}-\frac{1}{(1+a_1)(1+a_2)...(1+a_n)}$ (B) $1-\frac{1}{(1+a_1)(1+a_2)...(1+a_n)}$

(C) $1-\frac{a_n}{(1+a_1)(1+a_2)...(1+a_{n-1})}$ (D) $\frac{1}{2}-\frac{1}{2(1+a_1)(1+a_2)...(1+a_n)}$

36. $\frac{3}{1^2 2^2}+\frac{5}{2^2 3^2}+\frac{7}{3^2 4^2}$ + ... + upto n terms is equal to

(A) $1-\frac{1}{(n+1)^2}$ (B) $1-\frac{1}{n^2}$

(C) $1-\frac{1}{n(n+1)}$ (D) $\frac{n}{(n+1)^2}$

37. The sum $1 + (1 + 2)x + (1 + 2 + 3)x^2 + (1 + 2 + 3 + 4)x^3 + ... + ... + (1 + 2 + 3 + 4 + ... n)x^{n-1}$ is equal to

(A) $\frac{1-x^n}{(1-x)^3}-\frac{nx^n}{(1-x)^2}-\frac{n(n+1)x^n}{2(1-x)}$ (B) $\frac{(1-x^n)}{(1-x)^3}-\frac{nx^n}{(1-x)^2}+\frac{n(n+1)x^n}{2(1-x)}$

(C) $\frac{1-x^n}{(1-x)^3}+\frac{nx^n}{(1-x)^2}+\frac{n(n+1)x^n}{2(1-x)}$ (D) $\frac{1-x^n}{(1-x)^3}+\frac{nx^n}{(1-x)^2}-\frac{n(n+1)x^n}{2(1-x)}$

38. The sum 2·3 + 3·6 + 4·11 + 5·18 + ... + upto n terms is equal to

(A) $\frac{n}{12}(3n^3 + 7n^2 + 2ln + 36)$ (B) $\frac{n}{12}(3n^3 - 10n^2 + 22n - 36)$

(C) $\frac{n}{12}(3n^3 + 10n^2 + 21n + 38)$ (D) $\frac{n}{12}(3n^3 + 8n^2 + 20n + 37)$

39. For $x > 1$, $\lim_{n\to\infty}\prod_{k=0}^{n}\left(1+\frac{2}{x^{2^k}+x^{-2^k}}\right)$ is equal to

(A) $\frac{x^2-1}{x^2+1}$ (B) $\frac{x^2+1}{x^2-1}$

(C) $\frac{x-1}{x+1}$ (D) $\frac{x+1}{x-1}$

40. Let G_1, G_2 are two geometric means and A is the arithmetic mean between two positive numbers, then $\frac{G_1^2}{G_2}+\frac{G_2^2}{G_1}=$

(A) A (B) 2A

(C) 3A (D) 4A

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. For all permissible values of x, consider $y=\frac{\sin 3x(\cos 6x+\cos 4x)}{\sin x(\cos 8x+\cos 2x)}$ and range of y is (–∞, a) ∪ (b, ∞). If 2b is the first term of G.P. and 'a' is its common ratio, then (s_∞ denotes the sum of infinite terms of G.P.)

(A) $b-a=\frac{10}{3}$ (B) $3a + b = 4$

(C) $s_\infty = 9$ (D) $s_\infty=\frac{27}{10}(a+b)$

2. It is given that the sequence $\{a_n\}$ satisfies $a_1 = 0$, $a_{n+1}=a_n+1+2\sqrt{1+a_n}$ for $n \in N$. Then

(A) $a_{100} = 9999$ (B) $a_{2001} = 4004000$

(C) $a_{2001} = 4002000$ (D) $a_{19} = 360$

3. Let $\{a_n\}$ consists of positive numbers and for any positive integer n, $\frac{a_n+2}{2}=\sqrt{2s_n}$, where $s_n=\sum_{i=1}^{n}a_i$. Then

(A) $a_{21} = 82$ (B) $a_{12} = 48$

(C) $a_{13} = 50$ (D) $a_{14} = 54$

4. If x, y, z are three distinct positive real numbers and are in H.P., then

$\frac{3x+2y}{2x-y}+\frac{3z+2y}{2z-y}$ is greater than

(A) 9 (B) 10

(C) 12 (D) 15

5. The sequence $\{a_n\}$, $n \in N$ satisfies $a_1 = 1$ and $5^{a_{n+1}-a_n}=1+\frac{1}{n+\frac{2}{3}}$. Then

(A) $[a_{501}] = 3$ (B) $[a_{207}] = 3$

(C) $[a_{223}] = 4$ (D) $[a_{625}] = 4$

(where [·] denotes greatest integer function)

6. If a, b, c are three positive real numbers, then $\frac{a^2+1}{b+c}+\frac{b^2+1}{c+a}+\frac{c^2+1}{a+b}$ can never be equal to

(A) 1
(B) 2
(C) $\frac{8}{3}$
(D) 3

7. The sum of squares of 3 distinct real numbers which are in G.P. is s^2. If there sum is αs, then α^2 lie in interval is

(A) $\left(0, \frac{1}{3}\right)$
(B) $\left(\frac{1}{3}, 1\right)$
(C) (1, 3)
(D) (3, 9)

8. Let 'p' be the first of 'n' arithmetic means between two positive numbers and 'q' be first of 'n' harmonic means between same two numbers. Then $\frac{q}{p}$ can lie in interval(s)

(A) (–∞, I]
(B) $\left(1, \left(\frac{n+1}{n-1}\right)^2\right)$
(C) $\left(\left(\frac{n-1}{n+1}\right)^2, \left(\frac{n+1}{n-1}\right)^2\right)$
(D) $\left[\left(\frac{n+1}{n-1}\right)^2, \infty\right)$

9. A sequence of real numbers $a_1, a_2, a_3, \ldots, a_{n+1}$ is such that $a_1 = 0$, $|a_2| = |a_1 + 1|$, $|a_3| = |a_2 + 1|, \ldots\ldots\ldots |a_{n+1}| = |a_n + 1|$. Then $\frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$ can not be equal to

(A) –2
(B) –1
(C) $-\frac{3}{4}$
(D) $-\frac{4}{7}$

10. Let x, y, z are distinct positive integers and

$$m = \left(\frac{x^2+y^2+z^2}{x+y+z}\right)^{(x+y+z)}, \quad n = x^x y^y z^z, \quad P = \left(\frac{x+y+z}{3}\right)^{(x+y+z)}, \text{ then}$$

(A) m > n
(B) n > p
(C) m < n
(D) n < p

11. Let $f(x)=\lim_{n\to\infty}\left(\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{4}{x^4+1}+.....+\frac{2^n}{x^{2^x}+1}\right)$ for x > 1, then

(A) $\sum_{r=2}^{6}\frac{1}{f(r)}=20$

(B) $f(5)=\frac{1}{6}$

(C) $f(5)=\frac{1}{4}$

(D) $\sum_{r=2}^{6}\frac{1}{f(r)}=15$

12. For a positive integer 'n', let $S(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+....+\frac{1}{2^n-1}$. Then

(A) S(200) > 100

(B) S(200) < 100

(C) S(100) < 100

(D) S(100) > 100

13. Let $a_1, a_2, a_3,.....$ be a sequence of integers satisfying $a_n + a_{n-1} = 2n\ \forall\ n \geq 2, n \in N$. If $a_1 = 100$, then

(A) $a_7 = 106$

(B) $a_{11} = 110$

(C) $a_8 = -90$

(D) $a_{12} = -88$

14. Consider a sequence $\{a_n\}$ with $a_1 = 2$ and $a_n = \frac{a_{n-1}^2}{a_{n-2}}\ \forall\ n \geq 3, n \in N$, terms of the sequence being distinct. Given that a_2 and a_5 are positive integers and $a_5 \leq 200$, then possible value(s) of a_5 can be

(A) 8

(B) 32

(C) 162

(D) 200

15. Let $a_1, a_2, a_3,..., a_n$ be first 'n' terms of a G.P. with first term 'a' and common ratio 'r'. Then

(A) $\frac{1}{a_1^2-a_2^2}+\frac{1}{a_2^2-a_3^2}+...+\frac{1}{a_{n-1}^2-a_n^2}=\frac{(1-r^{2n})}{a^2r^{2n-4}(1-r^2)^2}$

(B) $\frac{1}{a_1^2-a_2^2}+\frac{1}{a_2^2-a_3^2}+...+\frac{1}{a_{n-1}^2-a_n^2}=\frac{(1-r^{2n-2})}{a^2r^{2n-4}(1-r^2)^2}$

(C) $\frac{1}{a_1^m+a_2^m}+\frac{1}{a_2^m+a_3^m}+...+\frac{1}{a_{n-1}^m+a_n^m}=\frac{(r^{mn-m}-1)}{a^m(1+r^m)(r^{mn-m}-r^{mn-2m})}$

(D) $\frac{1}{a_1^m+a_2^m}+\frac{1}{a_2^m+a_3^m}+...+\frac{1}{a_{n-1}^m+a_n^m}=\frac{(r^{mn-m}-1)}{a^m(1-r^m)(r^{mn-m}-r^{mn-2m})}$

16. Let the three non-zero real numbers x(y – z), y(z – x), z(y – x) form a geometric progression in that order, with common ratio 'r', then the possible value(s) of [r] is equal to ([.] denotes greatest integer function)

(A) –2
(B) –1
(C) 0
(D) 1

17. The sequence $\{a_n\}$ and $\{b_n\}$ satisfy $a_k b_k = 1$, k = 1, 2, 3,.... If the sum of first 'n' terms of $\{a_n\}$ is equal to $\frac{n(n+1)(n+2)}{3}$. Then

(A) $\sum_{r=1}^{n} b_r = \frac{n}{n+1}$
(B) $\lim_{n\to\infty} \sum_{r=1}^{n} b_r = 2$
(C) $\sum_{r=1}^{n} b_r = \frac{2n+1}{n+2}$
(D) $\lim_{n\to\infty} \sum_{r=1}^{n} b_r = 1$

18. Let the equation $x^3 + px^2 + qx - q = 0$, where $p, q \in R$, $q \neq 0$ has 3 real roots α, β, γ in H.P., then

(A) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} \geq \frac{1}{3}$
(B) $9p + 2q + 27 = 0$
(C) Maximum value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$ is $\frac{1}{3}$
(D) $\frac{p}{q} \geq -\frac{1}{3}$

19. The sum $\sum_{n=1}^{\infty} \sum_{k=1}^{n-1} \frac{k}{2^{n+k}}$ is equal to

(A) $\sum_{k=1}^{\infty} \frac{k}{2^k}$
(B) $\sum_{k=1}^{\infty} \frac{k}{4^k}$
(C) $\sum_{m=1}^{\infty} \left(\frac{m}{2^m} \sum_{n=m+1}^{\infty} \frac{1}{2^n} \right)$
(D) $\frac{4}{27}$

20. For any natural number n > 1, consider the sum S = $\frac{1}{n^2+1} + \frac{2}{n^2+2} + \frac{3}{n^2+3} + \ldots + \frac{n}{n^2+n}$, then

(A) $S < \frac{1}{2} + \frac{1}{2n}$
(B) $S > \frac{1}{2} + \frac{1}{2n}$
(C) $S > \frac{1}{2}$
(D) $S < 1$

21. If $n \in N$, $n > 5$ then which of the following holds true ?

(A) $n^n > 1\cdot3\cdot5...(2n-1)$

(B) $2^n > 1 + n\sqrt{2^{n-1}}$

(C) $\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+...\frac{1}{2n}>\frac{1}{2}$

(D) $2^n < 1 + n\sqrt{2^{n-1}}$

22. Let $\{a_n\}$ be a sequence of real numbers such that $a_1 = 2$, $a_{n+1} = a_n^2 - a_n + 1$ for $n = 1, 2, 3,....$ Let $S = \frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_{2018}}$, then

(A) $S < 1 - \frac{1}{(2018)^{2018}}$

(B) $S > 1 - \frac{1}{(2018)^{2018}}$

(C) $S < 1$

(D) $S > 1 - \frac{1}{(2017)^{2017}}$

23. Let $a_k = \frac{k}{(k-1)^{4/3}+k^{4/3}+(k+1)^{4/3}}$ and $S_n = \sum_{k=1}^{n} a_k$, then

(A) $S_{26} > \frac{17}{4}$

(B) $S_{26} < \frac{17}{4}$

(C) $S_{999} < 50$

(D) $S_{999} > 50$

24. Let $S = \frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\frac{1}{\sqrt{9}+\sqrt{11}}+...\frac{1}{\sqrt{9997}+\sqrt{9999}}$, then

(A) $S < 24$

(B) $S > 24$

(C) $S > 18$

(D) $S < 18$

25. When we divide the ninth term of an arithmetic progression by its second term, we get 5 as quotient and when we divide the thirteenth term of that progression by the sixth term, we get 2 as a quotient and 5 as a remainder, then

(A) First term is 3

(B) Seventh term is 27

(C) First term is 4

(D) Third term is 9

26. Define $f_n(x) = (1 + x)(1 + 2x)(1 + 4x) \dots (1 + 2^n x) = a_{n,0} + a_{n,1}x + a_{n,2}x^2 + \dots + a_{n,n}x^n$, where n is a positive integer, then

(A) $a_{100,\,2} = \dfrac{(2^{100}-1)(2^{102}-4)}{3}$

(B) $a_{100,\,2} = \dfrac{(2^{101}-1)(2^{101}-2)}{3}$

(C) $a_{100,\,2} - a_{99,\,2} = 2^{201} - 2^{101}$

(D) $a_{100,\,2} - a_{99,\,2} = 2^{200} - 2^{100}$

27. Let $\{a_n\}$ be a sequence of real number such that $a_1 = 2$, $a_{n+1} = a_n^2 - a_n + 1 \ \forall\, n \in N$ and $S = \dfrac{1}{a_1} + \dfrac{1}{a_2} + \dfrac{1}{a_3} + \dots + \dfrac{1}{a_{2019}}$, then

(A) $S < 1$

(B) $S < 1 - \dfrac{1}{(2018)^{2018}}$

(C) $S > 1 - \dfrac{1}{(2018)^{2018}}$

(D) $S > 1 - \dfrac{1}{(2019)^{2019}}$

28. Let a, b, c be positive integers such that $a + b + c = n$, then

(A) $(a^a b^b c^c)^{1/n} \le \dfrac{a^2 + b^2 + c^2}{n}$

(B) $(a^b\, b^c\, c^a)^{1/n} \le \dfrac{ab + bc + ca}{n}$

(C) $(a^b\, b^c\, c^a)^{1/n} \le \dfrac{a^2 + b^2 + c^2}{n}$

(D) $(a^a\, b^b\, c^c)^{1/n} + (a^b\, b^c\, c^a)^{1/n} + (a^c\, b^a\, c^b)^{1/n} \le n$

29. If a, b, c, d are four unequal positive numbers which are in A.P., then

(A) $\dfrac{1}{a} + \dfrac{1}{d} > \dfrac{1}{b} + \dfrac{1}{c}$

(B) $\dfrac{1}{a} + \dfrac{1}{d} < \dfrac{1}{b} + \dfrac{1}{c}$

(C) $ad < bc$

(D) $\dfrac{1}{b} + \dfrac{1}{c} > \dfrac{4}{a + d}$

30. Let $a_1, a_2, a_3, \dots, a_n$ are in A.P., then

(A) $a_1 - 2a_2 + a_3 = 0$

(B) $a_1 - 2a_2 + 2a_3 - 2a_4 + a_5 = 0$

(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$

(D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_6 = 0$

31. Let $S = 2016^2 + 2015^2 + 2014^2 - 2013^2 - 2012^2 - 2011^2 + 2010^2 + 2009^2 + 2008^2 - 2007^2 - 2006^2 - 2005^2 + ... + 6^2 + 5^2 + 4^2 - 3^2 - 2^2 - 1^2$, then S is divisible by

(A) 8
(B) 27
(C) 112
(D) 2017

32. It is given that the numbers 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ form an arithmetic progression. Then

(A) $x^6 = y^8$
(B) $y^3 = z^4$
(C) $x^6 = y^7$
(D) $x^9 = z^{14}$

33. Consider a geometric pregression of real numbers such that sum of the first four terms is equal to 15 and the sum of their squares is equal to 85, then the possible value of sixth terms of the G.P. is/are

(A) 32
(B) 16
(C) $\frac{1}{4}$
(D) $\frac{1}{8}$

34. Let $S_n = \frac{1}{3!} + \frac{5}{4!} + \frac{11}{5!} + \frac{19}{6!} + ... +$ upto n terms and $S = \lim_{n\to\infty} S_n$, then

(A) $S = \frac{1}{6}$
(B) $S_n = \frac{1}{3!} - \frac{1}{(n+1)!}$
(C) $S_n = \frac{1}{2!} - \frac{(n+1)}{(n+2)!}$
(D) $S = \frac{1}{2}$

35. Let $f_n(x) = \frac{x}{1-x^2} + \frac{x^2}{1-x^4} + \frac{x^4}{1-x^8} + \ldots + \frac{x^{2^{n-1}}}{1-x^{2^n}}$ where $x > 0$, $x \neq 1$ and

$f(x) = \lim_{n\to\infty} f_n(x)$, then

(A) $f\left(\frac{1}{4}\right) = \frac{1}{3}$
(B) $f\left(\frac{1}{2}\right) = 1$
(C) $f(3) = -\frac{1}{2}$
(D) $f(5) = -\frac{1}{6}$

36. Let

$$f(x)=\frac{1}{x+1}+\frac{2x}{(x+1)(x+2)}+\frac{3x^2}{(x+1)(x+2)(x+3)}+\ldots+\frac{nx^{n-1}}{(x+1)(x+2)(x+3)\ldots.(x+n)}$$

then

(A) $f(x)=\frac{x}{1+x}-\frac{x^n}{(x+1)(x+2)\ldots(x+n)}$

(B) $1-\frac{x^n}{(x+1)(x+2)\ldots(x+n)}$

(C) $f'(x)=\left(-\frac{x^n}{(x+1)(x+2)\ldots(x+n)}\right)\left(\sum_{r=1}^{n}\frac{r}{x+r}\right)$

(D) $f'(x)=\left(-\frac{x^{n-1}}{(x+1)(x+2)\ldots(x+n)}\right)\left(\sum_{r=1}^{n}\frac{r}{x+r}\right)$

37. The sum to first 'n' terms of the series $(n \geq 3)$ $1^2 + 2\cdot2^2 + 3^2 + 2\cdot4^2 + 5^2 + 2\cdot6^2 + \ldots +$ upto 'n' terms is equal to

(A) $\frac{n^2(n+1)}{2}$ if n is odd

(B) $\frac{n(n+1)^2}{2}$ if n is even

(C) $n^2(n-1)$ if n is odd

(D) $2(n+1)^2$ if n is even

38. Let $a_1, a_2, a_3, \ldots a_n$ be the first 'n' terms of an A.P. having common difference 'd' $(d \neq 0)$, then the greatest value of product of two terms equidistant from the extreme terms is

(A) $a_1a_n+\frac{d^2(n-1)^2}{4}$ if n is odd

(B) $a_1a_n+\frac{d^2(n+1)^2}{4}$ if n is odd

(C) $a_1a_n+\frac{d^2n(n+2)}{4}$ if n is even

(D) $a_1a_n+\frac{d^2}{4}n(n-2)$ if n is even

39. If three successive terms of a G.P. having common ratio 'r' form the sides of a triangle. Then the possible value(s) of [r] is/are (where [·] denotes greatest integer function)

(A) 2

(B) 0

(C) 1

(D) 3

40. If a, b, c be in A.P. and a^2, b^2, c^2 be in H.P., then which of the following(s) may be true?

(A) $\frac{a}{2}$, b, c are in G.P.

(B) a, $-\frac{b}{2}$, c are in G.P.

(C) $-\frac{a}{2}$, b, c are in G.P.

(D) $a = b = c$

SECTION-3

COMPREHENSION BASED QUESTIONS

COMPREHENSION: (Q.1 TO Q.3)

Let $\{a_n\}$ and $\{b_n\}$ be two sequence of numbers in A.P. each with common difference 2 such that $a_1 < b_1$ and $c_n = \sum_{k=1}^{n} a_k, d_n = \sum_{k=1}^{n} b_k$. Suppose that the points (a_n, c_n) and (b_n, d_n) $\forall$ $n \in N$ both lie on the curve $y = \alpha x^2 + \beta x + \gamma$ where α, β, γ are real constants, then :

1. α is equal to

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1

2. If $\gamma = 0$, then a_1 is equal to

(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

3. If $\gamma = 0$, then b_1 is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

COMPREHENSION (Q.4 TO Q.6) :

$\{a_n\}$, $n \in N$ is a sequence of real numbers and are arranged in the form

a_1

a_2 a_3

a_4 a_5 a_6

a_7 a_8 a_9 a_{10}

Let $\{b_n\}$, $n \in N$ is a sequence formed by $a_1, a_2, a_4, a_7, a_{11}, \ldots.$ where $b_1 = a_1 = 1$.

Consider the sum, $S_n = \sum_{k=1}^{n} b_k$ satisfies $\frac{2b_n}{b_n S_n - S_n^2} = 1$ for $n \ge 2$. Also in the above table starting from the third row, each row (from left to right) is a G.P. and their common ratio are all equal to the same positive numbers.

4. b_{12} is equal to

(A) $-\frac{1}{78}$ (B) $-\frac{1}{66}$ (C) $-\frac{1}{21}$ (D) $-\frac{1}{105}$

5. $\frac{1}{S_1}+\frac{1}{S_2}+\frac{1}{S_3}+.....+\frac{1}{S_n}$ is equal to

(A) $\frac{n(7n-3)}{4}$ (B) $\frac{n(n+1)}{2}$ (C) $\frac{n}{4}(5n-1)$ (D) $\frac{n}{5}(6n-1)$

6. The sum of all numbers in 10^{th} row, if $a_{81} = -\frac{4}{91}$ is equal to

(A) $-\frac{1358}{47}$ (B) $-\frac{729}{47}$ (C) $-\frac{1021}{55}$ (D) $-\frac{1023}{55}$

Comprehension (Q.7 to Q.9) :

Given that a_1, a_2, a_3 is an A.P. in that order satisfying $a_1 + a_2 + a_3 = 15$; b_1, b_2, b_3 is a G.P. in that order and $b_1b_2b_3 = 27$. If $a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$ are positive integers and form a G.P. in that order.

7. The greatest possible value of common difference of A.P. a_1, a_1, a_3, is equal to

(A) $\frac{55+7\sqrt{61}}{2}$ (B) $\frac{63+7\sqrt{61}}{2}$ (C) $\frac{65+7\sqrt{61}}{2}$ (D) $\frac{61+7\sqrt{61}}{2}$

8. The greatest possible value of common ratio of G.P. b_1, b_2, b_3, is equal to

(A) $\frac{57+7\sqrt{61}}{6}$ (B) $\frac{53+7\sqrt{61}}{6}$ (C) $\frac{61+7\sqrt{61}}{6}$ (D) $\frac{55+7\sqrt{61}}{6}$

9. The greatest value of a_3 is equal to

(A) $\frac{73+7\sqrt{61}}{2}$ (B) $\frac{55+7\sqrt{61}}{2}$ (C) $\frac{71+7\sqrt{61}}{2}$ (D) $\frac{75+7\sqrt{61}}{2}$

COMPREHENSION (Q.10 TO Q.12) :

Let $\{a_n\}$, $a_n \geq 0$, $n \in N$, $a_1 = 1$ be a sequence of real numbers and $S_n = a_1 + a_2 + a_3 + ... + a_n$. Also it is given that $S_n = \frac{1}{2}\left(a_n + \frac{1}{a_n}\right)$.

10. $S_{100} + S_{400} + S_{900}$ is equal to

(A) 10 (B) 20 (C) 30 (D) 60

11. $\lim_{n\to\infty}(\sqrt{n}\, a_n)$ is equal to

(A) 1 (B) 1/2 (C) $\frac{1}{\sqrt{2}}$ (D) 0

12. $\left[\sum_{k=1}^{100} \frac{1}{S_k}\right]$ is equal to ([.] = denotes greatest integer function)

(A) 18 (B) 19 (C) 20 (D) 17

COMPREHENSION (Q.13 TO Q.15) :

If $\phi(r) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{r}$ and $\sum_{r=1}^{n}(2r+1)\phi(r) = P(n)\,\phi(n+1) - Q(n)$, where P(n) and Q(n) are polynomial functions of 'n', then

13. $\sum_{r=0}^{10} P(r)$ is equal to

(A) 235 (B) 506 (C) 285 (D) 385

14. $\sum_{r=0}^{\infty} \frac{1}{Q(r)}$ is equal to

(A) 1 (B) 2 (C) 4 (D) 8

15. $P(13) - Q(13)$ is equal to

(A) 81 (B) 78 (C) 91 (D) 65

Comprehension (Q.16 to Q.17) :

An arithmetic progression has the following property : for any even number of terms, the ratio of the sum of first half of the terms to the sum of second half is always equal to a constant k. Let the first term of A.P is 1.

16. The sum of all possible values of k is

(A) 1
(B) $\frac{4}{3}$
(C) $\frac{5}{3}$
(D) 3

17. Let the number of terms of A.P. is 20, then the sum of sum of all terms of all possible A.P.'s is

(A) 400
(B) 420
(C) 440
(D) 480

Comprehension (Q.18 to Q.19) :

Let $\{a_n\}$ be a sequence of integers such that $a_1 = 1$, $a_2 = 1$ and $a_{n+1} = a_n + a_{n-1}$ for all $n \in N, n > 1$.

18. $\sum_{n=2}^{\infty} \frac{a_n}{a_{n-1}a_{n+1}}$ is equal to

(A) 1
(B) 2
(C) 4
(D) $\frac{1}{2}$

19. $\sum_{n=2}^{\infty} \frac{1}{a_{n-1}a_{n+1}}$ is equal to

(A) 1
(B) 2
(C) 4
(D) $\frac{1}{2}$

COMPREHENSION (Q.20 TO Q.22) :

Two A.P. s have the same number of terms. The ratio of the last term of the first A.P. to the first term of second A.P. is equal to the ratio of the last term of the second A.P to the first term of the first A.P. and is equal to 4. The ratio of the sum of the first 'n' terms of first A.P. to the sum of first 'n' terms of the second A.P. is equal to 2.

20. The ratio of first term of second A.P. to the first term of first A.P. is equal to

(A) 4 (B) $\frac{2}{7}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

21. The ratio of the common difference of first A.P. to the common difference of second A.P. is equal to

(A) 13 (B) 26 (C) 12 (D) 8

22. If the first term of first A.P. is equal to 1, then the sum of its first 14 terms is equal to

(A) 84 (B) 98 (C) 105 (D) 120

COMPREHENSION (Q.23 TO Q.25)

In the following arrangement of n^2 positive numbers,

$$\begin{matrix} a_{11}, a_{12}, \ldots\ldots\ldots a_{1n} \\ a_{21}, a_{22}, \ldots\ldots\ldots a_{2n} \\ \vdots \\ a_{n1}, a_{n2}, \ldots\ldots\ldots a_{nn} \end{matrix}$$

each horizontal row is an arithmetic progression and each vertical column is a geometric progression. It is known that geometric progressions have the same positive common ratio. Given that $a_{24} = 1$, $a_{42} = \frac{1}{8}$ and $a_{43} = \frac{3}{16}$.

23. Let d_i (i = 1, 2, 3, ...,n) be the common difference of i^{th} row, then $\lim_{n\to\infty} \sum_{i=1}^{n} d_i =$

(A) 1 (B) 2 (C) 4 (D) 6

24. The value of $\lim_{n\to\infty} (1.a_{11} + 2a_{22} + 3a_{33} + ... + na_{nn}) =$

(A) 2 (B) 4 (C) 6 (D) 8

25. The value of the sum of all n^2 elements is equal to

(A) $\frac{n(n-1)}{2}\left(1-\frac{1}{2^n}\right)$ (B) $\frac{n(n+1)}{4}\left(1-\frac{1}{2^{n-1}}\right)$

(C) $\frac{n(n+1)}{2}\left(1-\frac{1}{2^n}\right)$ (D) $\frac{n(n+1)}{2}\left(1-\frac{1}{2^{n+1}}\right)$

COMPREHENSION (Q.26 TO Q.28)

Let a_m (m = 1,2,3 .., p) be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$ meets at some point for all real values of b. Let $t_r = \prod_{m=1}^{p}(r - a_m)$ and $S_n = \sum_{r=1}^{n} t_r$, $n \in N$.

26. The minimum possible value of a is

(A) $\frac{1}{5}$ (B) $\frac{5}{26}$ (C) $\frac{3}{38}$ (D) $\frac{2}{43}$

27. The sum of values of n for which S_n vanishes is

(A) 8 (B) 9 (C) 10 (D) 11

28. The value of $\sum_{r=5}^{\infty} \frac{1}{t_r}$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{15}$ (D) $\frac{1}{18}$

COMPREHENSION (Q.29 TO Q.30)

Let x, $x^{\log_{10}x}$, $y^{\log_{10}y}$ and $(xy)^{\log_{10}(xy)}$ are four consecutive terms of a geometric progression, x, y > 0, then

29. The number of ordered pair (x, y) is equal to

(A) 1 (B) 2 (C) 3 (D) 4

30. If (x_i, y_i), $1 \le i \le n$ be the ordered pair satisfying the given condition, $x_i, y_i \notin N$, then $\prod_{i=1}^{n} \frac{x_i}{y_i}$ is equal to

(A) $\sqrt{10}$ (B) $100\sqrt{10}$ (C) 10 (D) $\sqrt{1000}$

SECTION-4

MATCH THE COLUMN

1.

	Column-I		Column-II
(A)	If the roots of equation $x^3 - 9x^2 + 26x - k = 0$ are positive and in A.P., then k is equal to	(P)	7
(B)	If the roots of equation $x^3 - 14x^2 + kx - 64 = 0$ are positive and in G.P., then k is equal to	(Q)	11
(C)	If the roots of equation $6x^3 - kx^2 + 6x - 1 = 0$ are positive and in H.P., then k is equal to	(R)	24
(D)	In the equation $x^3 - kx + 6 = 0$, the sum of two roots is 3, then the value of k is equal to	(S)	26
		(T)	56

:

2.

	Column-I		Column-II
(A)	If $A = \sum_{r=1}^{n} r^2, B = \sum_{m=1}^{n}\sum_{r=1}^{m} r - \frac{1}{2}\sum_{r=1}^{n} r$, then $\frac{A}{B}$ is equal to	(P)	1
(B)	For positive numbers a, b, c the minimum value of $\frac{a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)}{abc}$ is equal to	(Q)	2
(C)	If $x + y + z = 1$, x, y, z > 0, then the minimum value of $\frac{2x^2}{y+z}+\frac{2y^2}{z+x}+\frac{2z^2}{x+y}$ is equal to	(R)	3
(D)	If $\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx} = 3$ where x, y, z ∈ N, then x + y + z is equal to	(S)	4
		(T)	6

3.

	Column-I		Column-II
(A)	If $\log_2 x + 4\log_4 y = 4 - 6\log_8 z$, then x + y + z can be equal to	(P)	2
(B)	Let $x^2 - 3x + p = 0$ has two positive roots 'a' and 'b', then $\frac{4}{a}+\frac{1}{b}$ can be equal to	(Q)	3
(C)	For triangle ABC, if cosecA, cosecB, cosecC are in H.P., then possible value(s) of $\frac{2b}{c}$ will be (where a, b, c denote lengths of sides of ΔABC as in usual notation)	(R)	5
(D)	If $3^{\lvert\sin x\rvert} 2^{-\lvert\sec y\rvert} = a - 5\cos z$, where x, y, z ∈ R and $y \neq (2n+1)\frac{\pi}{2}$, n ∈ I, then possible value(s) of a can be	(S)	6
		(T)	7

4.

	Column-I		**Column-II**
(A)	Let a, b, c are positive real numbers such that $a^3b^2c = 12$, then the minimum value of $49a + 3b + c$ is equal to	(P)	1
(B)	The minimum value of $\left\|2x^3 - \frac{3}{x^2}\right\|$ for $x < 0$ is equal to	(Q)	5
(C)	The maximum value of $\frac{x^5(8-x^3)}{\sqrt[3]{25}}$ for $0 < x < 2$ is equal to	(R)	7
(D)	If $x^7y^5 = a$ and $7x + 5y \geq 12\ \forall\ x, y > 0$, then the minimum value of 'a' is equal to	(S)	15
		(T)	42

5. Consider a sequence $\{b_n\}$ of integers such that b_1, b_2, b_3 are in G.P. b_2, b_3, b_4 are in A.P., b_3, b_4, b_5 are in G.P., b_4, b_5, b_6 are in A.P., b_5, b_6, b_7 are in G.P. and so on. Also given that $b_1 = 1$ and $b_5 + b_6 = 198$. Then

	Column-I		**Column-II**
(A)	$\sqrt{b_7}$ is equal to	(P)	5
(B)	Sum of digits of b_8 is equal to	(Q)	15
(C)	$\sqrt{b_9}$ is equal to	(R)	9
(D)	Sum of digits of b_{10} is equal to	(S)	17
		(T)	13

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. If $\sum_{r=1}^{n} a_r = \sum_{k=1}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j} 2$ and $\lambda = \lim_{n\to\infty}\left(\sum_{r=1}^{n}\frac{1}{a_r}\right)^n$ then $\left[\frac{1}{\lambda}\right]$ is equal to (where [.] denotes greatest integer function).

2. If $a = 1^2 + \frac{2^2}{3} + \frac{3^2}{5} + \frac{4^2}{7} + ... + \frac{(1001)^2}{2001}, b = \frac{1^2}{3} + \frac{2^2}{5} + \frac{3^2}{7} + ... + \frac{(1001)^2}{2003}$ then $[a - b]$ is equal to ([.] denotes greatest integer function)

3. The sum of the first 2018 terms of the sequence 1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, ... is equal to

4. If a, b, c, d, e ∈ R satisfy $a + b + c + d + e = 8$ and $a^2 + b^2 + c^2 + d^2 + e^2 = 16$, then the complete range of e is [a, b]. Then 5(a + b) is equal to

5. Let $a_i + b_i = 1\ \forall\ i = 1, 2, ..., 6$ and $a = \frac{1}{6}(a_1 + a_2 + ... + a_6)$, $b = \frac{1}{6}(b_1 + b_2 + ... + b_6)$. Then
$a_1b_1 + a_2b_2 + ... + a_6b_6 = nab - (a_1 - a)^2 - (a_2 - a)^2 - ... - (a_6 - a)^2$ where n is equal to

6. It is given that for the sequence $\{a_n\}$, its sum of first 'n' terms $S_n = n^2 + 3n + 4$, $n \in N$. Find the value of $a_1 + a_3 + a_5 + a_7 + ... + a_{21}$.

7. The common difference 'd' of an A.P. $\{a_n\}$ is not zero, the common ratio of a G.P. $\{b_n\}$ is a positive rational less than 1. If $a_1 = d$, $b_1 = d^2$ and $\frac{a_1^2 + a_2^2 + a_3^2}{b_1 + b_2 + b_3} = m$, where m is a positive integer, then m is equal to :

8. The value of expression $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+...+[\sqrt{100^2}]$, (where [.] denotes greatest integer function) is equal to :

9. If the value of $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{3^i3^j3^k}$ $(i \neq j \neq k)$ is equal to $\frac{m}{n}$, where m, n are coprime natural numbers, then m + n is equal to

10. If the sum $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2n}{3^m(n3^m+m3^n)}=\frac{p}{q}$ where p, q are relatively prime natural numbers, then m + n is equal to :

11. Suppose x is a positive real number such that {x}, [x] and x are in a geometric progression. Find the least positive integer n such that $x^n > 100$. (Here [x] denotes the integral part of x and {x} = x – [x]).

12. Integers 1, 2, 3, ..., n, where n > 2 are written on a board. Two numbers m, k such that 1 < m < n, 1 < k < n are removed and the average of the remaining numbers is found to be 17. What is the maximum sum of the two removed numbers?

13. Five distinct 2-digit numbers are in a geometric progression. Find the middle term.

14. Let the equation $x^4 - 16x^3 + px^2 - 256x + q = 0$ has four positive real roots in G.P., then p + q is equal to

15. Let $x_1, x_2, x_3, ..., x_{2018}$ be real numbers different from 1, such that $x_1 + x_2 + ... + x_{2018} = 1$ and $\frac{x_1}{1-x_1}+\frac{x_2}{1-x_2}+.....+\frac{x_{2018}}{1-x_{2018}}=1$. Then the value of $\frac{x_1^2}{1-x_1}+\frac{x_2^2}{1-x_2}+....+\frac{x_{2018}^2}{1-x_{2018}}$ is equal to

16. The first term of a sequence is 2014. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2018th term of the sequence.

17. Let $a_1, a_2, a_3, ..., a_{2018}$ be an arithmetic progression of positive real numbers with common difference d. Let $a_1^2 + a_3^2 + ... + a_{2017}^2 = x$, $a_2^2 + a_4^2 + ... + a_{2018}^2 = y$ and $a_{1009} + a_{1010} = z$, then $\frac{y-x}{dz}$ is equal to

18. Let $x_1, x_2, ..., x_{2018}$ be positive real numbers such that $x_1 + x_2 + ... + x_{2018} = 1$. Determine the smallest constant k such that $k\sum_{i=1}^{2018}\frac{x_i^2}{1-x_i} \geq 1$

19. Let $S = \frac{1}{2\left[\sqrt{1}\right]+1} + \frac{1}{2\left[\sqrt{2}\right]+1} + \frac{1}{2\left[\sqrt{3}\right]+1} + ... + \frac{1}{2\left[\sqrt{1000}\right]+1}$ (where [.] denotes greatest integer function), then [S] is equal to

20. Let x, y, z are positive real numbers satisfy $2x - 2y + \frac{1}{z} = \frac{1}{2018}$, $2y - 2z + \frac{1}{x} = \frac{1}{2018}$, $2z - 2x + \frac{1}{y} = \frac{1}{2018}$ then $x + y - z$ is equal to

21. Let a, b, c be positive real numbers such that $\frac{a}{1+b} + \frac{b}{1+c} + \frac{c}{1+a} = 1$. Then the maximum value of abc is $\frac{p}{q}$ where p, q are relatively prime natural numbers, where $p + q$ is equal to

22. Let a, b, c be positive real numbers such that $a + b + c \geq 4$, then find the minimum value of $\frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-c)(b-a)} + \frac{c^3}{(c-a)(c-b)}$.

23. Let k be a positive real number such that $\frac{1}{k+a}+\frac{1}{k+b}+\frac{1}{k+c}<1$ for any positive real numbers a, b and c with abc = 1. Then the least integral value of k is equal to

24. Let $x_1, x_2, x_3, \ldots, x_{2018}$ be positive real numbers satisfying the condition $\frac{1}{1+x_1}+\frac{1}{1+x_2}+\ldots+\frac{1}{1+x_{2018}}=1$. Then the minimum value of $\prod_{r=1}^{2018} x_r$ is equals to $(k)^{k+1}$, where k is equal to

25. The value of $\left[\sum_{n=1}^{\infty}\frac{1}{(n+1)\sqrt{n}}\right]$ (where [.] denotes greatest integer function) is equal to

26. The product $\prod_{n=0}^{\infty}\left(1+\frac{1}{2^{2^n}}\right)$ is equal to

27. The sum $\sum_{k=1}^{\infty}\frac{6^k}{(3^k-2^k)(3^{k+1}-2^{k+1})}$ is equal to

28. The sequence $\{x_n\}$ is defined by $x_1=\frac{1}{2}$ and $x_{k+1}=x_k^2+x_k\ \forall\ k\in N$. Then $\left[\frac{1}{x_1+1}+\frac{1}{x_2+1}+\frac{1}{x_3+1}+\ldots+\frac{1}{x_{100}+1}\right]$, where [.] denotes greatest integer function, is equal to

29. Let n > 4 be a natural number and let P be a polygon with 'n' sides. Let $a_1, a_2, a_3, \ldots, a_n$ be the lengths of the sides of P and let 'p' be its perimeter. Then find the value of $\left[\sum_{i=1}^{n}\frac{a_i}{p-a_i}\right]$, where [.] denotes greatest integer function.

30. If S_1, S_2, S_3 denote the sums of first twenty terms of three non constant sequence in A.P., whose first terms are unity and common differences are in H.P. Then $\frac{2S_3S_1 - S_1S_2 - S_2S_3}{S_1 - 2S_2 + S_3}$ is equal to

31. Let 'p' is the coefficient of x^2 in the expansion $(1 + x)(1 - 3x)(1 + 5x)(1 - 7x)....(1 - 23x)(1 + 25x)$, then the sum of the digits of $|p|$ is equal to

32. Find the number of triplets (a, b, c) such that a, b, c are three distinct positive numbers and a, b, c, b + c – a, c + a – b, a + b – c and a + b + c form a seven term arithmetic progression in some order.

33. Given that $a_1, a_2, a_3, ...a_{15}, a_{16}$ are positive numbers constituting a geometric progression. If $a_1 + a_2 + a_3 + a_4 = 20$, $a_5 + a_6 + a_7 + a_8 = 320$, then $a_{13} + a_{14} + a_{15} + a_{16} = N$, find number of divisors of N.

34. The sum upto infinite terms of the series $1^2 + \frac{3^2}{2} + \frac{5^2}{2^2} + \frac{7^2}{2^3} + ...\infty$ is equal to

35. Let $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^3 + 6k^2 + 11k + 5}{(k+3)!} = \frac{p}{q}$, where p and q are coprime natural numbers, then p + q is equal to

36. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the G.M. between two distinct positive numbers a and b, then n is equal to

Answer Key

Single choice Questions

1. A	**2.** B	**3.** D	**4.** D	**5.** A	**6.** B
7. B	**8.** D	**9.** C	**10.** A	**11.** D	**12.** C
13. A	**14.** B	**15.** A	**16.** B	**17.** C	**18.** D
19. B	**20.** B	**21.** A	**22.** C	**23.** C	**24.** B
25. C	**26.** B	**27.** A	**28.** B	**29.** D	**30.** C
31. D	**32.** A	**33.** D	**34.** C	**35.** B	**36.** A
37. A	**38.** C	**39.** D	**40.** B		

One or More Than One Correct Questions

1. B,C,D	**2.** A,B,D	**3.** C,D	**4.** A,B	**5.** A,B,D	**6.** A,B,C
7. B,C	**8.** A,D	**9.** A,B,C,D	**10.** A,B	**11.** C,D	**12.** A,C
13. A,B,C	**14.** B,C	**15.** B,C	**16.** B,D	**17.** A,D	**18.** A,B,C,D
19. B,C	**20.** A,C,D	**21.** A,B,C	**22.** B,C,D	**23.** B,C	**24.** B,C
25. A,B	**26.** B,D	**27.** A,C,D	**28.** A,B,C,D	**29.** A,C,D	**30.** A,B
31. A,B,C,D	**32.** B,C,D	**33.** A,C	**34.** C,D	**35.** A,B,C	**36.** B,D
37. A,B	**38.** A,D	**39.** B,C	**40.** C,D		

Comprehension Based Questions

1. B	**2.** A	**3.** D	**4.** A	**5.** C	**6.** D
7. B	**8.** D	**9.** A	**10.** D	**11.** B	**12.** A
13. D	**14.** B	**15.** C	**16.** B	**17.** B	**18.** B
19. A	**20.** D	**21.** B	**22.** C	**23.** A	**24.** C
25. C	**26.** B	**27.** C	**28.** D	**29.** C	**30.** D

Match The Column

1. $A \to R$; $B \to T$; $C \to Q$; $D \to P$
2. $A \to Q$; $B \to T$; $C \to P$; $D \to R$
3. $A \to R, S, T$; $B \to Q, R, S, T$; $C \to P, Q$; $D \to P, Q, R, S$
4. $A \to T$; $B \to Q$; $C \to S$; $D \to P$
5. $A \to T$; $B \to P$; $C \to S$; $D \to Q$

Subjective Type Questions

1. 2	**2.** 500	**3.** 3974	**4.** 16	**5.** 6	**6.** 268
7. 8	**8.** 661750	**9.** 289	**10.** 41	**11.** 10	**12.** 69
13. 36	**14.** 352	**15.** 0	**16.** 370	**17.** 1009	**18.** 2017
19. 30	**20.** 2018	**21.** 9	**22.** 4	**23.** 3	**24.** 2017
25. 1	**26.** 2	**27.** 2	**28.** 1	**29.** 1	**30.** 20
31. 19	**32.** 0	**33.** 30	**34.** 34	**35.** 8	**36.** 0.5

Previous Year Questions

SECTION-1

1. If 1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P. then x equals **[AIEEE 2002]**

(A) $\log_3 4$
(B) $1 - \log_3 4$
(C) $1 - \log_4 3$
(D) $\log_4 3$

2. The value of $2^{1/4}.4^{1/8}.8^{1/16} \ldots \infty$ is **[AIEEE 2002]**

(A) 1
(B) 2
(C) 3/2
(D) 4

3. Fifth term of a GP is 2, then the product of its 9 terms is **[AIEEE 2002]**

(A) 256
(B) 512
(C) 1024
(D) None of these

4. Sum of infinite number of terms of GP is 20 and sum of their squares is 100. The common ratio of GP is **[AIEEE 2002]**

(A) 5
(B) 3/5
(C) 8/5
(D) 1/5

5. $1^3 - 2^3 + 3^3 - 4^3 + \ldots\ldots. + 9^3 =$ **[AIEEE 2002]**

(A) 425
(B) –425
(C) 475
(D) –475

6. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, m ≠ n, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals- **[AIEEE 2004]**

(A) 0 (B) 1

(C) 1/mn (D) $\frac{1}{m} + \frac{1}{n}$

7. The sum of the first n terms of the series $1^2 + 2.\ 2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n\,(n+1)^2}{2}$ when n is even. When n is odd the sum is- **[AIEEE 2004]**

(A) $\frac{3n\,(n+1)}{2}$ (B) $\frac{n^2\,(n+1)}{2}$

(C) $\frac{n\,(n+1)^2}{4}$ (D) $\left[\frac{n\,(n+1)}{2}\right]^2$

8. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and | a | < 1, | b | < 1, | c | < 1 then x, y, z are in - **[AIEEE 2005]**

(A) GP (B) AP

(C) Arithmetic - Geometric Progression (D) HP

9. Let a_1, a_2, a_3, ... be terms of an A.P. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, p ≠ q then $\frac{a_6}{a_{21}}$ equals – **[AIEEE 2006]**

(A) $\frac{7}{2}$ (B) $\frac{2}{7}$

(C) $\frac{11}{41}$ (D) $\frac{41}{11}$

10. If a_1, a_2, ... a_n are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to **[AIEEE 2006]**

(A) $(n-1)(a_1 - a_n)$ (B) na_1a_n

(C) $(n-1)\,a_1a_n$ (D) $n\,(a_1 - a_n)$

11. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- **[AIEEE 2007]**

(A) $\frac{1}{2}(1-\sqrt{5})$ (B) $\sqrt{5}$

(C) $\frac{1}{2}\sqrt{5}$ (D) $\frac{1}{2}(\sqrt{5}-1)$

12. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[AIEEE 2008]**

(A) –4 (B) –12

(C) 12 (D) 4

13. The sum to infinity of the series

$$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+.......$$

[AIEEE 2009]

(A) 2 (B) 3

(C) 4 (D) 6

14. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11},...$ are in an AP with common difference –2, then the time taken by him to count all notes is

[AIEEE 2010]

(A) 34 minutes (B) 125 minutes

(C) 135 minutes (D) 24 minutes

15. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[AIEEE 2011]**

(A) 19 months (B) 20 months

(C) 21 months (D) 18 months

16. **Statement-1 :** The sum of the series
$1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400)$ is 8000.

Statement-2 : $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n. **[AIEEE 2012]**

(A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is true, Statement-2 is true, Statement-2 is a **NOT** the correct explanation for Statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

17. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is :

[JEE (Mains)2013]

(A) $\frac{7}{9}(99 + 10^{-20})$ (B) $\frac{7}{81}(179 - 10^{-20})$

(C) $\frac{7}{9}(99 - 10^{-20})$ (D) $\frac{7}{81}(179 + 10^{-20})$

18. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is **[IIT Main 2014]**

(A) $\frac{2\sqrt{13}}{9}$ (B) $\frac{\sqrt{61}}{9}$

(C) $\frac{2\sqrt{17}}{9}$ (D) $\frac{\sqrt{34}}{9}$

19. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is :

[IIT Main 2014]

(A) $2 + \sqrt{3}$ (B) $\sqrt{2} + \sqrt{3}$

(C) $3 + \sqrt{2}$ (D) $2 - \sqrt{3}$

20. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to

[IIT Main 2014]

(A) 110
(B) $\frac{121}{10}$
(C) $\frac{441}{100}$
(D) 100

21. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + ...$ is :

[JEE Mains 2015]

(A) 192
(B) 71
(C) 96
(D) 142

22. If m is the A.M. of two distinct real numbers ℓ and n (ℓ, n > 1) and G_1, G_2 and G_3 are three geometric means between ℓ and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

[JEE Mains 2015]

(A) $4\ell^2m^2n^2$
(B) $4\ell^2mn$
(C) $4\ell m^2n$
(D) $4\ell mn^2$

23. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P., is : **[JEE Mains 2016]**

(A) $\frac{8}{5}$
(B) $\frac{4}{3}$
(C) 1
(D) $\frac{7}{4}$

24. If the sum of the first ten terms of the series **[JEE Mains 2016]**

$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...$, is $\frac{16}{5}$ m, then m is equal to :

(A) 102
(B) 101
(C) 100
(D) 99

25. If, for a positive integer, n the quadratic equation,

$x(x+1)+(x+1)(x+2)+ \ldots\ldots\ldots\ldots + (x+\overline{n-1})(x+n)=10n$

has two consecutive integral solutions, then n is equal to : **[JEE Mains 2017]**

(A) 10 (B) 11

(C) 12 (D) 9

26. For any three positive real numbers a, b and c

$9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$. Then **[JEE Mains 2017]**

(A) a, b and c are in A.P. (B) a, b and c are in G.P.

(C) b, c and a are in G.P. (D) b, c and a are in A.P.

27. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots\ldots\ldots$

If $B-2A=100\lambda$, then λ is equal to **[JEE Mains 2018]**

(A) 496 (B) 232

(C) 248 (D) 464

28. Let $a_1, a_2, a_3, \ldots\ldots\ldots a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1}=416$ and $a_9+a_{43}=66$.

If $a_1^2+a_2^2+\ldots.+a_{17}^2=140m$, then m is equal to **[JEE Mains 2018]**

(A) 33 (B) 66

(C) 68 (D) 34

SECTION-2

1. (a) The harmonic mean of the roots of the equation

$\left(5+\sqrt{2}\right) x^2 - \left(4+\sqrt{5}\right) x + 8 + 2\sqrt{5} = 0$ is

(A) 2 (B) 4

(C) 6 (D) 8

(b) Let $a_1, a_2, ..., a_{10}$, be in A.P. & $h_1, h_2, ..., h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$ then $a_4 h_7$ is: **[JEE '99, 2 + 2 out of 200]**

(A) 2 (B) 3

(C) 5 (D) 6

2. The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. If the inverse of its common ratio is an integer, find all possible values of the common ratio, n and the first terms of the series. **[REE '99, 6]**

3. (a) Consider an infinite geometric series with first term 'a' and common ratio r . If the sum is 4 and the second term is 3/4, then :

(A) $a = \frac{7}{4}, r = \frac{3}{7}$ (B) $a = 2, r = \frac{3}{8}$

(C) $a = \frac{3}{2}, r = \frac{1}{2}$ (D) $a = 3, r = \frac{1}{4}$

(b) If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation :

[JEE 2000, Screening, 1 + 1 out of 35]

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$

(C) $2 \le M \le 3$ (D) $3 \le M \le 4$

(c) The fourth power of the common difference of an arithmetic progression with integer entries added to the product of any four consecutive terms of it . Prove that the resulting sum is the square of an integer.

[JEE 2000, Mains, 4 out of 100]

4. Given that α, γ are roots of the equation, $Ax^2 - 4x + 1 = 0$ and β, δ the roots of the equation, $Bx^2 - 6x + 1 = 0$, find values of A and B, such that α, β, γ & δ are in H.P. **[REE 2000, 5 out of 100]**

5. The sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals. Find whether bc^2, ca^2 and ab^2 in A.P., G.P. or H.P.?

[REE 2001, 3 out of 100]

6. Solve the following equations for x and y

$$\log_2 x + \log_4 x + \log_{16} x + \ldots\ldots\ldots\ldots = y$$

$$\frac{5+9+13+\ldots\ldots\ldots+(4y+1)}{1+3+5+\ldots\ldots\ldots+(2y-1)} = 4\log_4 x$$

[REE 2001, 5 out of 100]

7. (a) Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are

(A) –2, –32 (B) –2, 3

(C) –6, 3 (D) –6, –32

(b) If the sum of the first 2n terms of the A.P. 2, 5, 8, ... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ..., then n equals

(A) 10 (B) 12

(C) 11 (D) 13

(c) Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are

[JEE 2001, Screening, 1 + 1 + 1 out of 35]

(A) NOT in A.P./G.P./H.P. (B) in A.P.

(C) in G.P. (D) H.P.

(d) Let a_1, a_2 .... be positive real numbers in G.P. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of $a_1, a_2, a_3, \ldots\ldots\ldots a_n$. Find an expression for the G.M. of $G_1, G_2, \ldots\ldots\ldots G_n$ in terms of $A_1, A_2 \ldots\ldots\ldots\ldots A_n, H_1, H_2, \ldots\ldots\ldots H_n$. **[JEE 2001 (Mains); 5]**

8. (a) Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is **[JEE 2002 (Screening), 3]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$

(C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

(b) Let a, b be positive real numbers. If a, A_1, A_2, b are in A.P. ; a, G_1, G_2, b are in G.P. and a, H_1, H_2, b are in H.P. , show that

$$\frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}$$ **[JEE 2002 , Mains , 5 out of 60]**

9. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P. , then prove that either $a = b = c$ or a, b, $-\frac{c}{2}$ form a G.P. **[JEE-03, Mains-4 out of 60]**

10. The first term of an infinite geometric progression is x and its sum is 5. Then **[JEE 2004 (Screening)]**

(A) $0 \le x \le 10$ (B) $0 < x < 10$

(C) $-10 < x < 0$ (D) $x > 10$

11. If a, b, c are positive real numbers, then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$. **[JEE 2004, 4 out of 60]**

12. (a) In the quadratic equation $ax^2 + bx + c = 0$, if $\Delta = b^2 - 4ac$ and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in G.P. where α, β are the roots of $ax^2 + bx + c = 0$, then **[JEE 2005 (Screening)]**

(A) $\Delta \ne 0$ (B) $b\Delta = 0$

(C) $c\Delta = 0$ (D) $\Delta = 0$

(b) If total number of runs scored in *n* matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by $k \cdot 2^{n+1-k}$, where $1 \le k \le n$. Find n. **[JEE 2005 (Mains), 2]**

13. If $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \ldots\ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$, then find the minimum natural number n_0 such that $B_n > A_n$. $\forall\, n > n_0$. **[JEE 2006, 6]**

Comprehension (Q.(a) to Q.(c)) :

14. Let V_r denote the sum of the first 'r' terms of an arithmetic progression (A.P.) whose first term is 'r' and the common difference is (2r – 1).

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ...

(a) The sum $V_1 + V_2 + \ldots\ldots + V_n$ is

(A) $\frac{1}{12}n(n + 1)(3n^2 - n + 1)$
(B) $\frac{1}{12}n(n + 1)(3n^2 + n + 2)$
(C) $\frac{1}{2}n(2n^2 - n + 1)$
(D) $\frac{1}{3}(2n^3 - 2n + 3)$

(b) T_r is always

(A) an odd number
(B) an even number
(C) a prime number
(D) a composite number

(c) Which one of the following is a correct statement?

[JEE 2007, 4 + 4 + 4]

(A) $Q_1, Q_2, Q_3, \ldots\ldots$ are in A.P. with common difference 5.
(B) $Q_1, Q_2, Q_3, \ldots\ldots$ are in A.P. with common difference 6.
(C) $Q_1, Q_2, Q_3, \ldots\ldots$ are in A.P. with common difference 11.
(D) $Q_1 = Q_2 = Q_3 = \ldots\ldots$

COMPREHENSION (Q.(A) TO Q.(C)) :

15. Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

(a) Which one of the following statements is correct?

(A) $G_1 > G_2 > G_3 > \ldots\ldots$

(B) $G_1 < G_2 < G_3 < \ldots\ldots$

(C) $G_1 = G_2 = G_3 = \ldots\ldots$

(D) $G_1 < G_3 < G_5 < \ldots\ldots$ and $G_2 > G_4 > G_6 > \ldots\ldots$

(b) Which one of the following statements is correct?

(A) $A_1 > A_2 > A_3 > \ldots\ldots$

(B) $A_1 < A_2 < A_3 < \ldots\ldots$

(C) $A_1 > A_3 > A_5 > \ldots\ldots$ and $A_2 < A_4 < A_6 < \ldots\ldots$

(D) $A_1 < A_3 < A_5 < \ldots\ldots$ and $A_2 > A_4 > A_6 > \ldots\ldots$

(c) Which one of the following statements is correct? **[JEE 2007, 4 + 4 + 4]**

(A) $H_1 > H_2 > H_3 > \ldots\ldots$

(B) $H_1 < H_2 < H_3 < \ldots\ldots$

(C) $H_1 > H_3 > H_5 > \ldots\ldots$ and $H_2 < H_4 < H_6 < \ldots\ldots$

(D) $H_1 < H_3 < H_5 < \ldots\ldots$ and $H_2 > H_4 > H_6 > \ldots\ldots$

16. (a) A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then **[JEE 2008, 4]**

(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$ (B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$ (D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

ASSERTION & REASON

(b) Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$. **[JEE 2008, 3 (–1)]**

STATEMENT-1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

STATEMENT-2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) Statement-1 is True, Statement-2 is True; statement-2 is a correct explana tion for statement-1

(B) Statement-1 is True, Statement-2 is True; statement-2 is **NOT** a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

17. Let S_k, k = 1, 2 ..... , 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then he value of $\frac{100^2}{100!}$ + $\sum_{k=1}^{100}|(k^2-3k+1)s_k|$ is **[JEE 2010]**

18. Let $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4....., 11. If $\frac{a_1^2+a_2^2+....+a_{11}^2}{11} = 90$, then the value of $\frac{a_1+a_2+....+a_{11}}{11}$ is equal to **[JEE 2010]**

19. Let $a_1, a_2, a_3, ..., a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[JEE 2011]**

20. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is **[JEE 2011]**

21. Let $a_1, a_2, a_3, ...$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is : **[JEE 2012]**

(A) 22 (B) 23

(C) 24 (D) 25

22. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s) **[JEE Advance 2013]**

(A) 1056 (B) 1088

(C) 1120 (D) 1332

23. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a,b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is **[JEE Advance 2014]**

24. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is **[JEE Advance 2015]**

25. Let $b_i > 1$ for i = 1, 2, ... , 101. Suppose $\log_e b_1, \log_e b_2, ..., \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, ..., a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$.

If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a_{51}$, then **[JEE Advance 2016]**

(A) $s > t$ and $a_{101} > b_{101}$ (B) $s > t$ and $a_{101} < b_{101}$

(C) $s < t$ and $a_{101} > b_{101}$ (D) $s < t$ and $a_{101} < b_{101}$

26. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side? **[JEE Advance 2017]**

Answer Key

SECTION-1

1. B	**2.** B	**3.** B	**4.** B	**5.** A	**6.** A
7. B	**8.** D	**9.** C	**10.** C	**11.** D	**12.** B
13. B	**14.** A	**15.** C	**16.** A	**17.** D	**18.** A
19. A	**20.** D	**21.** C	**22.** C	**23.** B	**24.** B
25. B	**26.** D	**27.** C	**28.** D		

SECTION-2

1. (a) B (b) D

2. $(r, n, a) \in \left\{\left(\frac{1}{3},4,108\right),\left(\frac{-1}{3},4,216\right),\left(\frac{1}{9},2,144\right),\left(\frac{-1}{9},2,180\right),\left(\frac{1}{81},1,160\right)\right\}$

3. (a) D (b) A **4.** A = 3 ; B = 8 **5.** A.P.

6. $x = 2\sqrt{2}$ and $y = 3$

7. (a) A, (b) C, (c) D , (d) $\left[(A_1, A_2, ...A_n)(H_1, H_2, ...H_n)\right]^{\frac{1}{2n}}$

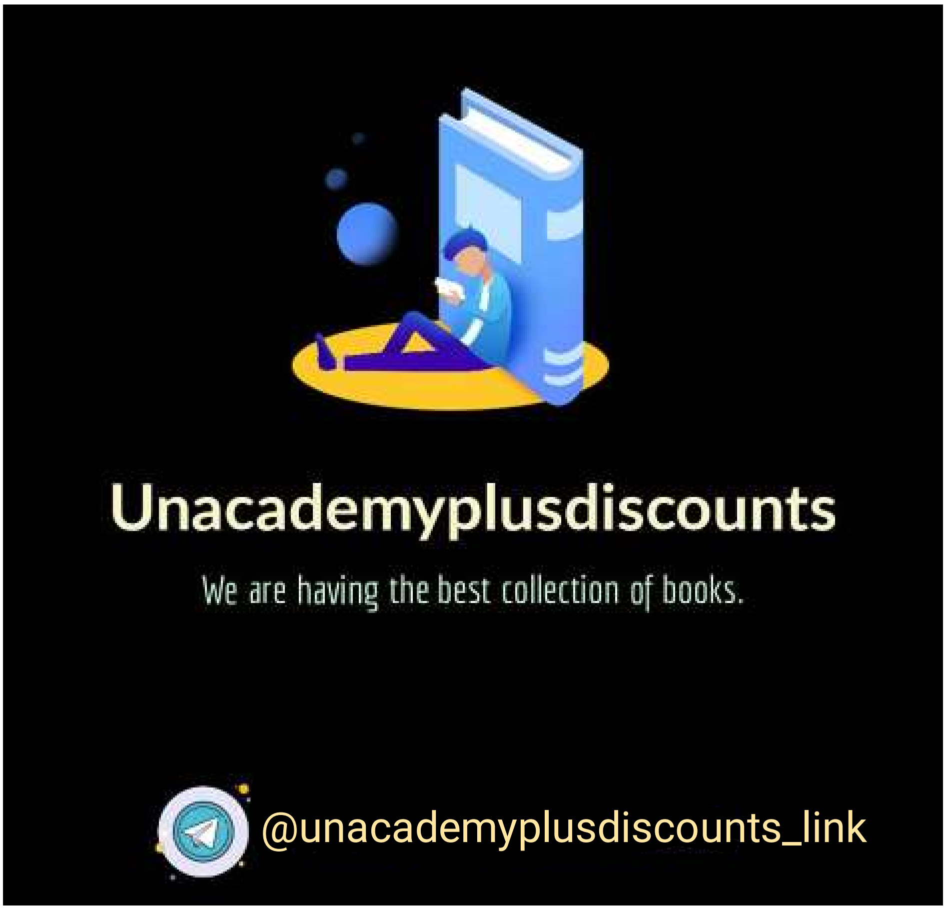
8. (a) D **10.** B **12.** (a) C, (b) n = 7

13. $n_0 = 5$ **14.** (a) B; (b) D; (c) B **15.** (a) C; (b) A; (c) B

16. (a) B, D; (b) C **17.** 3 **18.** 0

19. 9 or 3 **20.** 8 **21.** D **22.** A, D

23. 4 **24.** 9 **25.** B **26.** 6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3 CHAPTER

PERMUTATION AND COMBINATION

SECTION-1

SINGLE CHOICE QUESTIONS

1. From a group of 10 professors $A_1, A_2, \ldots A_{10}$ how many ways can a committee of 5 members be formed so that atleast one of professor A_1 and professor A_2 will be included.

(A) 196 (B) 252

(C) 140 (D) 212

2. Let S = {1, 2, 3,....., n}. Find number of unordered pairs (A, B) of subsets of S such that A and B are disjoint, where A or B both may be empty.

(A) 3^n (B) $\frac{3^n - 1}{2}$

(C) $\frac{3^n + 1}{2}$ (D) $2^n + 1$

3. In how many ways can we choose a black square and a white square from a chessboard so that they are neither in the same row nor the same column.

(A) 658 (B) 768

(C) 1024 (D) 972

4. Consider a rational number $\frac{a}{b}$ in its lowest form a, b are integers, with $0 < \frac{a}{b} < 1$, $b > 1$. How many of these have ab = 15!

(A) 64 (B) 32

(C) 256 (D) 16

5. How many permutations of the digits of the number 123456789 are there in which none of the blocks 12, 34 or 567 appear.

(A) $9! - 661 \times 5!$ (B) $9! - 110 \times 6!$

(C) $9! - 41 \times 7!$ (D) $9! - 55 \times 7!$

6. Five boys and 6 girls must sit around a round table. How many arrangements are possible if Ram (a boy) must always be adjacent to seeta and geeta (two girls).

(A) $10! - 2 \times 6!$ (B) $6 \times 8!$

(C) $9! \times 6$ (D) $2 \times 8!$

7. In how many ways may one seat 100 people into 20 distinct round tables in such a way that there are 5 people per table.

(A) $\frac{100! \times (4!)^5}{(20!)^5}$ (B) $\frac{100!}{(20!)^5}$

(C) $\frac{100!\,(4!)^5}{(20!)^5\,5!}$ (D) $\frac{100!\,(5!)^4}{(20!)^5}$

8. The number of ordered triplets (a, b, c) from the set {1, 2, 3, 4,.....,100} such that $a \le b \le c$ is equal to

(A) ${}^{102}C_3$ (B) ${}^{100}C_3$

(C) ${}^{101}C_2 + {}^{100}C_3$ (D) ${}^{100}C_1 + 3({}^{100}C_2) + 6({}^{100}C_3)$

9. Given 'n' different objects arranged in a row. Then the number of ways of choosing k of them so that no two of them are consecutive is equal to

(A) ${}^{n}C_k$ (B) ${}^{n-k}C_k$

(C) ${}^{n}C_k-(n - k + 1)$ (D) ${}^{n-k+1}C_k$

10. In how many ways can we distribute k identical balls into n different boxes so that each box contains atmost one ball and no two consecutive boxes are empty.

(A) ${}^{k+1}C_{n-k}$ (B) ${}^{k+(n-1)}C_{n-1}$

(C) ${}^{n}C_{k}$ (D) ${}^{n-k+1}C_{k}$

11. How many different sets of 'k' numbers $\{a_1, a_2, \ldots\ldots, a_k\}$ with integral $a_i \in \{1, 2, 3, \ldots\ldots n\}$ are there satisfying $1 \le a_1 \le a_2 \le a_3 \le \ldots\ldots \le a_k \le n$.

(A) ${}^{k+n-1}C_{n-1}$ (B) ${}^{n+k-1}C_{k-1}$

(C) ${}^{n-k+1}C_{n}$ (D) ${}^{n-k+1}C_{k}$

12. Find number of points (x, y) in xy plane with both x and y integers which satisfy $|x| + |y| < 100$

(A) 19801 (B) 20000

(C) 19404 (D) 19602

13. The number of three element subsets from the set {1,2,3,...., 298, 299, 300}, such that the sum of the elements is a multiple of 3 is equal to

(A) 4851 (B) 1004851

(C) 1014881 (D) 998624

14. How many integers are there between 0 and 10^5 having the digit sum equal to 8.

(A) 560 (B) 495

(C) 35 (D) 640

15. In how many ways 2n + 1 identical oranges can be distributed among 3 persons so that the sum of number of oranges received by any two persons should exceed the number of oranges received by the other.

(A) $n(2n - 1)$ (B) $\frac{n(n-1)}{2}$

(C) $\frac{n(n-5)}{2}$ (D) $\frac{n(n+1)}{2}$

16. The number of triples (A, B, C) where A, B, C are subsets of {1, 2, 3,....., n} such that $A \cap B \cap C = \phi$, $A \cap B \neq \phi$, $B \cap C \neq \phi$ is equal to

(A) $7^n - 2.6^n + 5^n$
(B) $7^n - 6^n + 5^n$
(C) $7^n - 5^n$
(D) $7^n - 3.6^n - 5^n$

17. A postman has to deliver five letters to five different houses. Mischievously, he posts one letter through each door without looking to see if it is the correct address. In how many different ways could he do this so that exactly two of the five houses receive the correct letters.

(A) 20
(B) 40
(C) 60
(D) 15

18. There are 10 bags $B_1, B_2, \ldots, B_{10}$ which contain 31, 32,....,40 distinct articles respectively. The total number of ways to draw 10 articles all from a single bag is

(A) $^{41}C_{30}$
(B) $^{41}C_{10}$
(C) $^{41}C_{30} - {}^{31}C_{20}$
(D) $^{40}C_{29}$

19. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{r}$ be a variable vector such that $\vec{r}.\hat{i}, \vec{r}.\hat{j}$ and $\vec{r}.\hat{k}$ are positive integers. If $\vec{r}.\vec{a} \leq 12$, then the number of values of $\vec{r}$ is

(A) $^{11}C_2$
(B) $^{14}C_2$
(C) $^{15}C_3$
(D) $^{12}C_3$

20. How many line segments have both their endpoints located at the vertices of a given cube.

(A) 36
(B) 28
(C) 45
(D) 21

21. At a couple dance party, each man danced with exactly four women and each woman danced with exactly three men. Nine men attended the party. How many women attended the party.

(A) 12 (B) 11

(C) 10 (D) 14

22. The number of all 6-digit natural numbers having exactly three odd digits and three even digits is equal to

(A) 321150 (B) 271250

(C) 182510 (D) 281250

23. There are eight rooms on the first floor of a hotel, with four rooms on each side of the corridor, symmetrically situated (that is each room is exactly opposite one other room). Four guests have to be accomodated in four of the eight rooms (that is, one in each) such that no two guests are in adjacent rooms or in opposite rooms. In how many ways can the guests be accomodated.

(A) 24 (B) 48

(C) 96 (D) 72

24. There are five cities A, B, C, D, E on a certain island. Each city is connected to every other city by road. In how many ways can a person starting from city A come back to A after visiting some cities without visiting a city more than once and without taking the same. road more than once. (The order in which he visits the cities also matters)

(A) 60 (B) 61

(C) 63 (D) 64

25. Find number of ways to distribute 8 distinct balls among 3 children so that every child receives atleast one ball is

(A) 6308 (B) 6561

(C) 5796 (D) 5793

26. The greatest of d so that 12^d divides in 100! is equal to $(d \in N)$

(A) 47 (B) 48

(C) 49 (D) 97

27. How many different 9 digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy odd positions.

(A) 300 (B) 60

(C) 240 (D) 360

28. Let $A = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ and $B = \{y_1, y_2, y_3\}$. The total number of functions $f : A \to B$ that are onto and there are exactly three elements x in A such that $f(x) = y_1$ is equal to

(A) $12 \times {}^7C_3$ (B) $15 \times {}^7C_3$

(C) $16 \times {}^7C_3$ (D) $14 \times {}^7C_3$

29. The number of interior points where diagonals of a convex polygon of n sides intersect if no three diagonal pass through the same interior point is equal to

(A) $\frac{n(n-1)(n-2)(n-3)}{8}$ (B) $\frac{n(n-1)(n-2)(2n-1)}{24}$

(C) ${}^nC_4$ (D) $\frac{n(n-1)(n-2)(n-3)}{12}$

30. The number of ways to select n objects from 3n objects of which n are identical and rest are different is equal to

(A) $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$ (B) $2^{2n-1} + \frac{(2n)!}{2(n!)^2}$

(C) $2^{2n-1} + \frac{(2n)!}{(n!)^2}$ (D) $2^{2n-1} - \frac{(2n)!}{(n!)^2 2}$

31. The number of common divisors of 10800 and 9000 is equal to

(A) 36 (B) 48

(C) 60 (D) 45

32. A, B are two students in a group of n students. If the number of ways of assigning the n students to a line of n single rooms such that A and B are not in adjacent rooms is 3600, then n is equal to

(A) 9 (B) 8

(C) 7 (D) 6

33. The number of three digit numbers of the form xyz such that $x < y$ and $z \leq y$ is

(A) 221 (B) 240

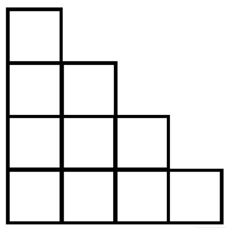
(C) 256 (D) 276

34. The number of all possible four digit numbers having exactly two fives and no two consecutive digits identical is equal to

(A) 216 (B) 225

(C) 228 (D) 235

35. The number of ways in which all the letters of the word ‘SAKSHAM’ can be placed in the squares of the figure shown, no that no row remains empty is equal to



(A) 95760 (B) 94640

(C) 97840 (D) 82120

36. If three dice are rolled and we make a set of numbers shown on the three dice. How many different sets are possible.

(A) 56 (B) 15

(C) 216 (D) 224

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. Let A = {1, 2, 3,.....,n}. The number of functions $f : A \to A$ which is strictly increasing such that $f(i) \le i \ \forall \ i \in A$ is equal to

(A) ${}^{2n-2}C_{n-1} - {}^{2n-2}C_n$
(B) ${}^{2n}C_n - {}^{2n}C_{n+1}$
(C) $\dfrac{{}^{2n-2}C_{n-1}}{n}$
(D) $\dfrac{{}^{2n}C_n}{n+1}$

2. Let S = {1, 2, 3, 4,......, (n + 1)}, where $n \ge 6$ and let $T = \{(x,y,z) \mid x, y, z \in S, x < z, y < z\}$, then number of members of T is equal to

(A) ${}^{n+1}C_2 + {}^{n+1}C_3$
(B) ${}^{n+1}C_2 + 2\ {}^{n+1}C_3$
(C) $\sum_{k=1}^{n} k^2$
(D) $\sum_{k=1}^{n-1} k^2$

3. A decimal code is declared legal if it has an even number of zeros. For example 1900200 is a legal code, but 10002 is not. Let a_n be the number of legal decimal codes of length 'n'. Then

(A) $a_n = 8a_{n-1} + 10^{n-1}$
(B) $a_n = \dfrac{8^n + 10^n}{2}$
(C) $a_n = 9a_{n-1} + 10^n$
(D) $a_n = 9^n + 10^n$

4. If Mr. A can climb either one step or two steps at a time. Let a_n is the number of ways, he climbs a n-step staircase then

(A) $a_8 = 35$
(B) $a_n = a_{n-1} + a_{n-2}$
(C) $a_9 = 55$
(D) $a_{11} = 144$

5. The number of ways to choose a subset of two elements {a, b} from the set {1, 2, 3, 4, ..., 49, 50} such that (The pair {a, b} is indistinguishble from the pair {b, a})

(A) $|a - b| \le 5$ is 245
(B) $|a - b| \le 5$ is 235
(C) $|a - b| = 5$ is 45
(D) $|a - b| \ge 5$ is 1035

6. The sequence of all positive palindromes are written in ascending order 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 22,..........

Then

(A) the 2018th positive palindrome is 1019101

(B) the 1998th positive palindrome is 1001001

(C) the rank of 2140412 is 3139

(D) the rank of 1999999 is 2998

7. Givne '2n' different objects arranged around a circle, the number of ways of choosing k of them so that no two of them are consecutive is equal to $(0 \le k \le n)$

(A) ${}^{2n-k+1}C_k - {}^{2n-k-1}C_{k-2}$

(B) $\frac{2n}{k}\,{}^{2n-k-1}C_{k-1}$

(C) $\frac{2n}{(2n-k)}\,{}^{2n-k}C_k$

(D) $\frac{n(4n-4k+1)}{(2n-2k+1)}\,{}^{2n-k}C_k$

8. The number of ordered triplets (x, y, z) of non negative integers satisfying the conditions.

$$x + y + z \le 100 \quad \text{and} \quad x \le y \le z$$

(A) if x is odd is 14724

(B) if x is odd is 14722

(C) if x is even is 16065

(D) if x is even is 16164

9. Let x be the number of 6 digit numbers, the sum of whose digits is even and y be the number of 6 digit numbers, the sum of whose digits is odd, then

(A) $x + y = 9 \times 10^5$

(B) $x < y$

(C) $x = y$

(D) $y = 450000$

10. There are n lines in a plane, no two of which are parallel and no three of them are concurrent. Let the plane be divided by n lines in a_n parts, then

(A) $a_n = a_{n-1} + (n-1)$

(B) $a_n = a_{n-1} + n$

(C) $a_6 = 22$

(D) $a_{10} = 56$

11. There are n married couples at a party. Each person shakes hand with every person other than her or his spouse. The total number of handshakes must be

(A) ${}^{2n}C_2 - 2n$ (B) ${}^{2n}C_2 - n$
(C) $2n(2n - 1)$ (D) $2n(n - 1)$

12. Let 5 letter words are formed using the letters of the word 'CALCULUS'. Then

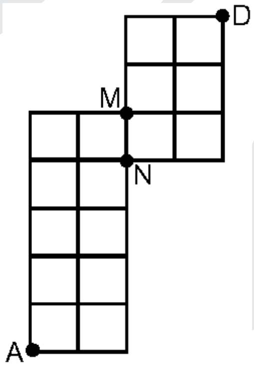
(A) The number of all such possible words is 1110.

(B) The number of words with exactly one alike pair of letters is 720.

(C) The number of words with exactly two alike pair of letters is 270.

(D) The number of words with exactly one alike pair of letters is 480.

13. Let a person has to go from A to D moving along horizontal and vertical grids using shortest path (as shown in figure). Then



(A) The number of paths is equal to 276.
(B) The number of paths is equal to 186.
(C) The number of paths in which he doesn't cross point N is equal to 36.
(D) The number of paths in which he doesn't cross point M is equal to 60.

14. Consider all possible permutations of all the letters of the word 'CONTINUITY', then the number of permutations.

(A) Which have 'COUNT' in all of them is 2(5!).
(B) Which have 'COUNT' in all of them is 3(5!).
(C) Which have all vowels separated is 15(7!).
(D) Which have all vowels separated is 7(7!).

15. Let a_n be the number of non empty subsets of S = {1, 2, 3, 4,....,n–1, n} such that there are no two consecutive numbers in one and the same set, then

(A) $a_n = a_{n-1} + a_{n-2}$
(B) $a_n = a_{n-1} + a_{n-2} + 1$
(C) $a_7 = 33$
(D) $a_7 = 21$

16. 'A' has cans of paint in eight different colours. He wants to paint the four unit squares of a 2 × 2 board in such a way that neighbouring unit squares are painted in different colours.

(A) The number of distinct colouring schemes 'A' can make is equal to 2072.
(B) The number of distinct colouring schemes 'A' can make is equal to 2036.
(C) The number of distinct colouring schemes 'A' can make in which two colouring schemes are considered the same if one can be obtained from the other by rotation is equal to 532.
(D) The number of distinct colouring schemes 'A' can make in which two colouring schemes are considered the same if one can be obtained from the other by rotation is equal to 616.

17. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth number is marked (i.e. 1, 16, 31, ...). This process is continued until a number is reached which has already been marked, then marked numbers are

(A) 11
(B) 17
(C) 986
(D) 996

18. The numberof selections of 4 letters taken from the word COLLEGE is equal to

(A) 24
(B) 18
(C) Coefficient of x^4 is $(1 + x)^2 (1 + x + x^2)^3$
(D) Coefficient of x^4 in $(1 + x)^3 (1 + x + x^2)^2$

19. All the five digit numbers in which each successive digit exceeds its predecessor are arranged in the increasing order. The 105th number does not contain the digit

(A) 1
(B) 2
(C) 3
(D) 4

SECTION-3

COMPREHENSION BASED QUESTIONS

COMPREHENSION: (Q.1 TO Q.3)

A library has 5 indistinguishable physics books, 4 indistinguishable mathematics books and 3 indistinguishable chemistry books. In how many distinguishable ways can a student take home.

1. 6 books

(A) 16 (B) 18 (C) 19 (D) 20

2. 6 books taking atleast one of each subject.

(A) 11 (B) 8 (C) 10 (D) 9

3. 6 books taking not more than 2 of each subject.

(A) 1 (B) 2 (C) 4 (D) 6

COMPREHENSION (Q.4 TO Q.5) :

Each of 5 woman who attend a banquet checks her coat and hat with the receptionist on arrival. Upon leaving, each woman is given a coat and a hat at random.

4. The number of ways these coats and hats may be distributed such that nobody gets back both her coat and her hat is

(A) 2048 (B) 1936 (C) 1956 (D) 1978

5. The number of ways these coats and hats may be distributed such that nobody gets back either her coat or her hat is

(A) 14400 (B) 118000 (C) 11842 (D) 11844

COMPREHENSION (Q.6 TO Q.8) :

A point in the x-y plane whose cordinates are integers is called a lattice point. Consider a path from the origin to the lattice point A(n, n), where n is non negative, that

(i) tats from origin.

(ii) is always parallel to the x-axis or y-axis.

(iii) makes turns only at a lattice point, either along positive x-axis or along the positive y-axis.

(iv) terminates of A.

Find the number of paths from (0, 0) to (n, n) such that

6. Either x > y at all interior lattice points or y > x at all interior lattice points.

(A) ${}^{2n-2}C_{n-1} - {}^{2n-2}C_n$

(B) $2\frac{{}^{2n-1}C_n}{(n+1)}$

(C) $\frac{2}{n}\,{}^{2n-2}C_{n-1}$

(D) $2\frac{{}^{2n}C_n}{n+1}$

7. $y \le x$ at every lattice point on the path

(A) $\frac{{}^{2n}C_n}{n+1}$

(B) $\frac{{}^{2n-2}C_{n-1}}{n}$

(C) ${}^{2n-2}C_{n-1} - {}^{2n-2}C_n$

(D) $\frac{{}^{2n-1}C_{n-1}}{n}$

8. The path never crosses the line y = x.

(A) $2\frac{{}^{2n-1}C_n}{n+1}$

(B) $2\left({}^{2n-2}C_{n-1} - {}^{2n-2}C_n\right)$

(C) $\frac{2}{n}\,{}^{2n-2}C_{n-1}$

(D) $\frac{2\,{}^{2n}C_n}{n+1}$

COMPREHENSION (Q.9 TO Q.11) :

Consider a binary string which consist only of digits 0 and 1. Let a_n be the number of binary strings of length 'n' that do not contain the sequence 11 and b_n be the number of binary strings of length 'n' that do not contain the string 111. Then

(For example 0001001010 is a binary string of length 10)

9. a_n is equal to (where $n \geq 5$)

(A) $a_{n-1} + 2a_{n-2}$
(B) $a_{n-1} + a_{n-3}$
(C) $2a_{n-1} + a_{n-2}$
(D) $a_{n-1} + a_{n-2}$

10. b_n is equal to (where $n \geq 5$)

(A) $b_{n-1} + 2b_{n-2} + 3b_{n-3}$
(B) $b_{n-3} + b_{n-2} + b_{n-1}$
(C) $4b_{n-1} + 2b_{n-2} + b_{n-3}$
(D) $b_{n-1} + b_{n-2}$

11. b_6 is equal to

(A) 64
(B) 58
(C) 26
(D) 44

COMPREHENSION (Q.12 TO Q.13) :

A bag has 5 white marbles, 3 red marbles, and 5 blue marbles, marbles are drawn one by one and without replacement till all marbles are drawn. Marbles of each colour are indistinguishable. Then

12. In how many ways may one draw the marbles out of the bag if all blue are drawn in odd numbered draws, or all blue are drawn in even numbered draw.

(A) 1202
(B) 1448
(C) 1512
(D) 1668

13. In how many ways will all red marbles come before any of the blue marbles.

(A) 1496
(B) 1287
(C) 1024
(D) 967

COMPREHENSION (Q.14 TO Q.16) :

Consider the word 'ASSASSINATION'. How many different arrangements are there of all the letters of the word given so that

14. The first N precedes O.

(A) $\frac{13!}{3!3!4!}$ (B) $\frac{13!}{3!3!4!2!}$ (C) $\frac{10!\times {}^{13}C_3}{3!}$ (D) $\frac{13!}{3!4!}$

15. The first A precedes the first S.

(A) $2700 \times {}^{13}C_7$ (B) $150 \times {}^{13}C_7$ (C) ${}^{13}C_7\,{}^{6}C_3$ (D) ${}^{13}C_7\,\frac{6!}{4}$

16. First A precedes the first S and the first N prercedes O

(A) $\frac{3\times 13!}{4\times 5!}$ (B) $\frac{13!}{7\times 4!}$ (C) $\frac{13!\times 11!}{7}$ (D) $\frac{5\times 13!}{2\times 7!}$

COMPREHENSION (Q.17 TO Q.18) :

Eleven criminals want to keep the location of their master criminal in a safe. They want to be able to open the safe only when any 6 of them are present. The safe is thus equipped with a number of different locks, and each criminal is given the keys to some of these locks.

17. What is the minimum number of locks required

(A) 256 (B) 504 (C) 462 (D) 420

18. What is the minimum number of keys each criminal must carry

(A) 216 (B) 252 (C) 126 (D) 504

Comprehension (Q.19 to Q.20) :

Let there is a large pile of red, white, green and blue balls, where balls are all alike except for the colour. Then find the number of ways to select 20 balls from them so that

19. The selection has atmost 3 green balls.

(A) 802 (B) 969

(C) 973 (D) 879

20. The selection has an even number of white balls

(A) 844 (B) 929

(C) 948 (D) 1017

Comprehension (Q.21 to Q.22) :

A committee of 10 person is to be selected from 9 women and 8 men consisting of atleast 4 women and 4 men. Find number of ways committee if can be formed if

21. Miss X refuses to work with Mr. Y

(A) 12182 (B) 5586

(C) 10876 (D) 10878

22. Miss X and Mr. Y insist to work together

(A) 9128 (B) 6486

(C) 10876 (D) 8232

COMPREHENSION (Q.23 TO Q.25) :

There are 'm' seats in the first row of a theatre, of which 'n' are to be occupied, where m is odd and n is even and $n < \frac{m}{2}$. Then find the number of ways of arranging 'n' persons so that.

23. N two persons sit side by side.

(A) ${}^{m-n+1}C_n (n-1)^1$ (B) ${}^{m-n+1}P_n$

(C) ${}^{m-n}P_n$ (D) ${}^{m}P_n$

24. Each person has exactly one neighbour

(A) ${}^{m-n+1}C_{\frac{n}{2}} n!$ (B) ${}^{m-n+1}C_{\frac{n}{2}} \left(\frac{n}{2}\right)^1$

(C) ${}^{m-n}C_{\frac{n}{2}} \left(\frac{n}{2}\right)!$ (D) ${}^{m-n+1}C_{\frac{n}{2}} \left(\frac{n}{2}\right)! \, 2^{\frac{r_1}{2}}$

25. Out of any two seats located symmetrically about the middle of the row, atleast one is empty

(A) ${}^{m+1}C_n 2^n n! + {}^{m+1}C_{n-1} n! \, 2^{n-1}$ (B) ${}^{m+1}C_n 2^n n!$

(C) ${}^{\frac{m-1}{2}}C_n 2^n n!$ (D) $2^{n-1} n! \left(2 \, {}^{\frac{m-1}{2}}C_n + {}^{\frac{m-1}{2}}C_{n-1}\right)$

COMPREHENSION (Q.26 TO Q.28) :

Consider the word 'MATHEMATICS'. Find the number of all possible words that can be formed using all letters of the given word so that.

26. The odd numbered places do not contain all distinct letters.

(A) $\frac{11!}{2!\,2!\,2!} - \frac{9!}{2!\,2!}$ (B) $\frac{(111)\,5!\,6!}{2!\,2!\,2!}$ (C) $\frac{11!}{2!\,2!\,2!}$ (D) $\frac{(113)\,5!\,6!}{2!\,2!\,2!}$

27. Two M's and two T's are together but 2A's are not gogether.

(A) (28)7! (B) $\frac{6!\,4!}{2!\,2!}$ (C) (24) 7! (D) $\frac{8!}{2!}$

28. Two vowels are never together.

(A) $115 \times 8!$ (B) $210 \times 7!$ (C) $220 \times 7!$ (D) $240 \times 7!$

SECTION-4

MATCH THE COLUMN

1. Five balls are to be placed in three boxes. Each can hold all the five balls. The number of different ways can we place the balls so that no box remain empty if

	Column-I		Column-II
(A)	balls and boxes are all different is equal to	(P)	2
(B)	balls are identical but boxes are different is equal to	(Q)	6
(C)	balls are different but boxes are identical is equal to	(R)	25
(D)	balls as well as boxes are identical is equal to	(S)	50
		(T)	150

2. The number of 10 letter permutations comprising 4a's, 3b's and 3c's such that

	Column-I		Column-II
(A)	a's are separated and all b's are together is equal to	(P)	18
(B)	a's are separated and exactly two b's are together is equal to	(Q)	20
(C)	no two adjacent letters are identical is equal to	(R)	150
(D)	no two b's are together, no two c's are together and all 'a's are	(S)	180
		(T)	248

3. A person moves in the x-y plane moving along points with integer coordinates x and y only. When she is at point (x, y), she takes a step based on the following rules:
(a) if $x + y$ is even, she moves to either $(x + 1, y)$ or $(x + 1, y + 1)$.
(b) if $x + y$ is odd, she moves to either $(x, y + 1)$ or $(x + 1, y + 1)$.
The number of distinct paths can she take to go from (0, 0) to (8, 8) given that

	Column-I		Column-II
(A)	she took exactly three steps to the right $((x, y) \text{ to } (x+1, y))$	(P)	210
(B)	she took exactly two steps up $((x, y) \text{ to } (x, y+1))$ is equal to	(Q)	320
(C)	she took exactly four steps to the right $((x, y) \text{ to } (x+1, y))$ is equal to	(R)	364
(D)	she took exactly three steps to the right $((x, y) \text{ to } (x+1, y))$ and exactly three steps up $((x, y) \text{ to } (x, y+1))$ is equal to	(S)	462
		(T)	495

4. A is set containing 'n' elements. A subset A_1 of A is chosen. The set A is reconstructed by replacing the elements of A_1. Then a subset A_2 of A is chosen and again set A is reconstructed by replacing elements of A_2. In this way we choose 'm' subsets $A_1, A_2, A_3, \ldots\ldots, A_m$, $m > 3$. Then the number of ways of choosing $A_1, A_2, A_3, \ldots\ldots, A_m$ such that

	Column-I		Column-II
(A)	$A_1 \cup A_2 \cup \ldots\ldots \cup A_m$ contains exactly 'r' elements of A ($r < n$) is	(P)	$(2^m - 1)^n$
(B)	$A_1 \cap A_2 \cap \ldots\ldots \cap A_m$ contains exactly 'r' elements of A ($r < n$) is	(Q)	${}^nC_r(2^m - 1)^{n-r}$
(C)	$A_i \cap A_j = \phi \quad \forall\, i \neq j$ is	(R)	${}^nC_r(m + 1)^r$
(D)	$A_1 \cup A_2 \cup A_3 \cup \ldots\ldots \cup A_m = A$ is	(S)	$(m + 1)^n$
		(T)	${}^nC_r(2^m - 1)^r$

5.

	Column-I		Column-II
(A)	The number of ways of selecting two parallel lines from the edges and face diagonals of a cube is equal to	(P)	18
(B)	The number of ways of selecting two skew face diagonals from the edges and face diagonals of a cube is equal to	(Q)	24
(C)	The number of ways of selecting an edge and a face diagonal from the edges and face diagonals of a cube such that they are skew is equal to	(R)	30
(D)	The number of ways of selecting two coplanar and perpendicular ines from the edges and face diagonals of a cube is equal to	(S)	54
		(T)	72

6. Let N be the number of 4 digit numbers, abcd (where a, b, c, d are digits) satisfying the condition in column-I and the last digit of N in column-II.

	Column-I		Column-II
(A)	a, b, c, d are distinct and $a > \max\{b, c, d\}$	(P)	0
(B)	a, b, c, d are distinct and $a < \min\{b, c, d\}$	(Q)	2
(C)	$a \le b \le c \le d$	(R)	4
(D)	$a \ge b \ge c \ge d$	(S)	5
		(T)	6

7. Let A = {1, 2, 3, 4}, B = {1, 2, 3, 4, 5, 6, 7, 8}

	Column-I		Column-II
(A)	The number of functions f : B → A, such that if $x_1 > x_2$, then $f(x_1) \geq f(x_2)$ $\forall$ $x_1 \in B$ is N. Then the sum of digits of N is	(P)	7
(B)	The number of onto functions f : B → A such that if $x_1 > x_2$, then $f(x_1) \leq f(x_2)$ $\forall$ $x_1 \in B$ is N. Then the sum of digits of N is	(Q)	8
(C)	The numbers of one-one function f : A → B such that $f(x) \neq x$ $\forall$ $x \in A$ is N. Then the sum of digits of N is	(R)	9
(D)	The number of one-one function f : A → B such that there are exactly two, x, $x \in A$ for which $f(x) = x$ is N. Then the sum of digits of N is	(S)	10
		(T)	12

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. Let $0 < a < b < c < d < e < f < g$ be a geometric sequence of integers. Let $*(k)$ denote the number of divisors of k. For example $*(6) = 4$ because 1, 2, 3, 6 are divisors of 6. If $*(a) = 7$, $*(g) = 13$ and $d-c = 432$, then find $\left[\frac{b}{100}\right]$ ([.] denote greatest integer function)

2. Let N is the number of times digit 5 is written when listing all natural numbers from 1 to 10^5. Then the sum of digits of N is

3. Let N be number of ways four different integers be chosen from the set $\{1, 2, 3, 4, ..., 104, 105\}$ so that their sum is divisible by 4, then $\left[\frac{N}{10^5}\right]$ is equal to ([.] denote greatest integer function)

4. Let N be the number of ordered 6-tuples $(x_1,x_2,x_3,x_4,x_5,x_6)$ of positive integers satisfying $x_1+x_2+x_3+3x_4+3x_5+5x_6 = 21$, then $\sqrt{N}$ is equal to

5. Let N is the number of 4 element subsets $\{a, b, c, d\}$ of $\{1, 2, 3, 4,....,20\}$ such that $a + b + c + d$ is divisible by 3. Then the sum of the digits of N is equal to

6. Suppose $A_1, A_2,....,A_6$ are six sets, each with four elements and $B_1, B_2,......,B_n$ are n sets each with two elements. Let $S = A_1 \cup A_2 \cup ... \cup A_6 = B_1 \cup B_2 \cup .... \cup B_n$. Given that each element of S belongs to exactly four of the A's and to exactly three of the B's, then n is equal to

7. Suppose $A_1, A_2,..., A_{20}$ is a 20 sided regular polygon. Let N be the number of non isosceles triangles that can be formed whose vertices are the vertices of the polygon but whose sides are not the sides of the polygon. Then $\sqrt{\frac{N}{10}}$ is equal to

8. Let $A_1A_2A_3...A_{200}$ be a regular 200-sided convex polygon. Make the diagonals A_iA_{i+9}, i = 1,2,3,...,200, where $A_{i+200} = A_i$ for $1 \le i \le 9$. Let N be the number of distinct points of intersection formed inside the polygon by these 200 diagonals, then the sum of the digits of N is

9. There are 6 given points on a circle and each two points are connected by a segment. Suppose that any three segments are not concurrent, so any three intersecting segments form a triangle inside the circle. Let N be the number of triangles formed, then sum of digits of N is

10. All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once and not divisible by 5, are arranged in the increasing order. Let N be the 2018^{th} number in this list, then the last digit of N is

11. An insect moves from (0, 0) to (6, 3) by moving through lattice points (lattice points are point (x, y) such that $x, y \in I$), moving either one unit right or one unit up at each step. Let N be the number of paths in which the lime segments joining (2, 1) and (2, 2) and (3, 2) and (4, 2) are avoided, then $\left[\frac{N}{6}\right]$ is equal to ([.] denote greatest integer function).

12. Let N be the number of all 5 digit numbers each of which contains the block 15 and is divisible by 15. Then the last digit of N is

13. Let N be the number of 6 digit numbers such that the digits of each number are all from the set {1, 2, 3, 4, 5} and any digit that appears in the number appears atleast twice. Then the last digit of N is

14. Let N be the number of all 6 digit numbers such that the sum of their digits is 10 and each of the digits 0, 1, 2, 3 occurs atleast once in them, then the last digit of N is

15. Let N be the number of all 4 digit numbers having non zero digits and which are divisible by 4 but not by 8. Then the last digit of N is

16. Let X = {1, 2, 3,.....,12} and N be the number of pairs {A, B} such that $A \subseteq X$, $B \subseteq X$, $A \neq B$ and $A \cap B = \{2, 3, 5, 7, 8\}$. Then the value of N is

17. Find the number of eight digit numbers the sum of whose digits is 4.

18. Find number of all 4-tuples (a, b, c, d) of natural numbers with $a \le b \le c$ and $a! + b! + c! = 3^d$.

19. Let N be the number of non empty subsets S of the set {0, 1, 2, 3,....,9} so that the sum of elements of 'S' is divisible by 3. Then the sum of the digits of N is

20. Let N be the number of ordered pairs (A, B) where A and B are subsets of {1, 2, 3, 4, 5} such that neither $A \subseteq B$ nor $B \subseteq A$. Then N is equal to

21. Suppose 32 objects are placed along a circle at equal distances. Let N be the number of ways can 3 objects be chosen from among them so that no two of the three chosen objects are adjacent nor diametrically opposite. Then the value of N is

22. Let N be the number of 3-digit numbers having atleast one 5 and atmost one 3. Then N is equal to

23. There are four basket ball players A, B, C, D. Initially, the ball is with A. The ball is always passed from one person to a different person. Let N be the number of ways the ball come back to A after seven passes, (For example $A \to C \to B \to D \to A \to B \to C \to A$ and so on). Then N is equal to

24. Let N be the number of positive integers less than 1000 have the property that the sum of the digits of each such number is divisible by 7 and the number itself is divisible by 3. Then N is equal to

25. Let N be the number of triplets (a, b, c), where a, b, c $\in$ {1, 2, 3, 4, 5} such that $2^a + 3^b + 5^c$ is divisible by 4. Then the sum of digits of N is

26. Let A = {1, 2, 3, 4, 5,......,99, 100} and B be a subset of A such that the sum of no two elements in B is divisible by 7. Let N be maximum number of elements in B, then $\frac{N}{5}$ is equal to

27. Let 5 letter words are formed using the letters of the word ‘CALCULUS’ and N is the rank of word CALCU among them as arranged in dictionary. Then the sum of digit of N is

28. Let N be the number of 5 letter words can you spell using the letters S, I and T only, if a ‘word’ is defined as any sequence of letters that doesn’t contain 3 consecutive consonants. Then the sum of digits of N is

29. A number has four divisors and sum of its divisors excluding 1 and itself is 30, then the number of such numbers is equal to

30. The number of order triplets (a, b, c) such that LCM(a, b) = 1000, LCM (b, c) = 2000 and LCM (c, a) = 2000 is

31. Let N be the number of eight digit numbers that can be formed using the digits 1, 2, 3, 4 only such, that sum of the eight digits is 12. Then the sum of digits of N.

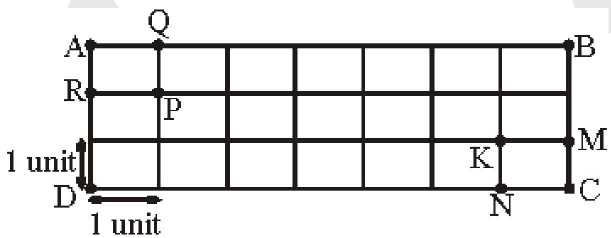
32. Consider all 5 digit numbers such that the sum of their digits is 7 and each of the digits 0, 1, 2 occurs atleast once in them, the number of such numbers is equal to

33. A class contains three girls and four boys. Every saturday, five students go on a picnic, a different group being sent each week. During the picnic, each girl in the group is given a doll by the accompanying teacher. After all possible groups have gone once, the total number of dolls the girls have got is equal to

34. The total number of divisors of $3^5 5^7 7^9$ which are of the form $4k + 1$, $k \geq 0$, $k \in I$ is equal to

35. A is the set of three digit numbers formed using the digits 1, 2, 3,.....,8, 9 using each digit at the most once. Also every number in A has exactly one even digit. Find the sum of all the numbers in A.

36. An ant has to travel from point C to A along the gridlines. It gets lost on its way but manages to reach A by travelling 12 units in all. Its first two moves are CM and MK and its last two moves are PQ and QA. In how many ways could the ant have travelled from C to A, if it never travels downwards.



37. Find the number of quadraples of positive integers (a, b, c, d) such that $(a + b)(c + d) = 2016$.

38. For any set S, let |S| denote the number of elements in S and let n(S) be the number of subsets of S, including the empty set and S itself. If A, B and C are sets for which $n(A) + n(B) + n(C) = n(A \cup B \cup C)$ and $|A| = |B| = 100$, then what is the minimum possible value of $|A \cap B \cap C|$.

39. Let N be the number of ordered triples of (A, B, C) such that $A \cup B \cup C = \{1, 2, 3,....,2003\}$ and $A \cap B \cap C = \phi$. Then number of divisors of N is equal to

40. For how many pairs of consecutive integers in {1000, 1001, 1002,.......,2000} is no carrying required when the two integers are added.

41. Find the number of ways 10 can be expressed as a sum of positive integer. For example '3' can be expressed as a sum of positive integers in 4 ways 3, 2 + 1, 1 + 2, 1 + 1 + 1.

Answer Key

Single Choice Questions

1. A	**2.** C	**3.** B	**4.** B	**5.** A	**6.** D
7. A	**8.** A	**9.** D	**10.** A	**11.** A	**12.** A
13. B	**14.** B	**15.** D	**16.** A	**17.** A	**18.** C
19. D	**20.** B	**21.** A	**22.** D	**23.** B	**24.** A
25. C	**26.** B	**27.** A	**28.** D	**29.** C	**30.** B
31. A	**32.** C	**33.** D	**34.** B	**35.** A	**36.** A

One or More than One Correct Questions

1. B,D	**2.** B,C	**3.** A,B	**4.** B,C,D	**5.** B,C,D	**6.** A,CD
7. A,B,C	**8.** B,C	**9.** A,C,D	**10.** B,C,D	**11.** B,D	**12.** A,B,C
13. B,C,D	**14.** B,C	**15.** B,C	**16.** A,D	**17.** A,C,D	**18.** B,D
19. A,C,D					

Comprehension Based Questions

1. B	**2.** D	**3.** A	**4.** B	**5.** D	**6.** C
7. A	**8.** D	**9.** D	**10.** B	**11.** D	**12.** C
13. B	**14.** A	**15.** A	**16.** D	**17.** C	**18.** B
19. A	**20.** C	**21.** D	**22.** D	**23.** B	**24.** A
25. D	**26.** D	**27.** A	**28.** B		

Match the column

1. $A \to T ; B \to Q ; C \to P ; D \to R$

2. $A \to Q ; B \to S ; C \to T ; D \to P$

3. $A \to S ; B \to P ; C \to T ; D \to S$

4. $A \to T ; B \to Q ; C \to S ; D \to P$

5. $A \to Q ; B \to R ; C \to T ; D \to S$

6. $A \to P ; B \to T ; C \to S ; D \to R$

7. $A \to T ; B \to Q ; C \to P ; D \to R$

Subjective Type Questions

1. 9	**2.** 5	**3.** 7	**4.** 9	**5.** 9	**6.** 9
7. 8	**8.** 7	**9.** 3	**10.** 1	**11.** 8	**12.** 9
13. 5	**14.** 0	**15.** 9	**16.** 2186	**17.** 120	**18.** 3
19. 9	**20.** 570	**21.** 3616	**22.** 249	**23.** 546	**24.** 28
25. 7	**26.** 9	**27.** 8	**28.** 6	**29.** 4	**30.** 70
31. 7	**32.** 100	**33.** 45	**34.** 240	**35.** 133200	**36.** 56
37. 59508	**38.** 98	**39.** 4016016	**40.** 156	**41.** 512	

Previous Year Questions

SECTION-1

1. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon.

If $T_{n+1} - T_n = 10$, then the value of n is **[IIT JEE Main 2013]**

(A) 8 (B) 7

(C) 5 (D) 10

2. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is: **[IIT JEE Main 2015]**

(A) 72 (B) 216

(C) 192 (D) 120

3. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is :

[IIT JEE Main 2015]

(A) 510 (B) 219

(C) 256 (D) 275

4. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary ; then the position of the word SMALL is : **[IIT JEE Main 2016]**

(A) 46^{th} (B) 59^{th}

(C) 52^{nd} (D) 58^{th}

5. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[IIT JEE Main 2017]**

(A) 469 (B) 484

(C) 485 (D) 468

6. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangmenets is : **[JEE Main 2018]**

(A) at least 750 but less than 1000

(B) at least 1000

(C) less than 500

(D) at least 500 but less than 750

SECTION-2

1. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ?

[JEE '2000, (Scr)]

(A) 16 (B) 36

(C) 60 (D) 180

2. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of 'n' sides. If $T_{n+1} - T_n = 21$, then 'n' equals:

[JEE '2001, (Scr)]

(A) 5 (B) 7

(C) 6 (D) 4

3. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is **[JEE 2002 (Screening), 3]**

(A) 40 (B) 60

(C) 80 (D) 100

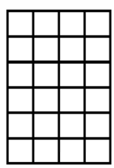
4. Number of points with integral co-ordinates that lie inside a triangle whose co-ordinates are (0, 0), (0, 21) and (21,0) **[JEE 2003 (Screening), 3]**

(A) 210 (B) 190

(C) 220 (D) None

5. Using permutation or otherwise, prove that $\frac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer. **[JEE 2004, 2 out of 60]**

6. A rectangle with sides 2m – 1 and 2n – 1 is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is **[JEE 2005 (Screening), 3]**



(A) $(m + n + 1)^2$ (B) 4^{m+n-1}

(C) m^2n^2 (D) $mn(m + 1)(n + 1)$

7. If *r*, *s*, *t* are prime numbers and p, q are the positive integers such that their LCM of p, q is is $r^2t^4s^2$, then the numbers of ordered pair of (p, q) is **[JEE 2006, 3]**

(A) 252 (B) 254

(C) 225 (D) 224

8. The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word **COCHIN** is **[JEE 2007, 3]**

(A) 360 (B) 192

(C) 96 (D) 48

9. Consider all possible permutations of the letters of the word ENDEANOEL

Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II**.

	Column-I		**Column-II**
(A)	The number of permutations containing the word ENDEA is	(P)	5!
(B)	The number of permutations in which the letter E occurs in the first and the last position is	(Q)	2 × 5!
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(R)	7 × 5!
(D)	The number of permutations in which the letters A, E, O occurs only in odd positions is	(S)	21 × 5!

[JEE 2008, 6]

10. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is **[JEE 2009]**

(A) 55 (B) 66

(C) 77 (D) 88

11. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to **[JEE 2010]**

(A) 25 (B) 34

(C) 42 (D) 41

12. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

[JEE 2012]

(A) 75 (B) 150

(C) 210 (D) 243

13. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

[IIT JEE Advance 2014]

14. Let n ≥ 2 be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is **[IIT JEE Adv. 2014]**

15. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6, and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is : **[IIT JEE Advance 2014]**

(A) 264 (B) 265

(C) 53 (D) 67

16. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

[IIT JEE Advance 2015]

17. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, them the number of ways of selecting the team is : **[IIT JEE Adv. 2016]**

(A) 380 (B) 320

(C) 260 (D) 95

18. Let S = {1, 2, 3, ..., 9}. For k = 1, 2, ... 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

[JEE Advance 2017]

(A) 125 (B) 252

(C) 210 (D) 126

19. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then $\frac{y}{9x} =$ **[JEE Advance 2017]**

20. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is _____________ .

[JEE Advanced 2018]

21. Let X be a set with exactly 5 elements and Y bet a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is _______.

[JEE Advanced 2018]

22. In a high school, a committee has to be formed from a group of 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

List-I	**List-II**
(P) The value of α_1 is	(1) 136
(Q) The value of α_2 is	(2) 189
(R) The value of α_3 is	(3) 192
(S) The value of α_4 is	(4) 200
	(5) 381
	(6) 461

The correct option is : **[JEE Advanced 2018]**

(A) $P \to 4; Q \to 6; R \to 2; S \to 1$ (B) $P \to 1; Q \to 4; R \to 2; S \to 3$

(C) $P \to 4; Q \to 6; R \to 5; S \to 2$ (D) $P \to 4; Q \to 2; R \to 3; S \to 1$

Answer Key

Section-1

1. C	**2.** C	**3.** B	**4.** D	**5.** C	**6.** B

Section-2

1. C	**2.** B	**3.** A	**4.** B	**6.** C	**7.** C
8. C	**9.** (A) P; (B) S; (C) Q; (D) Q			**10.** C	**11.** D
12. B	**13.** 7	**14.** 5	**15.** C	**16.** 5	**17.** A
18. D	**19.** 5	**20.** 625	**21.** 119	**22.** C	

4 CHAPTER

BINOMIAL THEOREM

SECTION-1

SINGLE CHOICE QUESTIONS

1. The number of rational terms in the expansion of $\left(\sqrt{2}+\sqrt[3]{3}+\sqrt[6]{5}\right)^{10}$ is equal to

(A) 0
(B) 1
(C) 2
(D) 3

2. If $(1+x)^n = \sum_{r=0}^{n} a_r x^r$, $n \in N$, then the value of $\sum_{r=0}^{n-1}(a_r + a_{r+1})^2$ is equal to

(A) ${}^{2n+2}C_{n+1} - 1$
(B) ${}^{2n+2}C_{n+1} - 2$
(C) ${}^{2n+2}C_{n+1}$
(D) ${}^{2n+2}C_{n} - 1$

3. For any $1 \le r \le n-1$, the value of ${}^nC_r - 2\,{}^nC_{r-1} + 3\,{}^nC_{r-2} - 4\,{}^nC_{r-3} + ... + (-1)^r(r+1)$ is equal to

(A) ${}^{n-1}C_r$
(B) $(-1)^r\,{}^{n-2}C_r$
(C) ${}^{n-2}C_r$
(D) $(-1)^r\,{}^{n-1}C_r$

4. The sum of coefficients of odd powers of x in the expansion of $(1+x+x^2+x^3)^5$ is equal to

(A) 256
(B) 512
(C) 1024
(D) 1026

5. If $2(1+x^3)^{100} = \sum_{r=0}^{100}\left(a_r x^r - \cos\left(\frac{\pi}{2}(x+r)\right)\right)$, then the value of $a_0 + a_2 + a_4 + ... + a_{100}$ is equal to:

(A) 2^{100}
(B) 2^{101}
(C) $2^{100} + 50$
(D) $2^{100} - 50$

6. If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $T_n = \sum_{r=1}^{n} \frac{r}{{}^nC_r}$, then $\frac{T_n}{S_n}$ is equal to

(A) $n - 1$
(B) $\frac{n}{2}$
(C) n
(D) $\frac{n}{2} - 1$

7. Let k and n be the positive integers and $S_k = 1^k + 2^k + 3^k + \ldots.. + n^k$. Then ${}^{m+1}C_1S_1 + {}^{m+1}C_2S_2 + {}^{m+1}C_3S_3 + \ldots.. {}^{m+1}C_mS_m$ is equal to

(A) $(n + 1)^{m+1}$
(B) $(n + 1)^m - n$
(C) $(n + 1)^{m+1} - 1$
(D) $(n + 1)^{m+1} - n - 1$

8. The sum of the series ${}^nC_1^2 + \frac{1+2}{2}{}^nC_2^2 + \frac{1+2+3}{3}{}^nC_3^2 + \ldots.. + \frac{1+2+3+\ldots..n}{n}{}^nC_n^2$ is equal to

(A) $\frac{1}{2}(n\,{}^{2n-1}C_n + {}^{2n}C_n)$
(B) $\frac{1}{2}(n\,{}^{2n-1}C_n + {}^{2n}C_n - 1)$
(C) $\frac{1}{2}((n + 1)\,{}^{2n-1}C_n - 1)$
(D) $\frac{1}{2}(n\,{}^{2n-1}C_n + {}^{2n}C_n - 2)$

9. The sum ${}^nC_0^2 + \frac{{}^nC_1^2}{2} + \frac{{}^nC_2^2}{3} + \ldots.. + \frac{{}^nC_n^2}{n+1}$ is equal to

(A) $\frac{(2n+1)!}{((n+1)!)^2}$
(B) $\frac{(2n+1)!}{n!(n+1)!}$
(C) $\frac{(2n+1)!}{(n-1)!\,(n+1)!}$
(D) $\frac{(2n+1)!}{(n+2)!\,(n+1)!}$

10. $2({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots.. + {}^{n-1}C_2 + {}^nC_2) + {}^{n+1}C_2$, $n \in N$ is equal to

(A) $\sum_{r=1}^{n} r(r-1)$
(B) $\sum_{r=1}^{n} r(r+1)$
(C) $\sum_{r=1}^{n} r$
(D) $\sum_{r=1}^{n} r^2$

11. $\frac{{}^nC_0}{1} - \frac{{}^nC_1}{4} + \frac{{}^nC_2}{9} - \frac{{}^nC_3}{16} + ... + (-1)^n \frac{{}^nC_n}{(n+1)^2}$ is equal to

(A) $\frac{1}{n}\left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}\right)$

(B) $\frac{1}{(n+1)}\left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{(n+1)}\right)$

(C) 0

(D) $\frac{(-1)^n}{n+1}$

12. If $n \in N$, $n > 5$, then ${}^nC_3 + 2\,{}^nC_4 + 3\,{}^nC_5 + ... + (n-2)\,{}^nC_n$ is equal to

(A) $(n-4)\,2^{n-1} + n + 1$

(B) $(n-4)\,2^n + n + 2$

(C) $(n-2)\,2^{n-1} + n + 1$

(D) $(n-4)\,2^{n-1} + n + 2$

13. $\frac{{}^nC_0}{2} - \frac{{}^nC_1}{6} + \frac{{}^nC_2}{10} - \frac{{}^nC_3}{14} + ... + \frac{(-1)^n\,{}^nC_n}{(4n+2)}$, $n \in N$ is equal to

(A) $\frac{2^{n-1}\,n!}{3\cdot 5\cdot 7.....(2n-1)(2n+1)}$

(B) $\frac{2^n\,n!}{1\cdot 3\cdot 5\cdot 7.....(2n-3)(2n-1)}$

(C) $\frac{2^n\,n!}{3\cdot 5.....(2n-1)(2n+1)}$

(D) $\frac{2^{n-1}(n-1)!}{1\cdot 3\cdot 5.....(2n-3)(2n-1)}$

14. The coefficient of x^n in $\left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + ... + \frac{(-1)^n x^n}{n!}\right)^2$ is equal to

(A) $\frac{2^n - 1}{n!}$

(B) $\frac{2^n}{n!}$

(C) $\frac{(-2)^n}{n!}$

(D) $\frac{(-1)^n}{n!}$

15. The coefficient of x^{64} in the expansion of $(1 + x)^{131}\,(x^2 - x + 1)^{130}$ is

(A) ${}^{130}C_{21}$

(B) ${}^{130}C_{21} + {}^{130}C_{20}$

(C) 0

(D) ${}^{129}C_{19} + {}^{129}C_{20}$

16. If in the expansion of $\left(2^x + \frac{1}{4^x}\right)^n$, $n \in N$ $\frac{T_3}{T_2} = 7$ and the sum of the coefficients of 2^{nd} and 3^{rd} terms is 36, then the value of x is equal to (T_i denote the i^{th} term of expansion)

(A) -1 (B) $-\frac{1}{3}$

(C) $-\frac{1}{2}$ (D) 0

17. $^nC_1 - \left(1+\frac{1}{2}\right)^nC_2 + \left(1+\frac{1}{2}+\frac{1}{3}\right)^nC_3 - \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)^nC_4 + ...$

$+(-1)^{n-1}\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)^nC_n =$

(A) $\frac{n-1}{n}$ (B) $\frac{1}{n}$

(C) $\frac{1}{n+1}$ (D) $\frac{2^n}{n+1}$

18. $\sum_{r=1}^{n} \frac{(-1)^{r-1}\,{}^nC_r(1-x)^r}{r} =$

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^n}{n}$ (B) $\frac{1-x}{1} + \frac{1-x^2}{2} + \frac{1-x^3}{3} + ... + \frac{1-x^n}{n}$

(C) $(x-1) + \frac{x^2-1}{2} + \frac{x^3-1}{3} + ... + \frac{x^n-1}{n}$ (D) $n - \frac{x}{1} - \frac{x^2}{2} - \frac{x^3}{3} ... - \frac{x^n}{n}$

19. $^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^{m-1}\,{}^nC_{m-1}$ is equal to

(A) $(-1)^{m-1}\,{}^nC_m$ (B) $(-1)^m\,{}^{n-1}C_{m-1}$

(C) $(-1)^{m-1}\,{}^{n-1}C_{m-1}$ (D) $(-1)^m\,{}^nC_m$

20. $\sum_{r=1}^{n} r^3\left(\frac{^nC_r}{^nC_{r-1}}\right)^2$ is equal to

(A) $\frac{n(n+1)^2(n+2)}{12}$ (B) $\frac{n^2(n+1)\,(n+2)}{12}$

(C) $\frac{n(n+1)\,(n+2)^2}{12}$ (D) $\frac{n(n+1)^2\,(n+2)}{24}$

21. The coefficient of x^n in the polynomial

$(x + {}^{2n+1}C_0)\,(x + {}^{2n+1}C_1)\,(x + {}^{2n+1}C_2) \ldots (x + {}^{2n+1}C_n)$ is equal to :

(A) 2^{2n+1} (B) 2^{2n}

(C) 2^n (D) $2^{2n+1} - 1$

22. $\sum_{i=0}^{n-1}\left(\sum_{j=i+1}^{n}(j\,{}^nC_i + i\,{}^nC_j)\right) =$

(A) $n\,2^{n-1}$ (B) $n^2\,2^n$

(C) $n^2\,2^{n-1}$ (D) $n\,2^n$

23. If n is an odd natural number, then $\sum_{r=0}^{n}\frac{(-1)^r}{{}^nC_r} =$

(A) -1 (B) $\frac{1}{n}$

(C) 1 (D) 0

24. ${}^nC_0\,x^{2n} + \frac{{}^nC_1}{2}\,x^{2n-2}(2 - x^2) + \frac{{}^nC_2}{3}\,x^{2n-4}(2-x^2)^2 + \ldots + \frac{{}^nC_n(2-x^2)^n}{(n+1)} =$

(A) $\frac{2^n - x^{2n+2}}{(n+1)(2-x^2)}$ (B) $\frac{2^n - x^{2n}}{(n+1)(2-x^2)}$

(C) $\frac{2^{n+1} - x^{2n+2}}{(n+1)(2-x^2)}$ (D) $\frac{2^{n+1} - x^{2n}}{(n+1)(2-x^2)}$

25. If $p + q = 1$, then $\sum_{r=0}^{n} r^3\,{}^nC_r p^r q^{n-r} =$

(A) $np(n^2p + 3(n-1)p + 1)$ (B) $np((n^2 - n)p^2 + 2(n-1)p + 1)$

(C) $np((n^2 - 3n + 2)p^2 + 2(n-1)p + 1)$ (D) $np((n^2 - 3n + 2)p^2 + 3(n-1)p + 1)$

26. The magnitude of the greatest coefficient in the expansion of $\left(\frac{1}{2}-\frac{x}{3}\right)^{18}$ is

(A) $\frac{^{18}C_8}{2^{10}3^8}$

(B) $\frac{^{18}C_5}{2^{13}3^5}$

(C) $\frac{^{18}C_6}{2^{12}3^6}$

(D) $\frac{^{18}C_7}{2^{11}3^7}$

27. If there are three successive coefficients in the expansion of $(1 + 2x)^n$ which are in the ratio 1 : 4 : 10, then 'n' is equal to

(A) 7

(B) 8

(C) 9

(D) 10

28. If 'a' is the sum of the coefficients of the two middle terms in the expansion of $(1 + x)^{2n-1}$ and b is the coefficient of the middle term in the expansion of $(1 + x)^{2n}$, $n \in N$, then

(A) $a > b$

(B) $a = b$

(C) $a < b$

(D) $a = 2b$

29. The coefficient of x^r in the expansion of $(x + a)^{n-1} + (x + a)^{n-2}(x + b) + (x + a)^{n-3}(x + b)^2 + ... + (x + b)^{n-1}$, where $a - b = 1$ is equal to

(A) $^nC_r(a^{n-r} + b^{n-r})$

(B) $^nC_r(a^r + b^r)$

(C) $^nC_r(a^r - b^r)$

(D) $^nC_r(a^{n-r} - b^{n-r})$

30. The greatest value of the term independent of x in the expansion $\left(x\sin\alpha+\frac{\cos\alpha}{x}\right)^{2n}$, $\alpha \in R$ is equal to

(A) $\frac{^{2n}C_{n+1}}{2^{n+1}}$

(B) $\frac{^{2n}C_{n-1}}{2^{n-1}}$

(C) $\frac{^{2n}C_n}{2^n}$

(D) $\frac{^{2n}C_n}{2^{2n}}$

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. $\sum_{r=1}^{n} r(n-r)\,{}^nC_r^2$ is equal to

(A) $n^2\,{}^{2n-2}C_{n-2}$ (B) $(n-1)^2\,{}^{2n-2}C_{n-1}$

(C) $n(n-1)^{2n-2}C_{n-2}$ (D) $n(n-1)^{2n-2}C_{n-1}$

2. The value of the sum ${}^nC_1^2 - 2.{}^nC_2^2 + 3.{}^nC_3^2 - 4.{}^nC_4^2 + ... + (-1)^n n.{}^nC_n^2$ where $n \in N$, $n > 3$ will be equal to

(A) $-n\,{}^{n-1}C_{\frac{n-2}{2}}$ if $n = 4k, k \in I$ (B) $n\,{}^{n-1}C_{\frac{n-1}{2}}$ if $n = 4k+1, k \in I$

(C) $n\,{}^{n-1}C_{\frac{n-2}{2}}$ if $n = 4k+2, k \in I$ (D) $-n\,{}^{n-1}C_{\frac{n-1}{2}}$ if $n = 4k+3, k \in I$

3. The expression $\frac{{}^nC_1}{n} - \frac{{}^nC_2}{n-1} + \frac{{}^nC_3}{n-2} - \frac{{}^nC_4}{n-3} + ... + (-1)^{n-1}\frac{{}^nC_n}{1}$ is equal to

(A) $\frac{1}{n+1}$ if n is even (B) 0 if n is even

(C) $\frac{2}{n+1}$ if n is odd (D) $\frac{4}{n+1}$ if n is odd

4. If $n \in N$ and $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $\sum_{r=0}^{n} (-1)^r a_r\,{}^nC_r$ is equal to :

(A) 0 if n = 57 (B) 0 if n = 77

(C) ${}^{24}C_8$ if n = 24 (D) ${}^{39}C_{13}$ if n = 39

5. If $\frac{{}^9C_0}{8} - \frac{{}^9C_1}{9} + \frac{{}^9C_2}{10} - \frac{{}^9C_3}{11} + ... + \frac{{}^9C_8}{16} - \frac{{}^9C_9}{17} = \frac{1}{n}$ then n is divisible by

(A) 7 (B) 11

(C) 13 (D) 17

6. In the expansion of $(x + a)^n$, $n \in N$, if the sum of odd numbered terms be α and the sum of even numbered terms be β, then

(A) $4\alpha\beta = (x + a)^{2n} - (x - a)^{2n}$ (B) $2(\alpha^2 + \beta^2) = (x + a)^{2n} + (x - a)^{2n}$

(C) $\alpha^2 - \beta^2 = (x^2 - a^2)^n$ (D) $\alpha^2 + \beta^2 = (x + a)^{2n} + (x - a)^{2n}$

7. If the middle term of the expression $(1 + x)^{24}$, $x > 0$, is the only greatest term of the expansion, then

(A) $x < 1$ (B) $x < \frac{13}{12}$

(C) $x > \frac{12}{13}$ (D) $x > 1$

8. ${}^{n}C_{m} + 3\,{}^{n-1}C_{m} + 5\,{}^{n-2}C_{m} + 7\,{}^{n-3}C_{m} + ... + (2(n-m)+1)\,{}^{m}C_{m}$ is equal to

(A) ${}^{n+2}C_{m+3} + {}^{n+3}C_{m+3}$ (B) ${}^{n+2}C_{m+2} + 2\,{}^{n+2}C_{m+3}$

(C) ${}^{n+1}C_{m+2} + {}^{n+2}C_{m+2}$ (D) ${}^{n+1}C_{m+1} + 2\,{}^{n+1}C_{m+2}$

9. Let $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, $n \in N$ and $\sum_{r=0}^{2n-1} (-1)^r a_r a_{r+1} = \lim_{p\to\infty} \lim_{q\to\infty} (\cos^2 q!\,\pi k)^p$, then k can be equal to :

(A) $\sqrt[3]{2}$ (B) $\sqrt{3}$

(C) 2 (D) 3

10. If $\frac{1}{\sqrt{4x+1}}\left[\left(\frac{1+\sqrt{4x+1}}{2}\right)^n - \left(\frac{1-\sqrt{4x+1}}{2}\right)^n\right] = a_5x^5 + a_4x^4 + ... + a_0$ where $a_5 \neq 0$, then possible values of 'n' can be $(n \in N)$

(A) 10 (B) 11

(C) 12 (D) 13

11. $\sum_{r=0}^{n} \frac{(-2)^r\,{}^{n}C_r}{{}^{r+2}C_r}$ is equal to $(n \in N)$

(A) $\frac{1}{n+1}$ if n is even (B) $\frac{1}{n+1}$ if n is odd

(C) $\frac{1}{n+2}$ if n is odd (D) $\frac{1}{n+2}$ if n is even

12. If $(1 + cx + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then

(A) $\sum_{r=0}^{2n} (2r+1)a_r = 2n(2 + c)^n$ (B) $\sum_{r=0}^{2n} (2r+1)a_r = (2n+1)(2+c)^n$

(C) $a_r = a_{2n-r}\ \forall\ r \in \{0, 1, 2, ..., n\}$ (D) $\sum_{r=1}^{2n} (-1)^r r a_r = n(2-c)^n$

13. Let n be a positive integer and $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n-1}x^{2n-1} + a_{2n}x^{2n}$, then

(A) $\sum_{r=0}^{n-1} a_r = \frac{1}{2}(3^n - a_n)$

(B) $\sum_{r=0}^{2n} (-1)^r a_r^2 = a_n$

(C) $a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + (-1)^{n-1}a_{n-1}^2 = \frac{a_n}{2}(1 - (-1)^n a_n)$

(D) $(r + 1)a_{r+1} = (n - r)a_r + (2n - r + 1)a_{r-1}$, $1 \le r \le 2n - 1$

14. Let $n \in N$, $n \ge 4$ and $P = \prod_{r=0}^{n} {}^nC_r$, then

(A) $P > \left(\frac{2^n}{n+1}\right)^{n+1}$

(B) $P < \left(\frac{2^n}{n+1}\right)^{n+1}$

(C) $P < \left(\frac{2^n - 2}{n-1}\right)^{n-1}$

(D) $P < \left(\frac{2^n - 2}{n-1}\right)^{n}$

15. Let $S_1 = \sum_{r=0}^{n} ({}^{2n+1}C_{2r})^2$ and $S_2 = \sum_{r=0}^{n} ({}^{2n+1}C_{2r+1})^2$, then

(A) $S_1 = \frac{1}{2}({}^{4n+2}C_{2n} + (-1)^n\, {}^{2n+1}C_n)$

(B) $S_2 = \frac{1}{2}({}^{4n+2}C_{2n} - (-1)^n\, {}^{2n+1}C_n)$

(C) $S_1 = \frac{1}{2}({}^{4n+2}C_{2n+1})$

(D) $S_2 = \frac{1}{2}({}^{4n+2}C_{2n+1})$

16. Let $n \in N$, $n > 3$, then $(25)^n - (20)^n - 8^n + 3^n$ is divisible by

(A) 5

(B) 14

(C) 2

(D) 17

17. The coefficient of x^m in $(1 + x)^k + (1 + x)^{k+1} + (1 + x)^{k+2} + ... + (1 + x)^n$, $k, m, n \in N$ is

(A) ${}^{n+1}C_{m+1}$ if $m < k$

(B) ${}^{n+1}C_{m+1} - {}^kC_{m+1}$ if $m < k$

(C) ${}^{n+1}C_{m+1}$ if $k < m < n$

(D) ${}^{n+1}C_{m+1} - {}^kC_{m+1}$ if $k < m < n$

18. The coefficient of x^{10} in the expansion

(A) $(1 + x + x^2)^{50} (1 - x)^{53}$ is $-3({}^{50}C_3)$

(B) $(1 + x + x^2)^{50} (1 - x)^{53}$ is $3({}^{50}C_3)$

(C) $\left(x^4 + 2 + \frac{1}{x^4}\right)^{15} (3 + 2x^2)$ is $3({}^{30}C_{17})$

(D) $\left(x^4 + 2 + \frac{1}{x^4}\right)^{15} (3 + 2x^2)$ is $2({}^{30}C_{17})$

19. The coefficient of x^{50} in the expression $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ... + 1001\, x^{1000}$ is

(A) ${}^{1000}C_{50} + 2.{}^{999}C_{49} + 3^{998}C_{48} + ... + 50\,{}^{951}C_1 + 51\,{}^{950}C_0$

(B) ${}^{1001}C_{50}$

(C) ${}^{1002}C_{51}$

(D) ${}^{1002}C_{50}$

20. The possible value(s) of x for which the 3^{rd} term in the expansion $(x + x^{\log_{10} x})^5$ is equal to 10^6 is/are

(A) $10^{-5/2}$
(B) $10^{-3/2}$
(C) 10
(D) 100

21. Let $P_n = \sum_{r=1}^{k+1} (-1)^{r-1}\ {}^nC_{2r-2}$ and $Q_n = \sum_{r=1}^{k} (-1)^{r-1}\ {}^nC_{2r-1}$ where $k = \frac{n}{2}$ and n is even, then

(A) $Q_8 = 0$
(B) $P_8 = 16$
(C) $Q_{10} = 32$
(D) $P_{10}^{\ 2} + Q_{10}^{\ 2} = 1024$

22. Let the coefficient of x^{20} in the expansions $(1 + x^2 - x^3)^{1000}$, $(1 - x^2 + x^3)^{1000}$, $(1 - x^2 - x^3)^{1000}$ and $(1 + x^2 + x^3)^{1000}$ be respectively a, b, c and d, then

(A) $a = d$
(B) $a > b$
(C) $a > c$
(D) $b < c$

23. Let $\sum_{r=0}^{200} \alpha_r (1+x)^r = \sum_{r=0}^{200} \beta_r x^r$, where $\alpha_r = 1\ \forall\ r \geq 98$, then the greatest coerfficient in the expansion of $(1 + x)^{201}$ is

(A) ${}^{201}C_{100}$
(B) β_{98}
(C) β_{99}
(D) β_{100}

24. Let $x = (5\sqrt{3} + 8)^{2n+1}$, $n \in N$, then

(A) $[x]$ is even
(B) $[x]$ is odd
(C) $x\{x\} = (11)^{2n+1}$
(D) $x\{x\} = (13)^{2n+1}$

where $[\cdot]$ denotes greatest integer and $\{\cdot\}$ denote fraction part function

25. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{39}x^{39} + a_{40}x^{40}$ and $N = a_0 + a_2 + a_4 + ... + a_{38}$, then N is divisible by

(A) 2^{20} (B) 2^{19}

(C) 3 (D) 25

26. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then the value of $a_1 - a_3 + a_5 - a_7 + ...$ is equal to

(A) 1 if $n = 4k$ (B) -1 if $n = 4k + 1$

(C) 0 if $n = 4k + 2$ (D) -1 if $n = 4k + 3$

where $k \in I$

27. If the unit digit of $13^n + 7^n - 3^n$, $n \in N$, is 3 then possible value(s) of n is/are

(A) 27 (B) 103

(C) 11 (D) 101

28. Let $n \in N$, $n > 3$ then $2^{4n} - 2^n (7n + 1)$ is divisible by

(A) 125 (B) 196

(C) 16 (D) 49

29. Let $S_n = \sum_{k=0}^{n} \frac{{}^{n+k}C_k}{2^k}$, then

(A) $S_{16} = 2S_{17}$ (B) $S_{18} = 2S_{17}$

(C) $S_{12} = 4096$ (D) $S_{14} = \frac{1}{2}S_{15}$

30. If the coefficients of x^t and x^{t+1} in $\sum_{r=0}^{n} (1+x)^r$ where $t < n - 2$ are equal, then

(A) n is odd

(B) n is even

(C) The sum of coefficients of x^t and $x^{t+1} = {}^{n+1}C_{t+2}$

(D) The sum of coefficients of x^t and $x^{t+1} = {}^{n+2}C_{t+2}$

SECTION-3

COMPREHENSION BASED QUESTIONS

COMPREHENSION: (Q.1 TO Q.2)

Consider a triangle ABC, where a, b, c denote the length of sides BC, CA, AB respectively. Then

1. $\sum_{r=0}^{n} {}^nC_r a^{n-r} b^r \sin(rA-(n-r)B)$ is equal to

(A) 0 (B) $(a+b)^n$ (C) $(a-b)^{2n}$ (D) c^n

2. $\sum_{r=0}^{n} {}^nC_r a^{n-r} b^r \cos(rA-(n-r)B)$ is equal to

(A) 0 (B) $(a+b)^n$ (C) $(a-b)^{2n}$ (D) c^n

COMPREHENSION-2 (Q.3 TO Q.5) :

If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r^2}{{}^nC_r} = P(n)\, a_{n+2} + Q(n)\, a_{n+1} + a_n + R(n)$, where P(n), Q(n), R(n) are the polynomial functions of n, then

3. P(5) =

(A) 56 (B) 42 (C) 36 (D) 30

4. Q(5) =

(A) –5 (B) –10 (C) –15 (D) –18

5. R(5) =

(A) –24 (B) –20 (C) –30 (D) –42

Comprehension-3 (Q.6 to Q.8) :

Let $f_1(x) = (x-2)^2$, $f_2(x) = ((x-2)^2-2)^2$, $f_3(x) = \left(\left((x-2)^2-2\right)^2-2\right)^2$,...... and so on ; so that $f_k(x) = \underbrace{\left(\ldots\left(\left((x-2)^2-2\right)^2-2\right)^2\ldots-2\right)^2}_{k \text{ times}} = A_k + B_kx + C_kx^2 + D_kx^3$ + ...

6. B_5 is equal to

(A) –2048 (B) –32 (C) –1024 (D) –512

7. C_3 is equal to

(A) 256 (B) 352 (C) 320 (D) 336

8. C_k is equal to

(A) $\dfrac{4^{2k-1}-4^{k-1}}{3}$ (B) 4^{2k-2} (C) $\dfrac{4^{2k-1}+4^{k-1}}{5}$ (D) 4^{k+1}

Comprehension-4 (Q.9 to Q.10) :

Consider a set $S = \{1, 2, 3, 4, ..., n-1, n\}$, $n \in N$. Let all possible distinct subsets of 'r' elements are formed using the elements of S. Then

9. Sum of all minima of all subsets is equal to

(A) ${}^{n+1}C_r$ (B) ${}^{n+2}C_r$ (C) ${}^{n+2}C_{r+1}$ (D) ${}^{n+1}C_{r+1}$

10. The arithmetic mean of the minima of all the subsets is

(A) $\dfrac{n}{r}$ (B) $\dfrac{n+1}{r+1}$ (C) $\dfrac{n-1}{r-1}$ (D) $\dfrac{n-1}{r+1}$

SECTION-4

MATCH THE COLUMN

1.

	Column-I		Column-II
(A)	$\frac{1}{n}\left(\sum_{0\le i<j\le n}\sum((i+j)({}^nC_i\ {}^nC_j)\right)$ is equal to	(P)	$n^2\,2^{n-1}$
(B)	$\left(\sum_{0\le i<j\le n}\sum(i\,{}^nC_i + j\,{}^nC_j)\right)$ is equal to	(Q)	$\frac{1}{2}\ (2^{2n} - {}^{2n}C_n)$
(C)	$\sum_{0\le i<j\le n}\sum\left((i+j)({}^nC_i - {}^nC_j)^2\right)$ is equal to	(R)	$n\sum_{r=1}^{n} r\,{}^nC_r$
(D)	$\sum_{0\le i<j\le n}\sum {}^nC_i\ {}^nC_j$ is equal to	(S)	$n((n+1)\,{}^{2n}C_n - 2^{2n})$
		(T)	$\sum_{r=0}^{n-1} {}^{2n}C_r$

2.

	Column-I		Column-II
(A)	$2^{(32)^{32}}$ is divided by 7, then the remainder is	(P)	0
(B)	5^{99} is divided by 13, then the remainder is	(Q)	2
(C)	$(20)^{13} + (13)^{20}$ is divided by 9, then the remainder is	(R)	4
(D)	$(32)^{(32)^{32}}$ is divided by 7, then the remainder is	(S)	6
		(T)	8

3.

	Column-I		Column-II
(A)	If the fourth term in the expansion of $\left(\frac{x}{a}+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then a^2 is divisible by	(P)	2
(B)	$\sum_{p=1}^{4}\sum_{r=p}^{4} {}^4C_r\, {}^rC_p$ is divisible by	(Q)	4
(C)	The coefficient of x^{13} in $(1-x)^5(1+x+x^2+x^3)^4$ is divisible by	(R)	5
(D)	$\sum_{r=0}^{4} {}^4C_r(r-2)^2$ is divisible by	(S)	8
		(T)	13

4.

	Column-I		Column-II
(A)	The coefficient of x^4 in $(2-x+3x^2)^6$ is	(P)	1024
(B)	${}^{11}C_0\,{}^{22}C_{11} - {}^{11}C_1\,{}^{20}C_{11} + {}^{11}C_2\,{}^{18}C_{11} - {}^{11}C_3\,{}^{16}C_{11} + \ldots =$	(Q)	2048
(C)	${}^5C_1\,{}^5C_5 - {}^5C_2\,{}^{10}C_5 + {}^5C_3\,{}^{15}C_5 - {}^5C_4\,{}^{20}C_5 + {}^5C_5\,{}^{25}C_5 =$	(R)	990
(D)	The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^{11}$ is	(S)	3125
		(T)	3660

5.

	Column-I		Column-II
(A)	We can express $(\sqrt{2}-1)^{10}$ in the form $\sqrt{k}-\sqrt{k-1}$, where k is a positive integer (the square root need not be irrational) then $[\sqrt{k}] =$	(P)	1351
(B)	The number $(2+\sqrt{5})^7$ can be expressed in the form $\sqrt{k}+\sqrt{k+1}$, where k is a positive integer (the square root need not be irrational), then $[\sqrt{k}] =$	(Q)	2702
(C)	Let the last three digits of the number $(17)^{256}$ is N, then 6N =	(R)	3363
(D)	The number $(2+\sqrt{3})^6$ can be expressed in the form $k+\sqrt{k^2-1}$ where k is a positive integer, then k = (where [.] denotes greatest integer function)	(S)	4086
		(T)	12238

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. Let $S = {}^{31}C_1 - \left(1+\frac{1}{2}\right){}^{31}C_2 + \left(1+\frac{1}{2}+\frac{1}{3}\right){}^{31}C_3 - \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right){}^{31}C_4 + \ldots$

$+ S\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{31}\right){}^{31}C_{31}$, then $\frac{1}{S}$ is equal to

2. Let $x = (15+\sqrt{220})^{19} + (15+\sqrt{220})^{82}$, then the units digit of [x] is equal to ([.] denotes greatest integer function)

3. Let N be the integer next above $(\sqrt{3}+1)^{2018}$. The greatest integer 'p' such that $(16)^P$ divides N is equal to

4. The value of ${}^{50}C_6 - {}^{5}C_1\,{}^{40}C_6 + {}^{5}C_2\,{}^{30}C_6 - {}^{5}C_3\,{}^{20}C_6 + {}^{5}C_4\,{}^{10}C_6$ is equal to

5. The remainder obtained when $6^{2007} + 8^{2007}$ is divided by 49 is equal to

6. ${}^{10}C_0\,{}^{20}C_{10} - {}^{10}C_1\,{}^{18}C_{10} + {}^{10}C_2\,{}^{16}C_{10} - {}^{10}C_3\,{}^{14}C_{10} + {}^{10}C_4\,{}^{12}C_{10} - {}^{10}C_5\,{}^{10}C_{10}$ is equal to

7. Let the 3^{rd}, 4^{th}, 5^{th} and 6^{th} terms in the expansion $(1 + x)^n$ be a, b, c, d respectively. Then $\frac{b^2 - ac}{c^2 - bd} = \frac{m\,a}{n\,c}$ where m and n are coprime natural numbers, then m + n is equal to

8. The coefficient of x^5y^5 in the expansion of $((1 + x + y + xy)(x + y))^5$ is equal to

9. The number of rational terms in the expansion of $\left(\sqrt[3]{7}+\sqrt[9]{11}\right)^{6561}$ is equal to :

10. Find the value of $\sum_{i=1}^{6}\left(\sum_{j=i}^{6} {}^{6}C_j \, {}^{j}C_i\right)$

11. $\sum_{i=0}^{50}\left(\sum_{j=i+1}^{51} {}^{50}C_i \, {}^{51}C_j\right) = 2^n$, where n is equal to

12. Find the number of integral values of x for which only the fourth term in the expansion $\left(2+\frac{3x}{8}\right)^{10}$ has the maximum numerical value.

13. The coefficient of $x^{\frac{n^2+n-14}{2}}$ in $(x-1)(x^2-2)(x^3-3)(x^4-4)\ldots(x^n-n)$, $n \geq 30$ is equal to

14. The value of $\left\{\frac{2^{2020}}{28}\right\}$, where $\{\cdot\}$ denotes the fractional part function is equal to $\frac{p}{q}$, where p and q are relatively prime natural numbers, then p + q is equal to

15. Using the identity $\frac{1}{{}^{2n+1}C_r}+\frac{1}{{}^{2n+1}C_{r+1}}=\frac{2n+2}{2n+1}\frac{1}{{}^{2n}C_r}$, then value of $\sum_{r=1}^{19}\frac{(-1)^r r}{{}^{20}C_r}=-\frac{p}{q}$, where p and q are relatively prime natural numbers, then p + q is equal to

16. $\sum_{r=1}^{\left(\frac{3n}{2}\right)}(-3)^{r-1}\, {}^{3n}C_{2r-1} =$ (where n is an even natural number)

17. ${}^{n}C_0\, {}^{2n}C_n - {}^{n}C_1\, {}^{2n-1}C_n + {}^{n}C_2\, {}^{2n-2}C_n - {}^{n}C_3\, {}^{2n-3}C_n + \ldots + \ldots + (-1)^n\, {}^{n}C_n\, {}^{n}C_n =$

18. If $n \in N$, $n > 4$ and $(1-x^3)^n = \sum_{r=0}^{n} a_r x^r (1-x)^{3n-2r}$, then $\sum_{r=0}^{n} a_r = \lambda^n$, find λ.

19. The sum of all rational terms in the expansion $\left(\sqrt{2}+3^{\frac{1}{5}}\right)^{10}$ is equal to

20. The value of $\dfrac{(18^3+7^3+3\cdot18\cdot7\cdot25)}{(3^6+6\cdot243\cdot2+15\cdot81\cdot4+20\cdot27\cdot8+15\cdot9\cdot16+6\cdot3\cdot32+64}$ is equal to

21. The sum of all values of x for which the 6th term in the expansion of the binomial $\left(\sqrt{2^{\log_{10}(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_{10}3}}\right)^m$ is equal to 21, if it is known that the binomial coefficient of the 2nd, 3rd and 4th terms in the expansion represent respectively the first, third and fifth terms of an A.P.

22. For what x is the 4th term in the expansion of $\left[\left(5^{\frac{1}{3}}\right)^{-\frac{1}{2}\log_{10}(6-\sqrt{8x})}+\left(\dfrac{5^{\log_{10}(x-1)}}{25^{\log_{10}5}}\right)^{\frac{1}{6}}\right]^m$ is equal to $\dfrac{84}{5}$, if it is known that $\dfrac{14}{9}$ of binomial coefficient of 3rd term, binomial coefficient of 4th term and binomial coefficient of 5th term in the expansion constitute a G.P.

23. Find x in the binomial $\left(2^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)^x$ if the ratio of 7^{th} term from the beginning of binomial expansion to 7^{th} term from its end is $\dfrac{1}{6}$ $(x \in N)$.

24. Let $x = (5 + 2\sqrt{6})^n$, $n \in N$, then find the value of $x - x^2 + x[x]$, where [.] denotes greatest integer function.

25. Let $x = (\sqrt{2} + 1)^6$, then [x] = (where [.] denotes greatest integer function)

26. If the number of terms in the expansion of $\left(2+\frac{1}{x}+\frac{1}{x^2}\right)^n$ is 13, then the sum of the coefficients is equal to

27. The total number of terms that are dependent of x in the expansion $\left(x^2-2+\frac{1}{x^2}\right)^{53}$ is equal to

28. If the coefficient of x^2 + coefficient of x in the expanssion of

$(1 + x)^m (1 - x)^n$, $(m \neq n)$ is equal to $-m$, then the value of $n - m$ is equal to

Answer Key

Single choice Questions

1. D	**2.** B	**3.** C	**4.** B	**5.** A	**6.** B
7. D	**8.** B	**9.** A	**10.** D	**11.** B	**12.** D
13. A	**14.** C	**15.** A	**16.** B	**17.** B	**18.** B
19. C	**20.** A	**21.** B	**22.** C	**23.** D	**24.** C
25. D	**26.** D	**27.** B	**28.** B	**29.** D	**30.** C

One or More Than One Correct Questions

1. A,D	**2.** A,B,C,D	**3.** B,C	**4.** B,C,D	**5.** B,C,D	**6.** A,B,C
7. B,C	**8.** C,D	**9.** A,B	**10.** B,C	**11.** B,D	**12.** B,C,D
13. A,B,C,D	**14.** B,C,D	**15.** C,D	**16.** A,C,D	**17.** B,C	**18.** B,D
19. A,D	**20.** A,C	**21.** A,B,C,D	**22.** A,B,C	**23.** A,C,D	**24.** A,C
25. B,C,D	**26.** C,D	**27.** A,B,C	**28.** B,C,D	**29.** B,C,D	**30.** B,D

Comprehension Based Questions

1. A **2.** D **3.** B **4.** D **5.** C **6.** C
7. D **8.** A **9.** D **10.** B

Match The Column

1. A → Q, T ; B → P, R ; C → S ; D → Q, T

2. A → Q ; B → T ; C → P ; D → R

3. A → P, Q ; B → R,T ; C → P, Q ; D → P, Q, S

4. A → T ; B → Q ; C → S ; D → R

5. A → R ; B → T ; C → S ; D → P

Subjective Type Questions

1. 31 **2.** 9 **3.** 252 **4.** 2250000 **5.** 21 **6.** 1024
7. 8 **8.** 2252 **9.** 730 **10.** 665 **11.** 100 **12.** 2
13. 13 **14.** 53 **15.** 21 **16.** 0 **17.** 1 **18.** 4
19. 41 **20.** 1 **21.** 2 **22.** 2 **23.** 9 **24.** 1
25. 197 **26.** 4096 **27.** 106 **28.** 3

Previous Year Questions

SECTION-1

1. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals : **[AIEEE 2004]**

(A) $-\frac{5}{3}$ (B) $\frac{3}{5}$

(C) $\frac{-3}{10}$ (D) $\frac{10}{3}$

2. If $S_n = \sum_{r=0}^{n} 1/{}^nC_r$ and $t_n = \sum_{r=0}^{n} r/{}^nC_r$, then t_n/S_n is equal to **[AIEEE 2004]**

(A) $\frac{1}{2}n$ (B) $\frac{1}{2}n - 1$

(C) $n - 1$ (D) $\frac{2n-1}{2}$

3. If the coefficients of r^{th}, $(r+1)^{th}$, and $(r+2)^{th}$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation

[AIEEE 2005]

(A) $m^2 - m(4r-1) + 4r^2 - 2 = 0$ (B) $m^2 - m(4r+1) + 4r^2 + 2 = 0$

(C) $m^2 - m(4r+1) + 4r^2 - 2 = 0$ (D) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

4. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is **[AIEEE 2005]**

(A) ${}^{55}C_4$ (B) ${}^{55}C_3$

(C) ${}^{56}C_3$ (D) ${}^{56}C_4$

5. If the coefficient of x^7 in $[ax^2 + (1/bx)]^{11}$ equals the coefficient of x^{-7} in $[ax -(1/bx^2)]^{11}$, then a and b satisfy the relation **[AIEEE 2005]**

(A) $a - b = 1$ (B) $a + b = 1$

(C) $\frac{a}{b} = 1$ (D) $ab = 1$

6. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as **[AIEEE 2005]**

(A) $1-\frac{3}{8}x^2$ (B) $3x + \frac{3}{8}x^2$

(C) $-\frac{3}{8}x^2$ (D) $\frac{x}{2}-\frac{3}{8}x^2$

7. If the expansion of powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then a_n is **[AIEEE 2006]**

(A) $\frac{b^n - a^n}{b-a}$ (B) $\frac{a^n - b^n}{b-a}$

(C) $\frac{a^{n+1} - b^{n+1}}{b-a}$ (D) $\frac{b^{n+1} - a^{n+1}}{b-a}$

8. For natural numbers m and n, if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ...$, and $a_1 = a_2 = 10$, then (m, n) is **[AIEEE 2006]**

(A) (20, 45) (B) (35, 20)

(C) (45, 35) (D) (35, 45)

9. In the binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of the fifth and sixth terms is zero, then a/b equals **[AIEEE 2007]**

(A) $\frac{5}{n-4}$ (B) $\frac{6}{n-5}$

(C) $\frac{n-5}{6}$ (D) $\frac{n-4}{5}$

10. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10}$ is : **[AIEEE 2007]**

(A) $-{}^{20}C_{10}$ (B) $\frac{1}{2}\,{}^{20}C_{10}$

(C) 0 (D) ${}^{20}C_{10}$

11. **Statement 1 :** $\sum_{r=0}^{n}(r+1)\,{}^{n}C_r = (n+2) \times 2^{n-1}$ **[AIEEE 2008]**

Statement 2 : $\sum_{r=0}^{n}(r+1)\,{}^{n}C_r\, x^r = (1+x)^n + nx(1+x)^{n-1}$

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true ; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(D) Statement 1 is true, statement 2 is false.

12. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is **[AIEEE 2009]**

(A) 0 (B) 2

(C) 7 (D) 8

13. Let $S_1 = \sum_{j=1}^{10} j(j-1)\ {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j.\,{}^{10}C_j$, and $S_3 = \sum_{j=1}^{10} j^2.\,{}^{10}C_j$ **[AIEEE 2010]**

Statement 1 : $S_3 = 55 \times 2^9$.

Statement 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true ; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 2.

(D) Statement 1 is true, statement 2 is false.

14. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is **[AIEEE 2011]**

(A) 132 (B) 144

(C) –132 (D) –144

15. If n is a positive integer then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is **[AIEEE 2012]**

(A) an irrational number

(B) an odd positive integer

(C) an even positive integer

(D) a rational number other than positive integers

16. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is : **[JEE Main 2013]**

(A) 310 (B) 4

(C) 120 (D) 210

17. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to **[JEE Main 2014]**

(A) $\left(16, \frac{272}{3}\right)$ (B) $\left(16, \frac{251}{3}\right)$

(C) $\left(14, \frac{251}{3}\right)$ (D) $\left(14, \frac{272}{3}\right)$

18. The sum of coefficents of integral powers of x in the binomial expansion of $(1-2\sqrt{x})^{50}$ is : **[JEE Main 2015]**

(A) $\frac{1}{2}(2^{50}+1)$ (B) $\frac{1}{2}(3^{50}+1)$

(C) $\frac{1}{2}(3^{50})$ (D) $\frac{1}{2}(3^{50}-1)$

19. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **[JEE Main 2016]**

(A) 64 (B) 2187

(C) 243 (D) 729

20. The value of

$$({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots\ldots + ({}^{21}C_{10} - {}^{10}C_{10}) \text{ is}$$

[JEE Main 2017]

(A) $2^{20}-2^{10}$ (B) $2^{21}-2^{11}$

(C) $2^{21}-2^{10}$ (D) $2^{20}-2^{9}$

21. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is : **[JEE Main 2018]**

(A) 2 (B) –1

(C) 0 (D) 1

SECTION-2

1. If in the expansion of $(1 + x)^m (1 - x)^n$, the co-efficients of x and x^2 are 3 and -6 respectively, then m is : **[JEE '99, 2 (Out of 200)]**

(A) 6 (B) 9

(C) 12 (D) 24

2. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$

(C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

3. For any positive integers m, n (with $n \ge m$), let $\binom{n}{m} = {}^nC_m$. Prove that

$$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \dots\dots + \binom{m}{m} = \binom{n+1}{m+1}$$

Hence or otherwise prove that ,

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \dots\dots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}.$$

[JEE '2000 (Mains), 6]

4. Find the largest co-efficient in the expansion of $(1 + x)^n$, given that the sum of co-efficients of the terms in its expansion is 4096 . **[REE '2000 (Mains)]**

5. In the binomial expansion of $(a - b)^n$, $n \ge 5$, the sum of the 5th and 6th terms is zero. Then a/b equals **[JEE '2001 (Screening), 3]**

(A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$

(C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

6. Find the coeffcient of x^{49} in the polynomial **[REE '2001 (Mains) , 3]**

$$\left(x-\frac{C_1}{C_0}\right)\left(x-2^2\cdot\frac{C_2}{C_1}\right)\left(x-3^2\cdot\frac{C_3}{C_2}\right)\ldots\ldots\ldots\ldots\left(x-50^2\cdot\frac{C_{50}}{C_{49}}\right) \text{ where } C_r={}^{50}C_r.$$

7. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q}=0$ if $P<q$) is maximum when m is

[JEE '2002 (Screening), 3]

(A) 5 (B) 10

(C) 15 (D) 20

8. (a) Coefficient of t^{24} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is

[JEE 2003, Screening 3 out of 60]

(A) ${}^{12}C_6+2$ (B) ${}^{12}C_6+1$

(C) ${}^{12}C_6$ (D) none

(b) Prove that :

$$2^K\cdot\binom{n}{0}\binom{n}{K}-2^{K-1}\binom{n}{1}\binom{n-1}{K-1}+2^{K-2}\binom{n}{2}\binom{n-2}{K-2}\ldots\ldots(-1)^K\binom{n}{K}\binom{n-K}{0}=\binom{n}{K}.$$

[JEE 2003, Mains-2 out of 60]

9. ${}^{n-1}C_r=(K^2-3).{}^{n}C_{r+1}$, if $K\in$ **[JEE 2004 (Screening)]**

(A) $[-\sqrt{3},\sqrt{3}]$ (B) $(-\infty,-2)$

(C) $(2,\infty)$ (D) $(\sqrt{3},2]$

10. The value of $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}+\ldots\ldots+\binom{30}{20}\binom{30}{30}$ is, where $\binom{n}{r}$ $={}^{n}C_r$. **[JEE 2005 (Screening)]**

(A) $\binom{30}{10}$ (B) $\binom{30}{15}$

(C) $\binom{60}{30}$ (D) $\binom{31}{10}$

11. For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r (B_{10}B_r - C_{10}A_r)$ is equal to **[JEE 2010]**

(A) $B_{10} - C_{10}$
(B) $A_{10} (B_{10}^2 - C_{10}A_{10})$
(C) 0
(D) $C_{10} - B_{10}$

12. The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n = **[JEE Advanced 2013]**

13. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is : **[JEE Advanced 2014]**

(A) 1051
(B) 1106
(C) 1113
(D) 1120

14. The coefficient of x^9 in the expansion of $(1 + x) (1 + x^2) (1 + x^3) ..... (1 + x^{100})$ is **[JEE Advanced 2015]**

15. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + ... + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\ {}^{51}C_3$ for some positive integer n.

Then the value of n is : **[JEE Advanced 2016]**

16. Let $X = \left({}^{10}C_1\right)^2 + 2\left({}^{10}C_2\right)^2 + 3\left({}^{10}C_3\right)^2 + ... + 10\left({}^{10}C_{10}\right)^2$,

where ${}^{10}C_r$, $r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is ______. **[JEE Advanced 2018]**

Answer Key

Section-1

1. C	**2.** A	**3.** C	**4.** D	**5.** D	**6.** C
7. D	**8.** D	**9.** D	**10.** B	**11.** B	**12.** B
13. B	**14.** D	**15.** A	**16.** D	**17.** A	**18.** B
19. Bonus	**20.** A	**21.** A			

Section-2

1. C	**2.** D	**4.** $^{12}C_6$	**5.** B	**6.** – 22100	**7.** C
8. **(a)** A	**9.** D	**10.** A	**11.** D	**12.** 6	**13.** C
14. 8	**15.** 5	**16.** 646			

5 CHAPTER

PROBABILITY

SECTION-1

SINGLE CHOICE QUESTIONS

1. A letter is chosen at random out of 'ASSININE' and one is chosen at random out of 'ASSASSIN'. Find the probability that the same letter is chosen on both occasions.

(A) $\frac{7}{16}$ (B) $\frac{7}{32}$

(C) $\frac{5}{16}$ (D) $\frac{9}{32}$

2. From a bag containing 10 balls, 7 balls are drawn simultaneously and replaced, then 5 balls are drawn. Find the probability that exactly 3 balls are common to the two drawings.

(A) $\frac{5}{12}$ (B) $\frac{7}{12}$

(C) $\frac{5}{18}$ (D) $\frac{13}{21}$

3. A man can take a step forward, backward, left or right with equal probability. Find the probability that after nine steps he will be just one step away from his initial position.

(A) $\frac{21347}{4^9}$ (B) $\frac{3865}{4^7}$

(C) $\frac{12345}{4^9}$ (D) $\frac{3969}{4^7}$

4. In a multiple choice question, there are five alternative answers, of which one or more than one are correct. A candidate will get marks on the question if he ticks all the correct answers. If he decides to tick answers at random, how many minimum chances should he be allowed so that the probability of his getting marks on the question exceeds $\frac{1}{8}$.

(A) 4 (B) 5

(C) 7 (D) 8

5. The probability that an item produced by a factory is defective is 'p'. From a certain lot, a sample of 'n' items is drawn with replacement. If it contains no defective items, the lot is accepted, while if it has more than two defective items, the lot is rejected. If the sample has one or two defective items, an independent sample of 'm' items is drawn with replacement from the lot and combined with previous sample. If the combined sample does not contain more than two defective items, the batch is accepted. Find the probability that the batch is accepted.

(A) $(1 - p)^n + np(1 - p)^{n+m}$

(B) $(1 - p)^n + np(1 - p)^{n+m-1}$

(C) $(1 - p)^n + np(1 - p)^{n+m-2}(1 + (m - 1)p)) + {}^nC_2(1-p)^{m+n-2}p^2$

(D) $(1 - p)^{n-2}\left((1-p)^2 + \frac{n(n-1)p^2}{2}\right) + np(1 - p)^{n+m-2}(1+(m-1)p)$

6. Each packet of blades sold contains a coupon which is equally likely to bear the letters A, B or C. If 'm' packets are purchased, what is the probability that the coupons can not be to used to spell BAC.

(A) $\left(\frac{2}{3}\right)^m$ (B) $\frac{3.2^m - 2}{3^m}$

(C) $\frac{2^m}{3^{m-1}}$ (D) $\frac{2^m - 1}{3^{m-1}}$

7. Suppose m, n are real numbers randomly chosen in [0, 1]. Determine the probability that the distance between the roots of the equation $x^2 + mx + n = 0$ is not greater than 1.

(A) $\frac{2}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

8. If 'n' different things are distributed among x boys and y girls. Find the probability that the number of things received by girls is even.

(A) $1-\left(\frac{x-y}{x+y}\right)^n$ (B) $\frac{1}{2}-\frac{1}{2}\left(\frac{x-y}{x+y}\right)^n$

(C) $1-\frac{1}{2}\left(\frac{x-y}{x+y}\right)^n$ (D) $\frac{1}{2}+\frac{1}{2}\left(\frac{x-y}{x+y}\right)^n$

9. Starting at (0, 0) an object moves in coordinate plane by a sequence of steps, each of length one. Each step is left, right up or down, all four equally likely. Find the probability that the object reaches (2, 2) in six or fewer steps.

(A) $\frac{3}{64}$ (B) $\frac{1}{64}$

(C) $\frac{5}{64}$ (D) $\frac{5}{32}$

10. A natural number 'x' is chosen at random from the first 1000 natural numbers. If [·] denotes the greatest integer function, then the probability that

$\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31x}{30}$ is

(A) $\frac{31}{1000}$ (B) $\frac{4}{125}$

(C) $\frac{33}{1000}$ (D) $\frac{67}{1000}$

11. If P(A) = x, P(B) = y and x < y, then

(A) $P(A/B) \in \left[\frac{x-1}{y}, \frac{x}{y}\right]$ (B) $P(A/B) \in \left[\frac{x+y-1}{y}, \frac{x}{y}\right]$

(C) $P(A/B) \in \left[\frac{y-1}{y}, \frac{x}{y}\right]$ (D) $P(A/B) \in \left[\frac{x}{y}, \frac{x+1}{y}\right]$

12. If the papers of 4 students can be checked by any one of the 7 teachers. Then the probability that all 4 papers are checked by exactly 2 teachers is

(A) $\frac{18}{343}$ (B) $\frac{12}{343}$

(C) $\frac{2}{49}$ (D) $\frac{6}{49}$

13. A bag consists of n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty. Then the probability that each pair consists of one white and one red ball is

(A) $\frac{2^n n!}{{}^{2n}C_n}$ (B) $\frac{2^n}{{}^{2n}C_n}$

(C) $\frac{2^n}{(2n)!}$ (D) $\frac{2^n}{(2n)!n!}$

14. From (4m +1) tickets numbered as 1, 2, 3,.....,4m + 1; three tickets are chosen at random. Then the probability that the numbers are in A.P. with even common difference is

(A) $\frac{2(2m-1)}{3(16m^2-1)}$ (B) $\frac{3(2m-1)}{m(16m^2-1)}$

(C) $\frac{3(2m-1)}{(16m^2-1)}$ (D) $\frac{3(2m-1)}{2(16m^2-1)}$

15. Consider a bag containing 10 balls of which 4 are black and remaining white . Now 5 balls are drawn from this bag and put in another bag (without noting the colour of balls). Finally one ball is drawn from the second bag and it is found to be white. Find the probability that 2 white balls had been drawn in the first draw.

(A) $\frac{10}{63}$ (B) $\frac{20}{121}$

(C) $\frac{4}{21}$ (D) $\frac{13}{63}$

16. There are two bags each containing 10 different books. A student draws out any number of books (atleast one) from first bag as well as from the second bag. The probability that the difference between the number of books drawn from the two bags does not exceed two is

(A) $\frac{{}^{20}C_{10} + 2\left({}^{20}C_{11} + {}^{20}C_{12}\right) - 110}{(2^{10}-1)^2}$ (B) $\frac{{}^{20}C_{10} + 2\left({}^{20}C_{11} + {}^{20}C_{12}\right)}{(2^{10}-1)^2}$

(C) $\frac{{}^{20}C_{10} + {}^{20}C_{11} + 2\left({}^{20}C_{12}\right)}{(2^{10}-1)^2}$ (D) $\frac{{}^{20}C_{10} + 2\times\left({}^{20}C_{12}\right) - 111}{(2^{10}-1)^2}$

17. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement.

Then the probability that $\lim_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{2/x} = 6$ is

(A) $\frac{1}{18}$ (B) $\frac{5}{36}$

(C) $\frac{1}{9}$ (D) $\frac{1}{6}$

18. 'A' writes a letter to his friend 'B' and does not receive a reply. It is known that one out of 'n' letters does not reach its destination. The probability that 'B' didn't receive the letters is (It is certain that 'B' would have replied, if he had received the letter)

(A) $\frac{1}{2}$ (B) $\frac{1}{2n-1}$

(C) $\frac{2n-1}{n^2}$ (D) $\frac{n}{2n-1}$

19. India plays 2, 3 and 5 matches against Pakistan, Srilanka and Australia respectively. Probability that India win a match against Pakistan, Srilanka and Australia is 0.6, 0.5 and 0.4 respectively. If India win a match, then the probability that it was against Pakistan is

(A) $\frac{12}{47}$ (B) $\frac{13}{47}$

(C) $\frac{14}{47}$ (D) $\frac{15}{47}$

20. A certain kind of bacteria either die, split into two or split into three bacteria. All splits are exact copies. The probability of dying is $\frac{1}{4}$, the probability of splitting into two is $\frac{1}{2}$ and splitting into three is $\frac{1}{4}$.

The probability that it survives for infinite length of time is

(A) $\frac{5-\sqrt{13}}{2}$ (B) $\frac{4}{7}$

(C) $\frac{\sqrt{13}-3}{2}$ (D) $\frac{6-\sqrt{13}}{2}$

21. A fair die is thrown until a score of less than 5 points is obtained. The probability of obtaining not less than 2 points on the last throw is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{4}{5}$ (D) $\frac{1}{5}$

22. From an urn containing 5 white and 5 black balls. 5 balls are transferred at random into an empty second urn from which one ball is drawn and it is found to be white. Then the probability that all balls transferred from the first urn are white is

(A) $\frac{1}{122}$ (B) $\frac{1}{125}$

(C) $\frac{1}{126}$ (D) $\frac{1}{131}$

23. Two distinct numbers are selected from the numbers {1, 2, 3,....,9}, then the probability that their product is a perfect square is

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$

(C) $\frac{1}{3}$ (D) $\frac{4}{9}$

24. A bag contains three white, two blue and four red balls. If four balls are drawn one by one with replacement, then the probability that sample contains just one white ball is

(A) $\frac{16}{81}$ (B) $\frac{8}{81}$

(C) $\frac{32}{81}$ (D) $\frac{64}{81}$

25. In a multiple choice questions, there are 4 alternative answers of which one or more may be correct. A candidate will get marks in the question only if he ticks all the correct answers. The candidate decides to tick answers at random. If he is allowed upto 5 chances to answer the question, the probability that he will get the marks in the question is

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$

(C) $\frac{1}{15}$ (D) $\frac{2}{3}$

26. P_1, P_2, P_3 and P_4 are four players playing in a knockout tournament and probability that P_1 will pair up with P_i is proportional to i and P_m wins with P_n if $m < n$; then the probability that P_2 will reach the second round is

(A) $\frac{2}{9}$ (B) $\frac{7}{9}$

(C) $\frac{4}{5}$ (D) $\frac{2}{3}$

27. Three distinct numbers are selected from numbers {1, 2, 3,....,15}, then the probability that their sum is a perfect cube of an integer is

(A) $\frac{5}{91}$ (B) $\frac{8}{91}$

(C) $\frac{39}{455}$ (D) $\frac{9}{91}$

28. A fair coin is tossed (2m + 1) times, the probability of getting at least 'm' consecutive heads is

(A) $\frac{m+1}{2^m}$ (B) $\frac{m+3}{2^{m+1}}$

(C) $\frac{(m+1)2^m-1}{2^{2m+1}}$ (D) $\frac{(m+3)2^m-1}{2^{2m+1}}$

29. When we throw a dice 4 times, the probability that the minimum number appearing on the dice is 2 and the maximum is 5 is

(A) $\frac{55}{648}$ (B) $\frac{47}{648}$

(C) $\frac{35}{648}$ (D) $\frac{16}{81}$

30. A seven digit number of $a_1a_2a_3a_4a_5a_6a_7$ (all digits distinct) is formed randomly. The probability that number formed satisfy $a_1 > a_2 > a_3 < a_4 < a_5 < a_6 < a_7$ is

(A) $\frac{11}{1572}$ (B) $\frac{1}{168}$

(C) $\frac{7}{1512}$ (D) $\frac{5}{1512}$

31. An urn contains 20 white marbles, 30 blue marbles and 50 red marbles. Ten marbles are selected, one at a time, with replacement. Then the probability that at least one colour will be missing from the 10 selected marbles is

(A) $\frac{8^{10}+7^{10}+5^{10}-3^{10}-2^{10}}{10^{10}}$ (B) $\frac{8^{10}+7^{10}-3^{10}-2^{10}}{10^{10}}$

(C) $\frac{7^{10}+5^{10}-3^{10}-2^{10}}{10^{10}}$ (D) $\frac{8^{10}+7^{10}+5^{10}-3^{10}-2^{10}-1}{10^{10}}$

32. Let A, B be independent events with P(A) = P(B) and $P(A \cup B) = \frac{1}{2}$, then P(A) is equal to

(A) $1-\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}-1$ (D) $2-\sqrt{2}$

33. Events A and B belong to common sample space S and have probabilities P(A) = P(B) = $\frac{1}{3}$. It is also known that $P(\overline{A} \cap \overline{B}) = \frac{7}{18}$, then P(A/B) is equal to

(A) $\frac{1}{18}$ (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{6}$

34. A pack of cards is counted, face downwards, and it is found that one card is missing. Two cards are drawn and are found to be spades. Then the odds against the missing card being a spade is

(A) 39 : 11 (B) 28 : 11

(C) 36 : 11 (D) 50 : 11

35. Let {x, y} be a subset of the set {1, 2, 3,....,9, 10}. Then the probability that $|x - y| \leq 5$ is equal to

(A) $\frac{2}{9}$ (B) $\frac{5}{9}$

(C) $\frac{7}{9}$ (D) $\frac{8}{9}$

36. A die is rolled three times, the probability of getting larger number than the previous number is

(A) $\frac{1}{36}$ (B) $\frac{11}{216}$

(C) $\frac{5}{208}$ (D) $\frac{5}{54}$

37. An unbiased coin is tossed 12 times. The probability that at least 7 consecutive heads show up is

(A) $\frac{5}{256}$ (B) $\frac{1}{32}$

(C) $\frac{3}{128}$ (D) $\frac{7}{256}$

38. From a group of 'n' persons arranged in a circle 3 persons are selected at random. If the probability that no two adjacent persons are selected is $\frac{2}{7}$, then n =

(A) 7 (B) 8

(C) 9 (D) 10

39. Two integers *x* and *y* are chosen from the set {0, 1, 2, 3,....,2n–1, 2n}, with replacement. The probability that $|x - y| \le n$, $n \in N$ is

(A) $\frac{3n^2+1}{(2n+1)^2}$ (B) $\frac{3n^2+3n-1}{(2n+1)^2}$

(C) $\frac{3n^2+3n+1}{(2n+1)^2}$ (D) $\frac{3n(n+1)}{(2n+1)^2}$

40. A letter is to come from either LONDON or CLIFTON. The postal mark on the letter legibly shows consecutive letters 'ON'. What is the probability that the letter has come from LONDON.

(A) $\frac{12}{17}$ (B) $\frac{11}{17}$

(C) $\frac{5}{17}$ (D) $\frac{13}{17}$

41. A man parks his car among 'n' cars standing in a row. His car not being parked at an end. On his return he finds that exactly *m* of the 'n' cars are still there. What is the probability that both the cars parked on two sides of his car have left.

(A) $\frac{(n-m)(n-m-1)(n-m-2)}{n(n-1)(n-2)}$ (B) $\frac{(n-m-1)(n-m-2)}{n(n-1)}$

(C) $\frac{m(m-1)}{n(n-1)}$ (D) $\frac{(n-m)(n-m-1)}{(n-1)(n-2)}$

42. A bag contains 6 white and 6 black balls. One by one the balls are drawn from the bag with replacement. Then the probability that 3rd time a white ball is drawn in 7th draw is

(A) $\frac{15}{64}$ (B) $\frac{3}{64}$

(C) $\frac{5}{128}$ (D) $\frac{15}{128}$

43. Two distinct numbers a and b are selected randomly from the set $\{2, 2^2, 2^3, ..., 2^{25}\}$. then the probability that $\log_a b$ is an integer is equal to

(A) $\frac{1}{10}$ (B) $\frac{31}{300}$

(C) $\frac{8}{75}$ (D) $\frac{11}{100}$

44. Twenty persons arrive in a town having 3 hotels A, B and C. If each person randomly chooses one of these hotels, then what is the probability that atleast two of them go in hotel A, atleast one in hotel B and atleast one in hotel C. (each hotel has capacity for more than 20 guests)

(A) $1-\left(\frac{12.2^{20}-42}{3^{20}}\right)$ (B) $1-\left(\frac{2^{20}-1}{3^{19}}\right)$

(C) $1-\left(\frac{10.2^{20}-40}{3^{20}}\right)$ (D) $1-\left(\frac{13.2^{20}-43}{3^{20}}\right)$

45. 7 persons are stopped on the road at random and asked about their birthdays. The probability that 3 of them are born on Wednesday, 2 on Thursday and remaining two on Friday is equal to

(A) $\frac{141}{7^7}$ (B) $\frac{45}{7^7}$

(C) $\frac{90}{7^6}$ (D) $\frac{30}{7^6}$

46. 3 distinct numbers are chosen from the set of first 15 natural numbers, then the probability that the numbers are in A.P. is equal to

(A) $\frac{51}{455}$ (B) $\frac{7}{65}$

(C) $\frac{10}{91}$ (D) $\frac{53}{455}$

47. A fair die is tossed 16 times, then the probability of getting prime outcomes as many times in the first 8 throws as in the last 8 throws is equal to

(A) $\frac{^{16}C_8}{2^{16}}$ (B) $\frac{^{15}C_7}{2^{16}}$

(C) $\frac{^{16}C_8}{2^{17}}$ (D) $\frac{^{15}C_7}{2^{17}}$

48. Let A, B, C be three events such that $P(A \cap B) = 0$, $P(A \cup B) = 1$, $P(A \cap C) = \frac{1}{5}$ and $P(C) = \frac{7}{15}$, then $P(B \cap C)$ is equal to

(A) $\frac{7}{15}$ (B) $\frac{4}{15}$

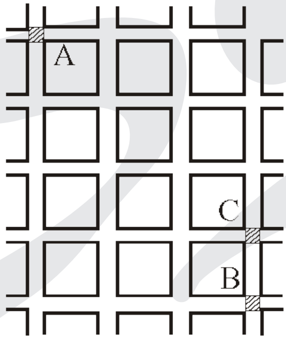
(C) $\frac{11}{15}$ (D) $\frac{1}{3}$

49. A person has to go through three successive tests. The probability of his passing first test is P. If he fails in one of the tests, then the probability of his passing next test is $\frac{P}{2}$, otherwise it remains the same. For selection, the person must pass at least two tests. The probability that the person is selected is

(A) $2P - P^3$ (B) $2P^2 - P^3$

(C) $P - P^3$ (D) $P^2 - P^3$

50. The adjoining figure is a map of a part of a city. The small rectangles are blocks and the spaces in between are streets. Each morning a student walks from intersection A to intersection B, always walking along the streets shown, always going east or south. For variety, at each intersection where he has choice, he chooses with probability $\frac{1}{2}$ (independent of all other choices) whether to go east or south. Find the probability that, on any given morning, he walks through intersection C.



(A) $\frac{2}{7}$ (B) $\frac{3}{7}$

(C) $\frac{1}{2}$ (D) $\frac{4}{7}$

51. A man throws a fair coin a number of times and get 2 points for each head he throws and 1 point for each tail. The probability that he gets exactly 6 points is

(A) $\frac{21}{32}$ (B) $\frac{23}{32}$

(C) $\frac{41}{64}$ (D) $\frac{43}{64}$

52. Two numbers x and y are chosen at random without replacement from the set {1, 2, 3, 4, ..., 100}. Then the probability that $x^4 - y^4$ is divisible by 5 is

(A) $\frac{67}{99}$ (B) $\frac{334}{495}$

(C) $\frac{62}{99}$ (D) $\frac{37}{99}$

53. A man sent 7 letters to his 7 friends. The letters are kept in the addressed envelopes at random. The probability that 3 friends receive correct letters and 4 letters go to wrong destinations is equal to

(A) $\frac{1}{8}$ (B) $\frac{1}{16}$

(C) $\frac{1}{24}$ (D) $\frac{1}{4}$

54. Three numbers are selected one by one at random without replacement from the set of numbers {1, 2, 3,....., n}. Then the probability that the third number lies between the first two, if the first number is known to be smaller than the second is equal to :

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

55. An unbaised coin is tossed. If the result is head, a pair of unbaised dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, card is drawn from a well shuffled pack of eleven cards numbered 2, 3, 4, 5,.....,11, 12 is picked and the number on the card is noted. Then the probability that the noted number is 7 or 8 is equal to

(A) $\frac{195}{792}$ (B) $\frac{185}{792}$

(C) $\frac{191}{792}$ (D) $\frac{193}{792}$

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. If two squares are chosen at random on a chess board, then find the probability so that

(A) they have a side common is $\frac{1}{9}$
(B) they have a side common is $\frac{1}{18}$
(C) they have contact at a corner is $\frac{7}{144}$
(D) they have contact at a corner is $\frac{7}{72}$

2. Of three independent events E_1, E_2, E_3, the chance that only E_1 occurs is a, that only E_2 occurs is b, and only E_3 occurs is c and probability of none of E_1, E_2, E_3 occuring is x. Then

(A) $P(E_2) = \frac{x}{b+x}$
(B) $P(E_2) = \frac{b}{b+x}$
(C) $x^2 = (a+x)(b+x)(c+x)$
(D) $P(E_3) = \frac{x}{c+x}$

3. $P_1, P_2, P_3, ..., P_8$ are 8 players participating in a tournament. If $i < j$, then P_i will win the match against P_j. Players are paired up randomly for first round and winners of this round again paired up for the second and so on. Then the probability that

(A) P_4 reaches the final is $\frac{4}{35}$
(B) P_6 reaches the final is 0
(C) P_1 wins the tournament is 1
(D) P_6 reaches the semifinal is $\frac{2}{7}$

4. In a city, a person own independently a sedan car with probability $\frac{3}{10}$ and a SUV with probability $\frac{4}{10}$. If he has sedan only, then he keeps a driver with probability $\frac{6}{10}$, whereas if he owns SUV only, then he keeps a driver with probability $\frac{7}{10}$, whereas if he keeps both type of cars then his probability of keeping a driver is $\frac{9}{10}$. Then

(A) Probability that person keeps a driver is $\frac{412}{1000}$

(B) Probability that person keeps a driver is $\frac{71}{125}$

(C) Given that person keeps driver, then probability that he owns SUV is $\frac{54}{103}$

(D) Given that person keeps driver, then probability that he owns SUV is $\frac{76}{103}$

5. Cards are drawn one by one at random without replacement from a well shuffled pack of 52 cards until 2 aces are obtained for the first time. The probability that 18 draws are required for this is

(A) $\frac{{}^{48}C_{16} \times {}^{4}C_{2}}{{}^{52}C_{17}} \times \frac{1}{35}$

(B) $\frac{{}^{48}C_{16} \times {}^{4}C_{1}}{{}^{52}C_{17}} \times \frac{3}{35}$

(C) $\frac{1}{78}$

(D) $\frac{17 \times {}^{34}C_{2}}{{}^{52}C_{4}}$

6. The probability that a student passes at least one of the three examinations A, B, C is $\frac{3}{4}$. The probability that he passes atleast two of them is $\frac{1}{2}$ and the probability he passes exactly two of them is $\frac{2}{5}$. If a, b, c are the probabilities of the student passing in A, B, C respectively, then

(A) $abc = \frac{1}{10}$

(B) $a + b + c = \frac{27}{20}$

(C) $a + b + c = \frac{23}{20}$

(D) $abc = \frac{1}{20}$

7. Let $A_1, A_2, \dots, A_n$ be independent events of the same sample space, then $P(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)$ is equal to

(A) $\prod_{i=1}^{n}(1 - P(A_i))$

(B) $1 - \prod_{i=1}^{n}(1 - P(A_i))$

(C) $1 - P(\overline{A_1 \cap A_2 \cap A_3 \cap \dots\dots \cap A_n})$

(D) $1 - P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap \dots\dots \cap \overline{A}_n)$

8. A, B, C are alternately and in this order, rolling a fair die. The first one to roll an even number wins and the game is ended, then

(A) $P(A) = \frac{4}{7}$ (B) $P(C) = \frac{1}{7}$

(C) $P(B)P(C) = \frac{1}{14}$ (D) $P(B) = \frac{3}{7}$

9. A fair die is rolled 'n' times. If P(n) denotes the probability that there are atleast two equal numbers among the result obtained, then

(A) $P(2) = \frac{1}{6}$ (B) $P(4) = \frac{13}{18}$

(C) $P(6) = \frac{319}{324}$ (D) $P(3) = \frac{4}{9}$

10. The probability of getting number of even numbered outcomes more than number of odd numbered outcomes by throwing a fair die 16 times is equal to

(A) $1 - \frac{^{16}C_8}{2^{16}}$ (B) $\frac{1}{2} - \frac{^{16}C_8}{2^{17}}$

(C) $\frac{1}{2} - \frac{^{15}C_7}{2^{16}}$ (D) $1 - \frac{^{15}C_7}{2^{15}}$

11. Ram has 4 bats namely B_1, B_2, B_3, B_4. He randomly selects one of the three bats which was not used in previous match. If for his first match the chosen bat was B_1 and P(n) denotes the probability that he chooses B_1 in n^{th} match, then

(A) $P(3) = \frac{1}{3}$ (B) $P(5) = \frac{7}{27}$

(C) $P(6) = \frac{10}{81}$ (D) $P(7) = \frac{61}{243}$

12. A certain coin lands head with probability P. Let Q denote the probability that when the coin is tossed four times, the number of heads obtained is even. Then

(A) There is no value of P, if $Q = \frac{1}{4}$.

(B) There is exactly one value of P if $Q = \frac{3}{4}$.

(C) There are exactly two values of P, if $Q = \frac{3}{5}$.

(D) There are exactly four values of P if $Q = \frac{4}{5}$.

13. A and B throw alternately with a pair of dice. A wins if he throws a sum 6 before B throws 7 and B wins if he throws a 7 before A throws 6. If A starts the game, then

(A) Probability that A wins is $\frac{31}{61}$

(B) Probability that B wins is $\frac{31}{61}$

(C) Probability that A wins is $\frac{30}{61}$

(D) Probability that B wins is $\frac{30}{61}$

14. Consider the system of equations $ax + by = 0$ and $cx + dy = 0$ where $a, b, c, d \in \{1, 2\}$. Then the probability that the system of equations has

(A) unique solution is $\frac{1}{2}$

(B) unique solution is $\frac{5}{8}$

(C) non trivial solutions is $\frac{3}{8}$

(D) non trivial solutions is $\frac{1}{2}$

15. A, B, C are three events for which $P(A) = 0.4$, $P(B) = 0.6$, $P(C) = 0.5$, $P(A \cup B) = 0.75$, $P(A \cap C) = 0.35$ and $P(A \cap B \cap C) = 0.2$. If $P(A \cup B \cup C) \geq 0.75$, then $P(B \cap C)$ can take values

(A) 0.15

(B) 0.2

(C) 0.3

(D) 0.4

16. If n different objects are distributed among 'n+2' persons, then

(A) Probability that exactly 2 persons will get nothing is $\frac{(n+1)!}{2(n+2)^{n-1}}$

(B) Probability that exactly 2 persons will get nothing is $\frac{(n+1)!}{(n+2)^{n-1}}$

(C) Probability that exactly 3 persons will get nothing is $\frac{{}^{n+2}C_3 {}^{n}C_2 (n-1)!}{(n+2)^n}$

(D) Probability that exactly 3 persons will get nothing is $\frac{(n+1)n^2(n-1)^2}{12(n+2)^{n-1}}$

17. Three missiles A, B and C whose probabilities of hitting the target are $\frac{2}{3}, \frac{1}{3}, \frac{2}{5}$ respectively are shot at an enemy ship simultaneously. A majority of hits is required to destroy the ship. If the ship is destroyed, then

(A) the probability that missile B failed to hit the target is $\frac{1}{5}$

(B) the probability that missile B failed to hit the target is $\frac{2}{5}$

(C) the probability that at least one of B or C hit the target is 1

(D) the probability that at least one of the missile B or C failed to hit the target is 7/10

SECTION-3

COMPREHENSION TYPE QUESTIONS

Comprehension (Q.1 to Q.3) :

A party of 'n' men of whom A, B are two are sitting in a row. what is the chance that

1. A, B are next to one another

(A) $\frac{1}{n(n-1)}$ (B) $\frac{2}{n(n-1)}$ (C) $\frac{1}{n}$ (D) $\frac{2}{n}$

2. Exactly 'm' men are between them

(A) $\frac{2(n-m)}{n(n-1)}$ (B) $\frac{2(n-m-1)}{n(n-1)}$ (C) $\frac{2(n-m-2)}{n(n-1)}$ (D) $\frac{2}{n-m}$

3. Not more than 'm' men are between them

(A) $\frac{(m+1)(2n-m-2)}{n(n-1)}$ (B) $\frac{(n-m-2)(n-m-1)}{n(n-1)}$

(C) $\frac{(n-m)(n-m-1)}{n(n-1)}$ (D) $\frac{m(n-2m-2)}{n(n-1)}$

Comprehension (Q.4 to Q.5) :

The values of a and b are equally possible in the square $|a| \le 1$, $|b| \le 1$. Consider the events

A = { The roots of quadratic expression $x^2 + 2ax + b$ are real }

B = { The roots of $x^2 + 2ax + b$ are positive }.

4. P(A) =

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{3}{8}$

5. P(B) =

(A) $\frac{3}{8}$ (B) $\frac{2}{9}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

COMPREHENSION (Q.6 TO Q.7) :

A player throws an ordinary die with faces numbered 1 to 6. Whenever he throws a 1, he gets a further throw. Let the sum of numbers obtained is their score.

6. The probability of obtaining a total score of exactly 'r' , where $2 \le r \le 6$ is

(A) $\frac{1}{5}\left(\frac{1}{6}\right)^{r-1}$ (B) $\frac{5}{6}\left(1-\frac{1}{6^r}\right)$ (C) $\frac{1}{5}\left(1-\frac{1}{6^r}\right)$ (D) $\frac{1}{5}\left(1-\frac{1}{6^{r-1}}\right)$

7. The probability of obtaining a total score of exactly 'r' where $r > 6$ is

(A) $\frac{1}{5}(6^5-1)\left(\frac{1}{6}\right)^{r-1}$ (B) $\frac{1}{5}\left(\left(\frac{1}{6}\right)^{r-5}-\left(\frac{1}{6}\right)^{r-1}\right)$

(C) $\frac{5}{6}\left(\left(\frac{1}{6}\right)^{r-6}-\left(\frac{1}{6}\right)^{r-1}\right)$ (D) $\frac{1}{5}\left(\frac{1}{6}\right)^{r-6}$

COMPREHENSION (Q.8 TO Q.9) :

A die is rolled and probability of showing any number is directly proportional to that number. If prime number appears then a ball is chosen from urn A containing 2 white and 3 black balls other wise a ball is chosen from urn B containing 3 white and 2 black balls. Then

8. The probability of drawing a black ball is

(A) $\frac{49}{105}$ (B) $\frac{10}{21}$ (C) $\frac{51}{105}$ (D) $\frac{52}{105}$

9. If a white ball is drawn, then the probability that it is from urn B is

(A) $\frac{32}{53}$ (B) $\frac{33}{53}$ (C) $\frac{35}{53}$ (D) $\frac{36}{53}$

COMPREHENSION (Q.10 TO Q.11) :

Initially a bag was known to contain some one rupee ('0' or more) and some fifty paisa ('0' or more) coins.

In all bag was known to have 4 coins. Two coins were randomly drawn from the bag and both found to be one rupee coin. (initially all number of rupee coins in the bag are equiprobable)

10. If these coins are replaced, what is the probability the next drawn coin is fifty paisa coin.

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{3}{8}$

11. If these coins are not replaced, what is the probability the next drawn coin is fifty paisa coin

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{3}{8}$

COMPREHENSION (Q.12 TO Q.14) :

A cube having all of its sides pointed is cut by two horizontal two vertical and other two also vertical in other direction so as to form 27 equal cubes. Of these cubes, a cube is randomly selected.

12. The probability that the cube selected has none of its side pointed is

(A) $\frac{2}{27}$ (B) $\frac{1}{9}$ (C) 0 (D) $\frac{1}{27}$

13. The probability that the cube selected has two sides pointed is

(A) $\frac{1}{27}$ (B) $\frac{2}{9}$ (C) $\frac{4}{9}$ (D) $\frac{8}{27}$

14. The probability that the cube selected has only one side pointed is

(A) $\frac{2}{9}$ (B) $\frac{8}{27}$ (C) $\frac{4}{9}$ (D) $\frac{1}{9}$

COMPREHENSION (Q.15 TO Q.16) :

There are two die D_1 and D_2 both having six faces. D_1 has 3 faces marked with 1, 2 faces marked with 2 and 1 face marked with 3. D_2 has 1 face marked with 1, 2 faces marked with 2 and 3 faces makred with 3. Both dice are thrown once. Let P(x) be the probability of getting the sum equal to x. Then

15. P(3) =

(A) $\frac{1}{12}$ (B) $\frac{1}{9}$ (C) $\frac{5}{36}$ (D) $\frac{2}{9}$

16. Which is the correct order

(A) $P(2) < P(6) < P(3) < P(4) < P(5)$

(B) $P(6) = P(2) < P(3) = P(5) < P(4)$

(C) $P(6) = P(2) < P(5) < P(3) < P(4)$

(D) $P(6) = P(2) < P(3) < P(5) < P(4)$

COMPREHENSION (Q.17 TO Q.19) :

A chess match between two grand masters X and Y is won by whoever first wins a total of two games. X's probability of winning, drawing or losing any particular game are p, q, r respectively. The games are independent and p + q + r = 1, then

17. The probability that X wins the match after (n) games (n ≥ 5) is

(A) $n\,p^2q^{n-3}(q + (n-1)r)$

(B) $(n-1)p^2\,q^{n-2}$

(C) $p^2(n-2)q^{n-3}(q + (n-1)r)$

(D) $p^2q^{n-3}(n-1)\,(q + (n-2)r)$

18. The probability that Y wins the match after 6^{th} game

(A) $5q^3r^2(q + 4p)$ (B) $6q^3r^2(q + 5p)$ (C) $4q^3r^2(q + 5p)$ (D) $5q^4r^2$

19. The probability that Y wins the match is

(A) $\frac{r^2(3p+r)}{(1-q)^3}$ (B) $\frac{r^2(2p+r)}{(1-q)^3}$ (C) $\frac{r^2(p+3r)}{(1-q)^3}$ (D) $\frac{r^2(p+2r)}{(1-q)^3}$

COMPREHENSION (Q.20 TO Q.22) :

A biased coin, for which probability of getting head is $\frac{2}{3}$ and that of tail is $\frac{1}{3}$, is tossed till the difference of number of head and tail is *r* and this event is denoted by A_r, $r \geq 2$.

20. If r = 2, then the probability that the game ends with more number of heads than tails is

(A) $\frac{4}{9}$ (B) $\frac{2}{9}$ (C) $\frac{7}{9}$ (D) $\frac{4}{5}$

21. For r = 2, it is given that game ends with a head then the probability it ends in minimum number of throws is

(A) $\frac{4}{5}$ (B) $\frac{5}{9}$ (C) $\frac{4}{9}$ (D) $\frac{7}{9}$

22. If B is the event that last two throws show either two consecutive head or tail, then $P(B/A_r)$ is

(A) $\frac{4}{5}$ (B) $1-\left(\frac{5}{9}\right)^r$ (C) $1-\left(\frac{4}{9}\right)^r$ (D) 1

COMPREHENSION (Q.23 TO Q.24) :

A simple board game has four fields A, B, C and D. Once you end up on field A you have won and once you end up on field B, you have lost. From fields C and D you move to other fields by tossing a coin. If you are on field C and you throw a head, then you move to field A, otherwise to field D. From field D, you move to field C if you throw a head, and otherwise you move to field B.

23. Suppose that you start in field D, then the probability that you will win is

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

24. Suppose that you start in field C; then the probability that you will win is

(A) $\frac{5}{6}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$

COMPREHENSION (Q.25 TO Q.26) :

Let $A = \{1, 2, 3, 4, \ldots\ldots, 25\}$ and $B = \{x, y\}$, $B \subset A$. Then

25. The probability that $x^2 - y^2$ is divisible by 7 is equal to

(A) $\frac{7}{30}$ (B) $\frac{71}{300}$ (C) $\frac{18}{75}$ (D) $\frac{73}{300}$

26. The probability that $x^2 - y^2$ is divisible by 5 is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{5}$

COMPREHENSION (Q.27 TO Q.29) :

Starting at (0, 0) an object moves in coordinate plane by a sequence of steps, each of length one. Each step is left, right, up or down, all four are equally likely. Find the probability that the object reaches (2, 2) in

27. Exactly 4 steps

(A) $\frac{3}{128}$ (B) $\frac{5}{256}$ (C) $\frac{3}{64}$ (D) $\frac{7}{128}$

28. Exactly 6 steps

(A) $\frac{3}{128}$ (B) $\frac{5}{256}$ (C) $\frac{3}{64}$ (D) $\frac{7}{128}$

29. Exactly 5 steps

(A) $\frac{3}{128}$ (B) $\frac{1}{32}$ (C) $\frac{5}{128}$ (D) 0

COMPREHENSION (Q.30 TO Q.31) :

Each face of a cube is to be coloured with exactly one colour from blue, green or red. Then the probability that

30. No. two adjacent faces are painted with the same colour is equal to

(A) $\frac{4}{243}$ (B) $\frac{2}{729}$ (C) $\frac{2}{243}$ (D) $\frac{2}{81}$

31. All the colours are used to paint the cube given that each pair of opposite faces are painted with a different colour is equal to

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{4}{9}$ (D) $\frac{8}{9}$

COMPREHENSION (Q.32 TO Q.33) :

A bag contains 3 red, 3 blue and 4 white balls. A ball is drawn at random and is replaced back together with a ball of a different colour (which can only be red, blue or white) so that there are now 11 balls in the bag. Then

32. The probability that the number of white balls will remain greater than the number of red balls is equal to

(A) $\frac{7}{20}$ (B) $\frac{2}{5}$ (C) $\frac{11}{20}$ (D) $\frac{13}{20}$

33. If another ball is drawn from the bag containing 11 balls and found to be red, then the probability that the first ball was also red is equal to

(A) $\frac{12}{61}$ (B) $\frac{16}{63}$ (C) $\frac{16}{67}$ (D) $\frac{18}{67}$

COMPREHENSION (Q.34 TO Q.36) :

Let a and b be two numbers chosen at random from the set {1, 2, 3,....,n} with replacement. Let $P_n(p)$ denotes the probability that $a^{p-1} - b^{p-1}$ is divisible by p, where p is a prime number. Then

34. Let [·] represents greatest integer function, then $P_n(p)$ is equal to

(A) $1-\frac{1}{n}\left[\frac{n}{p}\right]+\frac{2}{n^2}\left[\frac{n}{p}\right]^2$

(B) $1-\frac{2}{n}\left[\frac{n}{p}\right]+\frac{1}{n^2}\left[\frac{n}{p}\right]^2$

(C) $1+\frac{1}{n}\left[\frac{n}{p}\right]+\frac{2}{n^2}\left[\frac{n}{p}\right]^2$

(D) $1-\frac{2}{n}\left[\frac{n}{p}\right]+\frac{2}{n^2}\left[\frac{n}{p}\right]^2$

35. $P_{25}(3)$ is equal to

(A) $\frac{353}{625}$ (B) $\frac{357}{625}$ (C) $\frac{361}{625}$ (D) $\frac{364}{625}$

36. $\lim\limits_{n\to\infty} P_n(p)$ is equal to

(A) $1-\frac{2}{p}+\frac{2}{p^2}$ (B) $1+\frac{1}{p}+\frac{2}{p^2}$ (C) $1-\frac{2}{p}+\frac{1}{p^2}$ (D) $1-\frac{1}{p}+\frac{2}{p^2}$

COMPREHENSION (Q.37 TO Q.39) :

There are two sets of well shuffled pack of 52 cards namely set P and set Q such that each set contains 52 cards.

37. Two cards are drawn from set P and two cards are drawn from set Q, then the probability that exactly one card is identical among the cards drawn is equal to

(A) $\frac{25}{663}$ (B) $\frac{50}{663}$ (C) $\frac{1}{13}$ (D) $\frac{32}{867}$

38. Two cards are drawn from set P and other two are drawn from set Q, then the probability that exactly one of them is a card of heart is equal to

(A) $\frac{301}{876}$ (B) $\frac{245}{576}$ (C) $\frac{247}{578}$ (D) $\frac{251}{578}$

39. If both the sets are mixed together such that now there are 104 cards and 3 cards are drawn at random, then the probability that they all belong to same set is equal to.

(A) $\frac{25}{103}$ (B) $\frac{25}{206}$ (C) $\frac{51}{206}$ (D) $\frac{27}{103}$

Comprehension (Q.40 to Q.41) :

There are 'n' blacks balls and 2 white balls in a bag. An uninvolved person Mr. C draws balls one by one from the bag without replacement. Mr. A wins as soon as 2 black balls are drawn and Mr. B wins as soon as 2 white balls are drawn. Let A(n) and B(n) represent the probability that Mr. A and Mr. B wins respectively. Then

40. The value of $\lim_{n\to\infty}$ (A(2)A(3)A(4)......A(n)) is equal to

(A) $\frac{1}{5}$ (B) $\frac{1}{10}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

41. The value of $\lim_{n\to\infty}$ (B(1) + B(2) + B(3) + ... + B(n)) is equal to

(A) 1 (B) 2 (C) 3 (D) 5

Comprehension (Q.42 to Q.43) :

A slip of paper is given to A, who marks it with either a plus sign or a minus sign, the probability of his writing a plus sign is $\frac{1}{3}$. He then passes the slip to B, who may leave it unchanged or change the sign before passing it to C.
Next C passes the slip to D after perhaps changing the sign. Finally D passes it to an honest judge after perhaps changing the sign. It is known that B, C, D each change the sign with probability $\frac{2}{3}$.

42. The probability that judge see a plus sign on the slip is equal to

(A) $\frac{4}{9}$ (B) $\frac{38}{81}$ (C) $\frac{40}{81}$ (D) $\frac{41}{81}$

43. If the judge see a plus sign on the slip, then the probabilitythat A originally wrote a plus sign is equal to

(A) $\frac{13}{41}$ (B) $\frac{14}{41}$ (C) $\frac{13}{40}$ (D) $\frac{11}{40}$

Comprehension (Q.44 to Q.46) :

An urn contains 3 white balls, 5 black balls and 2 red balls. Two persons draw balls in turn, without replacement. The first person to draw a white ball wins the game. If a red ball is drawn, the game is a tie. Then

44. The probability that player who begins the game is the winner is equal to

(A) $\frac{67}{168}$ (B) $\frac{333}{840}$

(C) $\frac{73}{210}$ (D) $\frac{83}{210}$

45. The probability that the second participant is the winner is equal to

(A) $\frac{43}{210}$ (B) $\frac{87}{420}$

(C) $\frac{77}{420}$ (D) $\frac{11}{70}$

46. The probability that the game is a tie is equal to

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

SECTION-4

MATCH THE COLUMN :

1. In a tournament, there are twelve players $S_1, S_2, \ldots, S_{12}$ and divided into six pairs at random. From each game a winner is decided on the basis of a game played between the two players of the pair. Assuming all the players are of equal strength, then match the following :

	Column-I		Column-II
(A)	Probability that S_2 is among the losers is	(P)	$\frac{5}{2}$
(B)	Probability that exactly one of S_3 and S_4 is among the losers is	(Q)	$\frac{1}{2}$
(C)	Probability that both S_2 and S_4 are among the winners is	(R)	$\frac{6}{11}$
(D)	Probability that S_4 and S_5 not playing against each other is	(S)	$\frac{10}{11}$
		(T)	$\frac{3}{11}$

2. Triangle is formed by joining the vertices of a cube.

	Column-I		Column-II
(A)	Probability that the triangle is isosceles or equilateral is greater than	(P)	$\frac{3}{14}$
(B)	Probability that the triangle is isosceles but not equilateral is smaller than	(Q)	$\frac{2}{7}$
(C)	Probability that the triangle is scalene is greater than	(R)	$\frac{3}{7}$
(D)	Probability that the triangle is right angled is greater than or equal to	(S)	$\frac{4}{7}$
		(T)	$\frac{6}{7}$

3. There are 16 equally skilled players $S_1, S_2, S_3, ...,S_{16}$ playing a knockout tournament. They are divided into 8 pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair, and they enter into the next round. the tournament proceed in the similar way.

	Column-I		Column-II
(A)	Probability that exactly one of S_1 & S_2 is among the 4 winners of 2[nd] round is	(P)	$\frac{1}{30}$
(B)	The probability that S_1 wins the tournament given that S_2 enters in the semifinals is	(Q)	$\frac{1}{20}$
(C)	The probability that S_1 wins the tournament given that S_2 enters into final is	(R)	$\frac{1}{10}$
(D)	The probability that S_1 is among the 4 winners of 2nd round given that S_2 wins the first round is	(S)	$\frac{7}{30}$
		(T)	$\frac{2}{5}$

4.

	Column-I		Column-II
(A)	A 7 digit number is formed using any 7 distinct digits selected from 1, 2, 3,....,9. The probability that it is divisible by 9 is less than or equal to	(P)	$\frac{1}{16}$
(B)	A pack of cards contain 4 aces, 4 kings, 4 queens and 4 jacks. Two cardsare drawn, then probability that there is atleast one ace is greater than or equal to	(Q)	$\frac{1}{9}$
(C)	Integers m and n are chosen at random between 1 and 98 (both inclusive).Then the probability that $7^m + 3^n$ is greater than or equal to	(R)	$\frac{1}{5}$
(D)	A coin is tossed 5 times. The probability that the result in the fifth toss is different from all the first four tosses is less than or equal to	(S)	$\frac{1}{4}$
		(T)	$\frac{9}{20}$

5. There are 6 pairs of children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to 6 children, and then the left gloves also are distributed to them at random. Then

	Column-I		Column-II
(A)	The probability that no child gets a matching pair is	(P)	$\frac{11}{30}$
(B)	The probability that every body get a matching pair is	(Q)	$\frac{19}{30}$
(C)	The probability that exactly one child gets a matching pair is	(R)	$\frac{691}{720}$
(D)	Atleast 2 children gets matching pairs is	(S)	$\frac{1}{720}$
		(T)	$\frac{53}{144}$

6. Three distinct numbers are selected from $\{1, 2, 3, 4, 5, 6\}$. Then the probability that

	Column-I		Column-II
(A)	they are in A.P. is equal to	(P)	$\frac{1}{20}$
(B)	they are in G.P. is equal to	(Q)	$\frac{1}{10}$
(C)	they are in H.P. is equal to	(R)	$\frac{3}{20}$
(D)	they form the 3 sides of a triangle is equal to	(S)	$\frac{3}{10}$
		(T)	$\frac{7}{20}$

7. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12. Four balls are drawn at random from the bag without replacement. Then

	Column-I		Column-II
(A)	The probability that all the four balls are black is equal to	(P)	$\frac{1}{5}$
(B)	If the bag contains 10 black and 2 white balls, then the probability that all four balls are black is equal to	(Q)	$\frac{70}{429}$
(C)	The probability that two balls are black and two balls are white is equal to	(R)	$\frac{1}{4}$
(D)	If all the four balls are black, then the probability that the bag contains 10 black balls is equal to	(S)	$\frac{14}{33}$

8. Five unbaised cubical dice are rolled simultaneously. Let 'm' and 'n' respectively denote the smallest and the largest number appearing on the upper faces of the dice. Now match the probabilities given in the column-II corresponding to the events given in column-I.

	Column-I		Column-II
(A)	P(m = 3) is equal to	(P)	$\left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^4 + \left(\frac{1}{3}\right)^5$
(B)	P(n = 4) is equal to	(Q)	$\left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5 + \left(\frac{1}{3}\right)^5$
(C)	P(m = 2 and n = 5) is equal to	(R)	$\left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$
(D)	P(2 ≤ m ≤ 4) is equal to	(S)	$\left(\frac{5}{6}\right)^5 - \left(\frac{1}{2}\right)^5$
		(T)	$\left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$

9. A box contains 25 tickets, each with a different number from 1 to 25. Four tickets are drawn at random and without replacement. Let a,b, c, d be the numbers of the tickets drawn.

	Column-I		Column-II
(A)	The probability that min (a, b, c, d) < 10 is equal to	(P)	$\frac{189}{2530}$
(B)	The probability that the second smallest number chosen is 10 is equal to	(Q)	$\frac{286}{575}$
(C)	The probability that the sum of a + b + c + d is odd is equal to	(R)	$\frac{207}{230}$
(D)	The probability that the product of abcd is even is equal to	(S)	$\frac{217}{230}$
		(T)	$\frac{1083}{1265}$

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. If a, b, c are numbers obtained by throwing a dice thrice, such that $a^2 + b^2 + c^2 \leq ab + bc + ca$, then the probability that a point (a, b, c) lies inside the tetrahedran formed by coordinate planes and $x + y + z = 12$ is p. Then 16p is equal to

2. Let $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are chosen respectively by throwing a die three times. The probability that f(x) is an strictly increasing function $\forall x \in R$ is equal to $\frac{a}{b}$, where a, b are coprime natural numbers, then b – a is equal to

3. 8 players $P_1, P_2, ...P_8$ of equal strength play in a knockout tournament. Assuming that players in each round are paired randomly. Then the probability that the player P_1 losses to the eventual winner is $\frac{m}{n}$, where m, n are relatively prime natural numbers, then n-m is equal to

4. A pack of playing card consist of 51 cards. Cards are drawn one by one without replacement. If first 4 cards drawn consist of only one face card, then the probability that the missing card being face card is $\frac{a}{b}$, where a, b are relatively prime natural numbers. Then b – 4a is equal to

5. A 7 digit number with all digits distinct is randomly formed using digits {1, 2, 3,....7}. The probability that formed number is such that product of any four consecutive digits is divisible by 10 is P, then the value of [10P] is (where [·] denote greatest integer function)

6. On each evening a boy either watches Doordarshan channel or Ten sports. The probability that he watches Ten sports is $\frac{4}{5}$. If he watches Doordarshan, there is a probability of $\frac{3}{4}$ that he will fall asleep, while it is $\frac{1}{4}$ when he watches Ten sports. On one evening, the boy is found to be asleep while watching TV. The probability that the body watched Doordarshan is $\frac{m}{n}$, where m, n are coprime, then $|m - n|$ is equal to

7. A fair coin is fossed 10 times and the outcomes are listed. Let E_i be the event that the i^{th} outcome is a head and E_m be the event that the list contains exactly 'm' heads. If E_i and E_m are independent, then m is equal to

8. A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all black. If the probability that the missed one is red is equal to $\frac{p}{q}$ where p, q are relatively prime natural numbers, then p + q is equal to

9. In an organisation number of women are μ times that of men. If α things are to be distributed among them. Then the probability that the number of things received by men are odd is $\frac{1}{2}-\left(\frac{1}{2}\right)^{\alpha+1}$. Find μ

10. A positive integer is randomly selected. Let the probability that $x^{n+1} - x^n + 1$ is divisible by $x^2 - x + 1$ be P, then the value of $\frac{1}{P}$ is

11. If p, q are chosen randomly with replacement from the set {1, 2, 3,.......,9, 10}. The probability that the roots of the equation $x^2 + px + q = 0$ are real is equal to $\frac{m}{n}$, where m, n are relatively prime natural numbers then $|m - n|$ is equal to

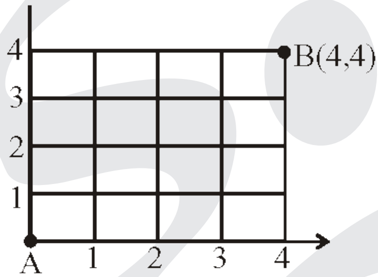
12. If all the letters of the word 'MATHEMATICS' are arranged arbitrarily. The probability that C comes before E, E before H, H before I and I before S is $\frac{1}{N}$, then N is equal to

13. If {x, y} is a subset of the first 30 natural numbers. Then the probability that $x^3 + y^3$ is divisible by 3 is equal to $\frac{m}{n}$ where m, n are relatively prime natural numbers, then m + n is equal to

14. In a tournament, there are five teams participating such that each team plays one game with every other team. Each team has a 50% chance of winning any game it plays. (There are noties). Then the probability that the tournament will produces neither an undefeated team nor a winless team is $\frac{m}{n}$, where m, n are coprime natural numbers. Then m + n is equal to

15. A box contains 3 red balls, 4 white balls and 3 blue balls. Balls are drawn from the box one at a time, at random, without replacement. Then the probability that all three red balls will be drawn before any white ball is obtained is $\frac{m}{n}$, where m, n are coprime natural numbers, then n – m is equal to :

16. Two persons P and Q are respectively located at A(0, 0) and B(4, 4) in the given figure. They start moving simultaneously toward each other and at a speed of one segment per minute. 'P' moves either to the right or up and 'Q' moves either left or down. The probability that they will meet on their path is $\frac{m}{n}$, where m, n are relatively prime natural numbers, then m + n is equal to :



17. A fair red die has three faces numbered 1, two faces numbered 2, and one faces numbered 3. A fair blue die has one face numbered 1, two faces numbered 2, and three faces numbered 3. The probability of the sum which is most likely to occur upon throwing both the dice is $\frac{m}{n}$, where m, n are relatively prime natural numbers, then m + n is equal to

18. A tetrahedral fair dice whose faces are numbered 1, 2, 3 and 4. The die is thrown and the face on which the pyramid lands is considered the 'winning' number. If the dice is thrown four times and the scores noted. The probability of obtaining a score of 10 is $\frac{m}{n}$ where m, n are relatively prime natural numbers, then m + n is equal to

19. A fair coin is flipped 15 times and it is observed that there are 10 heads and 5 tails. By a run of heads we mean a consecutive sequence of heads. For example, the sequence of outcomes

HHTHTHHHHHHTHTT

has four runs of heads, the first has 2 heads, the second 1 head, the third 6 heads and the fourth 1 head. The probability that there are 4 runs of heads is $\frac{m}{n}$ where m, n are relatively prime natural numbers, then m + n is equal to

20. 200 moviegoers quene up for tickets at the box office. The price of admission is Rs. 100 and the cashier has no change initially. Of the moviegoers, 100 have rupee 100 note only and 100 have rupee 200 note only. The probability that all of the moviegoers can collect their tickets without facing any problem of change is $\frac{1}{p}$, then p is equal to

21. A bag has 10 balls, 6 balls are drawn in an attempt and replaced. Then another draw of 5 balls is made from the bag, the probability that exactly two balls are common to both the draws is $\frac{m}{n}$, where m, n are coprime natural numbers, then n is equal to

22. 32 players ranked 1 to 32 are playing a knochout tournament. Assume that in every match between two players, the better ranked player wins. The probability that ranked 1 and ranked 2 players are winner and runner up respectively is $\frac{m}{n}$ where m, n are relatively prime natural numbers, then m + n is equal to

23. Let a bag contains 5 white and 10 black balls and 4 balls are selected randomly from it and kept aside. Now a next draw of a ball is made. Then the probability that this drawn ball is white is $\frac{m}{n}$, where m, n are coprime natural numbers then m + n is equal to :

24. In the above problem, the drawn 4 balls are put in a empty bag. Now a ball is drawn randomly from this new bag and it is found white. The probability that this was the only white which was transferred from first bag to second one is $\frac{m}{n}$, where m, n are coprime natural numbers, then n – m is equal to :

Answer Key

Single Choice Questions:

1. B	2. A	3. D	4. A	5. C	6. D
7. C	8. D	9. A	10. C	11. B	12. D
13. B	14. D	15. B	16. D	17. C	18. D
19. A	20. A	21. B	22. C	23. A	24. C
25. A	26. B	27. A	28. D	29. A	30. D
31. B	32. A	33. D	34. A	35. C	36. D
37. D	38. B	39. C	40. A	41. D	42. D
43. B	44. D	45. D	46. B	47. A	48. B
49. B	50. D	51. D	52. A	53. B	54. C
55. D					

One or More Than One Correct Questions

1. B,C	2. B,C	3. B,C,D	4. A,C	5. B,D	6. A,B
7. B,D	8. A,BC	9. A,B,C,D	10. B,C	11. A,B,D	12. A,C
13. B,C	14. B,C	15. A,B,C	16. A,C,D	17. B,C,D	

Comprehension Type Questions

1. D	2. B	3. A	4. A	5. D	6. D
7. A	8. D	9. B	10. C	11. B	12. D
13. C	14. A	15. D	16. B	17. D	18. A
19. A	20. D	21. B	22. D	23. C	24. D
25. D	26. A	27. A	28. A	29. D	30. C
31. D	32. D	33. D	34. D	35. A	36. A
37. B	38. C	39. A	40. B	41. C	42. D
43. A	44. D	45. A	46. B		

Match the Column

1. (A) → (Q) ; (B) → (R) ; (C) → (P) ; (D) → (S)
2. (A) → (P, Q, R) ; (B) → (S, T) ; (C) → (P, Q) ; (D) → (P, Q, R, S, T)
3. (A) → (T) ; (B) → (Q) ; (C) → (P) ; (D) → (S)
4. (A) → (Q, R, S, T) ; (B) → (P, Q, R, S, T) ; (C) → (P, Q, R, S) ; (D) → (P)
5. (A) → (T) ; (B) → (S) ; (C) → (P) ; (D) → (R)
6. (A) → (S) ; (B) → (P) ; (C) → (Q) ; (D) → (T)
7. (A) → (P) ; (B) → (S) ; (C) → (P) ; (D) → (Q)
8. (A) → (T) ; (B) → (T) ; (C) → (P) ; (D) → (R)
9. A → T ; B → P ; C → Q ; D → S

Subjective Type Questions

1. 8	**2.** 5	**3.** 5	**4.** 4	**5.** 9	**6.** 4
7. 5	**8.** 5	**9.** 3	**10.** 6	**11.** 19	**12.** 120
13. 4	**14.** 49	**15.** 34	**16.** 163	**17.** 25	**18.** 75
19. 203	**20.** 101	**21.** 21	**22.** 47	**23.** 4	**24.** 61

Previous Year Questions

SECTION-1

1. A multiple choice examinaton has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is: **[IIT JEE Main 2013]**

(A) $\frac{10}{3^5}$ (B) $\frac{17}{3^5}$

(C) $\frac{13}{3^5}$ (D) $\frac{11}{3^5}$

2. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A.

Then the events A and B are **[IIT JEE Main 2014]**

(A) Independent and equally likely.
(B) Mutually exclusive and independent.
(C) Equally likely but not independent.
(D) Independent but not equally likely.

3. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : **[IIT JEE Main 2015]**

(A) $22\left(\frac{1}{3}\right)^{11}$ (B) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

(C) $55\left(\frac{2}{3}\right)^{10}$ (D) $220\left(\frac{1}{3}\right)^{12}$

4. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ? **[IIT JEE Main 2016]**

(A) E_1 and E_2 are independent
(B) E_2 and E_3 are independent
(C) E_1 and E_3 are independent
(D) E_1, E_2 and E_3 are independent

5. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is : **[IIT JEE Main 2017]**

(A) 4
(B) $\frac{6}{25}$
(C) $\frac{12}{5}$
(D) 6

6. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10} ; then the probability that their sum as well as absolute difference are both multiple of 4, is : **[IIT JEE Main 2017]**

(A) $\frac{14}{45}$
(B) $\frac{7}{55}$
(C) $\frac{6}{55}$
(D) $\frac{12}{55}$

7. For three events A, B and C, P(Exactly one of A or B occurs)
= P(Exactly one of B or C occurs)
= P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is : **[IIT JEE Main 2017]**

(A) $\frac{7}{64}$
(B) $\frac{3}{16}$
(C) $\frac{7}{32}$
(D) $\frac{7}{16}$

8. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is : **[JEE Main 2018]**

(A) $\frac{3}{4}$ (B) $\frac{3}{10}$

(C) $\frac{2}{5}$ (D) $\frac{1}{5}$

SECTION-2

1. (a) If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$

(C) $\frac{1}{8}$ (D) $\frac{1}{49}$

(b) The probability that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% hance of passing in exactly two, which of the following relations are true?

(A) $p + m + c = \frac{19}{20}$ (B) $p + m + c = \frac{27}{20}$

(C) $pmc = \frac{1}{10}$ (D) $pmc = \frac{1}{4}$

(c) Eight players P_1, P_2, P_3, ...P_8 play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final. **[JEE ' 99, 2 + 3 + 10 (out of 200)]**

2. Four cards are drawn from a pack of 52 playing cards. Find the probability of drawing exactly one pair. **[REE'99, 6]**

3. A coin has probability ' p ' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that,

$$p_1 = 1, \; p_2 = 1 - p^2 \; \& \; p_n = (1 - p)\, p_{n-1} + p\,(1 - p)\, p_{n-2}, \text{ for all } n \geq 3.$$

[JEE ' 2000 (Mains), 5]

4. A and B are two independent events. The probability that both occur simultaneously is 1/6 and the probability that neither occurs is 1/3. Find the probabilities of occurance of the events A and B separately.

[REE ' 2000 (Mains), 3]

5. Two cards are drawn at random from a pack of playing cards. Find the probability that one card is a heart and the other is an ace. **[REE ' 2001 (Mains), 3]**

6. (a) An urn contains 'm' white and 'n' black balls. A ball is drawn at random and put back into the urn along with K additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white.

(b) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and he list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list.

[JEE ' 2001 (Mains), 5 + 5]

7. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **[JEE '2002 (mains)]**

8. (a) A person takes three tests in succession. The probability of his passing the first test is p, that of his passing each successive test is p or p/2 according as he passes or fails in the preceding one. He gets selected provided he passes at least two tests. Determine the probability that the person is selected.

(b) In a combat, A targets B, and both B and C target A. The probabilities of A, B, C hitting their targets are 2/3 , 1/2 and 1/3 respectively. They shoot imultaneously and A is hit. Find the probability that B hits his target whereas C oes not. **[JEE' 2003, Mains-2 + 2 out of 60]**

9. (a) Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(A) $\frac{4}{25}$ (B) $\frac{4}{35}$

(C) $\frac{4}{55}$ (D) $\frac{4}{1155}$

(b) If A and B are independent events, prove that $P(A \cup B) \cdot P(A' \cap B') \le P(C)$, where C is an event defined that exactly one of A or B occurs.

(c) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast 4 balls are white. Find the probability that in the next two draws exactly one white ball is drawn (leave the answer in terms of ${}^{n}C_{r}$). **[JEE 2004, 3 + 2 + 4]**

10. (a) A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even number of trials is **[JEE 2005 (Scr)]**

(A) 5/11 (B) 5/6

(C) 6/11 (D) 1/6

(b) A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car.

[JEE 2005 (Mains), 2]

COMPREHENSION (Q.11 TO Q.13) :

There are n urns each containing $n + 1$ balls such that the i^{th} urn contains i white balls and $(n + 1 - i)$ red balls. Let u_i be the event of selecting i^{th} urn, $i = 1, 2, 3, ..., n$ and w denotes the event of getting a white ball.

11. (a) If $P(u_i) \propto i$ where $i = 1, 2, 3,....., n$ then $\lim\limits_{n\to\infty} P(w)$ is equal to

(A) 1 (B) 2/3

(C) 3/4 (D) 1/4

(b) If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to

(A) $\dfrac{2}{n+1}$ (B) $\dfrac{1}{n+1}$

(C) $\dfrac{n}{n+1}$ (D) $\dfrac{1}{2}$

(c) If n is even and E denotes the event of choosing even numbered urn

$(P(u_i) = \dfrac{1}{n})$, then the value of $P(w/E)$, is **[JEE 2006, 5 marks each]**

(A) $\dfrac{n+2}{2n+1}$ (B) $\dfrac{n+2}{2(n+1)}$

(C) $\dfrac{n}{n+1}$ (B) $\dfrac{1}{n+1}$

12. (a) One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(A) 1/2 (B) 1/3

(C) 2/5 (D) 1/5

(b) Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c \mid G)$ equals

(A) $P(E^c) + P(F^c)$ (B) $P(E^c) - P(F^c)$

(C) $P(E^c) - P(F)$ (D) $P(E) - P(F^c)$

(c) Let $H_1, H_2, \ldots\ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

Statement-1: $P(H_i / E) > P(E / H_i) \cdot P(H_i)$ for $i = 1, 2, \ldots, n$.

because

Statement-2: $\sum_{i=1}^{n} P(H_i) = 1$ **[JEE 2007, 3+3+3]**

(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

13. (a) An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8 (B) 3, 6, or 9

(C) 4 or 8 (D) 5 or 10

(b) Consider the system of equations

$$ax + by = 0,\ cx + dy = 0, \text{ where } a, b, c, d \in \{0, 1\}.$$

STATEMENT-1 : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

and

STATEMENT-2 : The probability that the system of equations has a solution is 1. **[JEE 2008, 3+3]**

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explana tion for statement-1

(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

COMPREHENSION (Q.14 TO Q.16) : **[JEE 2009]**

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

14. The probability that x = 3 equals

(A) $\frac{25}{216}$
(B) $\frac{25}{36}$
(C) $\frac{5}{36}$
(D) $\frac{125}{216}$

15. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$
(B) $\frac{25}{36}$
(C) $\frac{5}{36}$
(D) $\frac{5}{216}$

16. The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$
(B) $\frac{25}{216}$
(C) $\frac{5}{36}$
(D) $\frac{5}{36}$

17. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is **[JEE 2010]**

(A) $\frac{1}{18}$
(B) $\frac{1}{9}$
(C) $\frac{2}{9}$
(D) $\frac{1}{36}$

18. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is 3/4. If the signal received at station B is green, then the probability that the original signal was green is **[JEE 2010]**

(A) $\frac{3}{5}$
(B) $\frac{6}{7}$
(C) $\frac{20}{23}$
(D) $\frac{9}{20}$

COMPREHENSION (Q.19 TO Q.20) : **[JEE 2011]**

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

19. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$

(C) $\frac{19}{30}$ (D) $\frac{11}{30}$

20. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$

(C) $\frac{15}{23}$ (D) $\frac{12}{23}$

21. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then **[JEE 2011]**

(A) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}$, $P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}$, $P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}$, $P(F) = \frac{4}{5}$

22. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ?

[JEE 2012]

(A) $P\left[x_1^c \middle| X\right] = \frac{3}{16}$

(B) P (Exactly two engines of the ship are functioning $|X) = \frac{7}{8}$

(C) $P[X|X_2] = \frac{5}{16}$

(D) $P[X|X_1] = \frac{7}{16}$

23. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1, 2, 3, 4, 5, and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on D_1, D_2 and D_3 is : **[JEE 2012]**

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$

(C) $\frac{125}{216}$ (D) $\frac{127}{216}$

24. Let X and Y be two events such that $P(X \mid Y) = \frac{1}{2}$, $P(Y \mid X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$.

Which of the following is (are) correct? **[JEE 2012]**

(A) $P(X \cup Y) = \frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) $P(X^C \cap Y) = \frac{1}{3}$

25. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is : **[JEE 2013]**

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$

(C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Comprehension (Q.26 to Q.27) **[JEE 2013]**

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red ball and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

26. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$

(C) $\frac{65}{181}$ (D) $\frac{55}{181}$

27. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$

(C) $\frac{558}{648}$ (D) $\frac{566}{648}$

28. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1)

Then $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ **[JEE 2013]**

29. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is :

[IIT JEE Advance 2014]

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

COMPREHENSION (Q.30 TO Q.31) **[IIT JEE ADVANCE 2014]**

Box 1 contains three cards bearing numbers 1, 2, 3 ; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 ; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_1 be the number on the card drawn from the i^{th} box, i = 1, 2, 3.

30. The probability that $x_1 + x_2 + x_3$ is odd, is :

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$

(C) $\frac{57}{105}$ (D) $\frac{1}{2}$

31. The probability that x_1, x_2, x_3 are in an arithmetic progression, is :

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$

(C) $\frac{11}{105}$ (D) $\frac{7}{105}$

32. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is **[IIT JEE Advance 2015]**

COMPREHENSION (Q.33 TO Q.34) **[IIT JEE ADVANCE 2015]**

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

33. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$
(B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$
(D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

34. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

(A) $n_1 = 4$ and $n_2 = 6$
(B) $n_1 = 2$ and $n_2 = 3$
(C) $n_1 = 10$ and $n_2 = 20$
(D) $n_1 = 3$ and $n_2 = 6$

35. Let X and Y be two events such that $P(X) = \frac{1}{3}, P(X \mid Y) = \frac{1}{2}$ and $P(Y \mid X) = \frac{2}{5}$. Then

[IIT JEE Advance 2017]

(A) $P(Y) = \frac{4}{15}$
(B) $P(X' \mid Y) = \frac{1}{2}$
(C) $P(X \cap Y) = \frac{1}{5}$
(D) $P(X \cup Y) = \frac{2}{5}$

36. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x + y + z = 10.0$ Then the probability that z is even, is :

[IIT JEE Advance 2017]

(A) $\frac{36}{55}$
(B) $\frac{6}{11}$
(C) $\frac{5}{11}$
(D) $\frac{1}{2}$

Comprehension (Q.37 to Q.38) :

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i , i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats. **[JEE Advanced 2018]**

37. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her is

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$

(C) $\frac{7}{40}$ (D) $\frac{1}{5}$

38. For i = 1, 2, 3, 4 let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$

(C) $\frac{7}{60}$ (D) $\frac{1}{5}$

Answer Key

Section-1

1. D	**2.** D	**3.** B	**4.** D	**5.** C	**6.** C
7. D	**8.** C				

Section-2

1. **(a)** A **(b)** B, C **(c)** 4/35

2. 0.304 **4.** $\frac{1}{2}$ & $\frac{1}{3}$ or $\frac{1}{3}$ & $\frac{1}{2}$

5. $\frac{1}{26}$ **6.** **(a)** $\frac{m}{m+n}$; **(b)** $\frac{{}^6C_3\left(3^n - 3.2^n + 3\right)}{6^n}$ **7.** $\frac{9m}{m+8N}$

8. (a) $p^2(2-p)$; (b) 1/2

9. (a) D , **(c)** $\dfrac{{}^{12}C_2\,{}^6C_4\,{}^{10}C_1\,{}^2C_1 + {}^{12}C_1\,{}^6C_5\,{}^{11}C_1\,{}^1C_1}{{}^{12}C_2\left({}^{12}C_2\,{}^6C_4 + {}^{12}C_1\,{}^6C_5 + {}^{12}C_0\,{}^6C_6\right)}$

10. **(a)** A, **(b)** $\dfrac{1}{7}$

11. (a) B, (b) A, (c) B

12. **(a)** C; **(b)** C; **(c)** D

13. **(a)** D, **(b)** B		**14.** A	**15.** B	**16.** D	**17.** C
18. C	**19.** B	**20.** D	**21.** AD	**22.** B, D	**23.** A
24. A, B	**25.** A	**26.** D	**27.** A	**28.** 6	**29.** A
30. B	**31.** C	**32.** 8	**33.** A, B		
34. C, D	**35.** AB	**36.** B	**37.** A	**38.** C	

6 CHAPTER

MATRICES

SECTION-1

SINGLE CHOICE QUESTIONS

1. If A and B are symmetric matrices of the same order and P = AB + BA and Q = AB – BA, then $(PQ)^T$ is equal to

(A) PQ (B) QP

(C) –PQ (D) –QP

2. If $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, then $A^{-1} + A^2 =$

(A) 3A – I (B) 3A + I

(C) 2A – I (D) 2A + I

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{bmatrix}$, then |adj(adj 2A)| is equal to

(A) 2^8 (B) 2^{12}

(C) 2^{13} (D) 2^{14}

4. If matrix $A_\lambda = \begin{bmatrix} \lambda+1 & \lambda-2 \\ \lambda-1 & \lambda \end{bmatrix}$, $\lambda \in N$, then $|A_1| + |A_2| + \ldots\ldots + |A_{300}|$ is equal to

(A) $(300)^2$ (B) $2(300)^2$

(C) $4(300)^2$ (D) $(301)^2$

5. If matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, $xyz = 50$ and $8x + 4y + 3z = 30$, then A(adjA) is equal to

(A) 16I (B) 48I

(C) 64I (D) 32I

6. Let M and N be two 3 × 3 non singular symmetric matrix such that MN = NM. If P^T denotes the transpose of a matrix P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2 (B) $-M^2$

(C) MN (D) NM

7. Let A and B be square matrices of same order satisfying AB = A and BA = B, then A^2B^2 equals.

(A) A (B) B

(C) I (D) Null matrix

8. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and $B = \sum_{r=1}^{2017} A^r$, then |B| is equal to

(A) 2017 (B) –2017

(C) 0 (D) 2015

9. If A, B are symmetric matrices of the same order then AB – BA is

(A) skew symmetric (B) Symmetric

(C) Orthogonal (D) Null matrix

10. If A and B are two square matrices which satisfy AB = A and BA = B, then $(A + B)^6$ is equal to

(A) 6(A + B) (B) 36(A + B)

(C) 32(A + B) (D) 64(A + B)

11. Let A and B are square matrices of same order and A is non singular matrix, then $(A^{-1}BA)^n$, $n \in N$, $n > 5$ is equal to

(A) $A^{-1}B^nA$
(B) $A^{-1}BA$
(C) B^n
(D) $A^{-n}B^nA^n$

12. If for a matrix A, $A^2 = A$ and $B = I - A$, then $AB + BA + I - (I - A)^2$ is equal to

(A) I
(B) A
(C) I + A
(D) I – A

13. A is a matrix such that $A^2 = A$. If $(I + A)^n = I + \lambda A$, $n \in N$, then λ is equal to

(A) 2^n
(B) $2^n - 2$
(C) $2^n - 1$
(D) n

14. If A is a diagonal matrix of order 3 × 3 is commutative with every square matrix of order 3 × 3 and trace(A) = 18, then |A| is equal to

(A) 6
(B) 36
(C) 216
(D) 27

15. If A and B are two matrices of order 3 such that AB = 0 and $A^2 + B = I$, then $|A^2 + B^2|$ is equal to :

(A) 0
(B) 1
(C) 9
(D) 3

16. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $AA^T = I$, then x + y is equal to :

(A) –3
(B) –2
(C) –1
(D) 0

17. If $A^TBA = O$, where A is 3×1 column matrix, then the value of $|B|$ is equal to :

(A) 1
(B) –1
(C) 3
(D) 0

18. If $A = \begin{bmatrix} 5 & 109 \\ 1 & 22 \end{bmatrix}$, then $|A^{100} - 5A^{99}|$ is equal to :

(A) 0
(B) 1
(C) 107
(D) –109

19. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $|A + A^2 + A^3 + A^4 + A^5|$ is equal to :

(A) 1
(B) 25
(C) 30
(D) 32

20. Let a non singular square matrix A satisfy $A^3 - 8A = O$, then the inverse of matrix $A^2 + I$ is equal to :

(A) $\frac{1}{9}(A^2 + 3I)$
(B) $\frac{1}{9}(I - A^2)$
(C) $\frac{1}{3}(8I - A^2)$
(D) $\frac{1}{9}(9I - A^2)$

21. A is a real, non zero, skew symmetric matrix of order 3×3, λ is a non zero scalar and X be a non zero column matrix such that $AX = \lambda X$, then λ must be.

(A) An integer
(B) A rational
(C) An irratioinal
(D) Imaginary

22. If A and B are two square matrices of the same order and

$(A + B)^m = {}^mC_0A^m + {}^mC_1A^{m-1}B + {}^mC_2A^{m-2}B^2 + ... + {}^mC_{m-1}AB^{m-1} + {}^mC_mB^m$

(m is a positive integer), then

(A) $AB = BA$
(B) $AB + BA = O$
(C) $A^m = O, B^m = O$
(D) $|A| = 0$ or $|B| = 0$

23. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $C = ABA^T$, then A^TC^nA is equal to :

$(n \in N, n > 5)$

(A) $\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 \\ -n & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 \\ -n & 0 \end{bmatrix}$

24. If A is a skew symmetric matrix, then

$$B = (I - A)(I + A)^{-1} \text{ is}$$

(A) Idempotent matrix

(B) Involutary matrix

(C) Orthogonal matrix

(D) Symmetric matrix

25. The number of 2 × 2 matrices A, having real numbers as elements satisfying

$A + A^T = I$ and $AA^T = I$

(A) 0

(B) 1

(C) 2

(D) Infinite

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. Let B be the set of all possible 2 × 2 matrices A of integer entries such that $AA^T = I$,

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then

(A) The number of matrices in set B is 4.

(B) The number of matrices in set B is 8.

(C) The number of matrices in set B such that $|A - I| \neq 0$ is 2.

(D) The number of matrices in set B such that $|A - I| \neq 0$ is 3

2. If A and B are square matrices such that $AB = \lambda A + \mu B$, where λ, μ are non zero real constants, then

(A) $A - \mu I$ is invertible

(B) $AB = BA$

(C) $AB = -BA$

(D) $B - \lambda I$ is invertible

3. Let A and B are two orthogonal matrices of order 3, then

(A) $(AB)^n$ is orthogonal, if n is odd

(B) $(AB)^n$ is orthogonal if n is even

(C) $\left||AB|B\right|$ can be 1 or –1 only

(D) $\left||AB|A\right|$ can be 1 or –1 only

4. Consider three square matrices A, B and C each of order n, where n is odd satisfying $A^T = A - B$ and $B^T = B - C$, then

(P^T denotes transpose of matrix P)

(A) $|C| = 2^n |B|$

(B) $|A + B| = |A - 2B|$

(C) $|A - C| = |A + 2B|$

(D) $|B| = 0$

5. Let $A = \begin{bmatrix} a & 1 \\ b & c \end{bmatrix}$ be a matrix with non zero real entries such that $A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

If $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ such that area of triangle OPQ $= \frac{1}{2}$, where

$P(x_1, y_1)$, $Q(x_2, y_2)$ and '0' is origin, then

(A) $|A| = -1$

(B) $|A| = 1$

(C) $b^2 + c^2 = 25$

(D) $A^{-1} = \begin{bmatrix} 3 & -1 \\ 4 & -1 \end{bmatrix}$

6. Which of the following statement(s) is/are Incorrect.

(A) If A is any non singular matrix, then $(tr(A)A)^{-1}$ always exists.

(B) If A is any square matrix of order 'n' such that $A = A^{-1}$, then $|A^2 + A^4 + A^6 + ... + A^{50}| = 625$ holds good for some n, $n \in N$.

(C) If A is a non singular matrix of order 2, then $|adj(|A|A)| = |A|^3$

(D) If A is a non singular matrix of order 2, then $|adj(|A|A)| = |A|^2$

7. Let A and B are two square matrices such that AB = A, BA = B, then $(A^2 - AB + B^2)$ is always equal to

(A) A

(B) B

(C) A^2

(D) B^2

8. Let $A = [a_{ij}]_{3 \times 3}$ be a matrix such that $a_{ij} = \dfrac{|i - j|}{2}$, then

(A) A is a symmetric matrix

(B) $2A^3 - 3A = I$

(C) $2A^3 - 3A + I = 0$

(D) A is not an orthogonal matrix

9. If A and B are square matrices such that AB = B and BA = A, then

(A) A is an idempotent matrix

(B) $A^3 + B^2 = A^2 + B^3$

(C) $A^3 + B^3 = A + B$

(D) A is an involutary matrix

10. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = [b_{ij}]$ is a 3 × 3 matrix such that $A^{100} - B = I$, then

(A) $\dfrac{b_{31} + b_{32}}{b_{21}} = 102$

(B) $b_{32} = b_{21}$

(C) B is lower triangular matrix

(D) B is upper triangular matrix

11. Let $A = [a_{ij}]$ be a 3×3 invertible matrix, where $a_{ij} \in \{0, 1\}$ and exactly four elements of A are 1. If N be the number of possible matrices A, then

(A) Number of divisors of N is even
(B) Sum of divisors of N is 91
(C) |adjA| can be –1
(D) |adjA| can be 1

12. Let A and B are commutative square matrices such that A is symmetric and B is skew symmetric.

If A – B is non singular matrix and $C = (A + B)^T (A + B)(A-B)^{-1}$, then

(A) $A + B + C = 0$
(B) $A - B - C = 0$
(C) $A + B - C = 0$
(D) $2A - C = C^T$

13. Consider a 3×3 matrix A whose elements $a_{ij} = \tan^{-1}(\tan(i + j))$, $1 \le i, j \le 3$, then

(A) $A^T = A$
(B) $Tr(A) = 12 - 3\pi$
(C) $Tr(A) = 12 - 4\pi$
(D) $(adjA)^3$ is a symmetric matrix

14. Let A be a square matrix of order 3 such that |A| = 2, then

(A) |adj (adj A)| = 8
(B) |adj A| = 4
(C) $|adj(adj\ 3A)| = 3^{12} 2^4$
(D) $|adj\ 2A| = 2^8$

15. If $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, then

(A) $A^{12} = \begin{bmatrix} -64 & 0 \\ 0 & -64 \end{bmatrix}$
(B) $A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$
(C) $A^{10} = \begin{bmatrix} 0 & 32 \\ -32 & 0 \end{bmatrix}$
(D) $A^{14} = \begin{bmatrix} 0 & 128 \\ -128 & 0 \end{bmatrix}$

16. Suppose a square matrix A satisfies $A^2 - 5A + 7I = O$. If $A^5 = aA + bI$, then

(A) a = 149
(B) a = 151
(C) b = –365
(D) b = –385

17. The product of 5 × 3 matrix and 3 × 5 matrix contains a variable entry x in exactly two places. If D(x) is the determinant of the matrix product, such that D(0) = 1, D(–1) = 1 and D(2) = 7. Then

(A) D(1) = 3 (B) D(–3) = 7

(C) D(–2) = 3 (D) D(4) = 20

18. A is a real skew symmetric matrix such that $A^2 + I = O$, then

(A) A is orthogonal matrix (B) A is of even order

(C) |A| = ±4 (D) A is of odd order

19. If A and B are non singular matrices of the same order 'n' and A is symmetric matrix, then

(A) $\text{adj}(\text{adj } B) = |B|^{n-2} B$ (B) adj A is a symmetric matrix

(C) $|\text{adj}(\text{adj } B)| = |B|^{(n-1)^2}$ (D) $(\text{adj } A)^T = (\text{adj } A^T)$

20. If P and Q are symmetric non singular matrices and PQ = QP, then

(A) PQ is symmetric (B) $P^{-1}Q$ is symmetric

(C) $QP^{-1} = P^{-1}Q$ (D) $Q^{-1}P$ is symmetric

21. If A and B are two non singular matrices of order 3 such that $2A + 3BB^T = I$ and $B^{-1} = A^T$, then
(Tr(P) = trace of matrix P)

(A) $|B^T - 2B + 3B^3 + 3BA| = 1$ (B) $\text{Tr}(A^{-1} + I - AB - 3B^3) = 6$

(C) $|A^{-1} - 3B^3| = 27$ (D) $|A^{-1} - 3B^3| = 8$

22. If $A(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, $\alpha \in R$, then

(A) $A(\pi)A\left(\frac{3\pi}{2}\right)A\left(\frac{5\pi}{2}\right)A\left(\frac{7\pi}{8}\right)A\left(\frac{9\pi}{16}\right)....\infty = 1$

(B) $(A(\alpha))^{-1} = A(-\alpha)$

(C) $\text{adj}(\text{adj}(\text{adj } A(\alpha))) = A(-\alpha)$

(D) $\text{adj}(\text{adj}(\text{adj}(\text{adj}(A(\alpha))))) = A(\alpha)$

23. If A is non singular matrix satisfying A = AB – BA, then

(A) $|B| = 0$ (B) $|A| = 1$

(C) $|B + I| = |B – I|$ (D) $|B + I| = |B|$

24. Which of the following statement(s) is/are true

(A) If A is non singular matrix of order n × n and k is a non zero scalar, then $|\text{adj}(kA)| = (k)^{n^2-n}|A|^{n-1}$.

(B) If A is non singular matrix of order n × n and k is a non zero scalar, then $\text{adj}(kA)| = k^{n-1}|A|^{n^2-n}$.

(C) If A & B are two square matrices of order n × n, where A is non singular and AB = 0, then B must be null matrix.

(D) If A and B are not null matrices, such that AB = 0, then both A, B must be singular.

25. Let B is a skew symmetric matrix of order 'n' and A is a n × 1 column matrix, then for matrix $M = A^TBA$, which of the following statement(s) is/are true.

(A) M is invertible (B) M is singular

(C) M is non singular (D) M is a null matrix.

26. Let $AoB = A^2B^2 + B^2A^2$, then

(A) If A and B are involutary, then inverse of AoB is $\frac{1}{2}$ AoB.

(B) If A and B are involutary, then inverse of AoB is $\frac{1}{4}$ AoB.

(C) If A and B are symmetric, then AoB is also symmetric.

(D) AoB = BoA

27. If A is a non singular periodic matrix with period 3 and $A^6 + B = I$, then

(A) AB = O (B) $|A| = 1$

(C) $|B| = 0$ (D) $|B| = 1$

28. Let A_{ij} be a 2 × 2 non singular matrix and

$$B = \begin{bmatrix} |A_{11}| & |A_{12}| & |A_{13}| & \cdots\cdots & |A_{1n}| \\ 0 & |A_{22}| & |A_{23}| & \cdots\cdots & |A_{2n}| \\ 0 & 0 & |A_{33}| & \cdots\cdots & |A_{3n}| \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots\cdots & |A_{nn}| \end{bmatrix},$$

then the values of λ satisfying the equation $|B - \lambda I| = 0$ is/are

(A) $|A_{11}|$ (B) $|A_{22}|$

(C) $|A_{23}|$ (D) $|A_{nn}|$

29. If P, Q, R are square matrices such that $|P| = |Q| = |R| = 1$, then $\text{adj}(Q^{-1}R^nP^{-1})$ is equal to, $(n \in N, n > 5)$

(A) $Q(R^{-1})^nP$ (B) $P(R^{-1})^nQ$

(C) $(\text{adj } P)^{-1}(\text{adj } R)^n (\text{adj } Q)^{-1}$ (D) $(\text{adj } P^{-1}) (\text{adj } R)^n (\text{adj } Q^{-1})$

30. Let a square matrix A satisfy $A^2 + A + I = 0$, then

(A) $A^{-1} = -(A^4 + A^9)$ (B) $A^7 + A^8 = -I$

(C) $A^{-1} = A^5$ (D) $A^6 + A^9 = -I$

31. If all elements of a 2 × 2 matrix A are real and distinct such that $|A + A^T| = 0$, then

(A) $|A| \le 0$ (B) $|A| > 0$

(C) $|AA^T| > 0$ (D) $|A - A^T| > 0$

32. Let A, B, C are square matrices of same order satisfy $|A - B| \ne 0$, $A^4 = B^4$, $C^3A = C^3B$ and $B^3A = A^3B$, then $A^3 + B^3 + C^3$ is equal to

(A) C^3 (B) $(A^3 + B^3)$

(C) I (D) 0

33. Let A is a non singular matrix such that $3ABA^{-1} + A = 2A^{-1}BA$, then

(A) A + B is singular matrix (B) A + B is non singular matrix.

(C) $ABA^{-1} - A^{-1}BA$ is non singular matrix (D) $ABA^{-1} - A^{-1}BA$ is singular matrix.

34. Let A, B are square matrices of same order such that $AB + BA = 0$, then $A^3 - B^3$ is equal to

(A) $(A - B)(A^2 + AB + B^2)$ (B) $(A + B)(A^2 - AB - B^2)$

(C) $(A + B)(A^2 + AB - B^2)$ (D) $(A - B)(A^2 - AB - B^2)$

SECTION-3

COMPREHENSION TYPE QUESTIONS

COMPREHENSION (Q.1 TO Q.3) :

If A is a 3 × 3 matrix, then a non trivial solution $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that $AX = \lambda X, \lambda \in R$ yields 3 values of λ say $\lambda_1, \lambda_2, \lambda_3$. For any such matrix A, λ's are called eigen values and corresponding X's are called eigen vectors. It is known that for any 3 × 3 marix 'A', $Tr(A) = \lambda_1 + \lambda_2 + \lambda_3$ and $\det(A) = \lambda_1\lambda_2\lambda_3$. Answer the following questions for matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

1. $Tr(A^{-1}) =$

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

2. $Tr(A^3)$

(A) 101 (B) 128

(C) 133 (D) 149

3. Which of the following is false.

(A) There exist a non trivial solution X such that $AX = (2 + \sqrt{7})X$.

(B) There exist a non trivial solution X such that $AX = X$.

(C) There exist a non trivial solution X such that $A^{-1}X = (2 - \sqrt{7})X$.

(D) The total number of real values of λ for which the equation $AX = \lambda X$ has non trivial solution X is 3.

COMPREHENSION (Q.4 TO Q.5) :

Let $\Delta = \begin{vmatrix} (10^4+2) & (10^7+3) & (10^8+8) \\ (10^9+9) & (10^2+8) & (10^3-4) \\ (10^3-5) & (10^8+b) & (10^6+a) \end{vmatrix}$, where a, b $\in$ {1, 2, 3, 4, 5, 6, 7, 8, 9}.

Then

4. The probability that Δ is odd is equal to

(A) $\frac{4}{9}$

(B) $\frac{5}{9}$

(C) $\frac{40}{81}$

(D) $\frac{25}{81}$

5. Let k $\in$ N such that the least prime factor of k is 7 and a = 3, b = 2. Then the least prime factor of k + Δ is

(A) 2

(B) 3

(C) 5

(D) 7

COMPREHENSION (Q.6 TO Q.7) :

Consider two 3 × 3 matrices A and B satisfying $A = adj(B) - B^T$ and $B = adj(A) - A^T$. Also given that A is a non singular matrix.
(where P^T denotes transpose of matrix P).

6. |A| + |B| is equal to

(A) 4

(B) 8

(C) 16

(D) 128

7. AB + BA is equal to

(A) 16I

(B) 2I

(C) 4I

(D) 8I

COMPREHENSION (Q.8 TO Q.10) :

Let A be a m × n matrix. If there exists a matrix P of order n × m such that PA = I, then P is called left inverse of A. Similarly, if there exists a matrix Q of order n × m such that AQ = I, then Q is called right inverse of A. Then

8. Which of the following matrices is not the left inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$.

(A) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -7 & 3 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 3 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

9. The number of right inverses for the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ is equal to

(A) 0 (B) 1 (C) 2 (D) Infinite

10. For which of the following matrices, number of left inverses is greater than the number of right inverses.

(A) $\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 4 & 4 \end{bmatrix}$

COMPREHENSION (Q.11 TO Q.13) :

Let A be the 2 × 2 matrices given by $A = [a_{ij}]$, where $a_{ij} \in \{0, 1, 2, 3, 4\}$ such that $a_{11} + a_{12} + a_{21} + a_{22} = 4$.

11. The number of matrices A such that A is invertible are

(A) 12 (B) 18 (C) 17 (D) 19

12. The absolute value of difference between maximum value and minimum value of det(A) is equal to

(A) 4 (B) 5 (C) 6 (D) 8

13. The number of matrices A such that A is either symmetric or skew symmetric or both and det (A) is divisible by 2 are

(A) 3 (B) 5 (C) 7 (D) 9

COMPREHENSION (Q.14 TO Q.15) :

Let $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ and U, V, W be 3 unit column vectors such that $AU = \lambda U$, $AV = \mu V$, $AW = \nu W$ and $\lambda < \mu < \nu$.

14. If θ is the angle between the vectors U and V, then $|\cos\theta| =$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{2}{\sqrt{3}}$

15. $\left|(\vec{U} \times \vec{V}).\vec{W}\right| =$

(A) $\frac{1}{3\sqrt{2}}$ (B) $\frac{1}{5\sqrt{2}}$ (C) $\frac{1}{7\sqrt{2}}$ (D) $\frac{1}{9\sqrt{2}}$

COMPREHENSION (Q.16 TO Q.18) :

Let A be the set of all 3 × 3 symmetric matrices whose entries are 1, 1, 1, 0, 0, 0, –1, –1, –1. B is one of the matrix in set A and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $U = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $V = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

16. Number of matrices B in set A is equal to

(A) 12 (B) 24 (C) 36 (D) 54

17. Number of matrices B in set A such that the system of equation BX = U has infinite solution is

(A) 6 (B) 8 (C) 12 (D) 15

18. Number of matrices B in set A such that the system of equation BX = V is inconsistent is

(A) 4 (B) 6 (C) 8 (D) 12

COMPREHENSION (Q.19 TO Q.21) :

Let $A = [a_{ij}]_{3\times3}$, $B = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $B^T = [1\ 1\ 1]$. Let M(x) be a matrix defined by $M(x) = I + xABB^T$. Let k = the element of singleton matrix B^TAB.

19. M(x) M(y) =

(A) M(x + y + xy) (B) M(x + y + kxy)

(C) M(k(x + y) + xy) (D) M(k + x + y + xy)

20. Inverse of matrix M(x) is

(A) $M\left(\frac{x}{1+kx}\right)$ (B) $M\left(\frac{k}{1+kx}\right)$ (C) $M\left(\frac{-k}{1+kx}\right)$ (D) $M\left(\frac{-x}{1+kx}\right)$

21. Let $R = [r_{ij}]_{3\times3}$ be defined as $r_{ij} = 1\ \forall\ i = j$ and $r_{ij} = p\ \forall\ i \neq j$, where p is any scalar, then R can also be written as

(A) $pI + (1 - p)BB^T$ (B) $(1 - p)I - pBB^T$

(C) $pI - (1 - p)BB^T$ (D) $(1 - p)I + pBB^T$

COMPREHENSION (Q.22 TO Q.24) :

Let there exists a matrix B such that $ABA^T = N$, where $A = \begin{bmatrix}1 & 2 & 0\\2 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ and N is a diagonal matrix of form $N = \text{diag}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are three values of n satisfying the equation $|A - nI| = 0$, $n_1 < n_2 < n_3$.

22. |B| is equal to

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{9}$ (D) $-\frac{1}{9}$

23. Trace of matrix A^{20} is equal to

(A) $2^{20} + 2$ (B) $3^{20} + 2^{20} - 2$ (C) $3^{20} + 2$ (D) 3^{20}

24. Matrix A satisfies

(A) $A^3 - (n_1 + n_2)A^2 + n_1n_3A - n_1n_2n_3I = O$

(B) $A^3 - n_2A^2 + n_1A - n_1n_2n_3I = O$

(C) $A^3 - n_2A^2 + n_1A + n_2n_3I = O$

(D) $A^3 - n_3A^2 + n_1A + n_2n_3I = O$

COMPREHENSION (Q.25 TO Q.26) :

Consider matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix}, D = \begin{bmatrix} 13 \\ 11 \\ 14 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

such that solutions of equation AX = C and BX = D represent two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ respectively in three dimensional space. Let a plane $\Pi : x + y + z = 9$.

25. If P'Q' is the reflection of line segment PQ in the plane Π ; then the point which does not lie on P'Q' is

(A) (3, 4, 2) (B) (5, 3, 4) (C) (7, 2, 3) (D) (1, 5, 6)

26. The value of |adj(adj A)| is equal to

(A) 16^4 (B) 16^2 (C) –16 (D) -16^3

COMPREHENSION (Q.27 TO Q.29) :

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and R_1, R_2, R_3 be the row matrices satisfying the relations $R_1A = [1\ \ 0\ \ 0]$, $R_2A = [2\ \ 3\ \ 0]$ and $R_3A = [2\ \ 3\ \ 1]$. If B is a square matrix of order 3 with rows R_1, R_2, R_3 in order, then

27. |B| is equal to

(A) 3 (B) 2 (C) 1 (D) 0

28. $|2A^{100}B^3 - A^{99}B^4|$ is equal to

(A) –81 (B) –27 (C) 27 (D) 81

29. $Tr(B^{-1})$ is equal to

(A) 5 (B) 4 (C) 7 (D) 9

SECTION-4

MATCH THE COLUMN:

1. Let A be the matrix of order 3 such that $A = [a_{ij}]_{3\times 3}$, where

$$a_{ij} = \begin{cases} 0 & \text{for } i = j \\ 1 & \text{for } i \neq j \end{cases}, \text{ then}$$

	Column-I		Column-II
(A)	$\lvert A^{-1} + I\rvert =$	(P)	0
(B)	$\lvert A^3 - A^2 + 2A\rvert =$	(Q)	4
(C)	$\lvert A^4 - 4A - 7I\rvert =$	(R)	8
(D)	$\lvert A^4 - 8A^{-1} - A^2 - 6I\rvert =$	(S)	64
		(T)	128

2. Let $A(t) = [a_{ij}]$ is a matrix of order 3 × 3 given by

$$a_{ij} = \begin{cases} 2\cos t & \text{if } i = j \\ 1 & \text{if } |i - j| = 1, \text{ then} \\ 0 & \text{otherwise} \end{cases}$$

	Column-I		Column-II
(A)	The number of t in interval $[-2\pi, 4\pi]$ such that $\lvert A(t)\rvert = 4$ is equal to	(P)	0
(B)	$\left\lvert A\left(\frac{\pi}{17}\right)\right\rvert \left\lvert A\left(\frac{4\pi}{17}\right)\right\rvert$ is equal to	(Q)	1
(C)	The maximum value of $\lvert A(t)\rvert + \lvert A(2t)\rvert$, $\forall$ t $\in$ R is equal to	(R)	4
(D)	$\int_0^{\pi} \lvert A(t)\rvert\lvert A(4t)\rvert\, dt$ is equal to	(S)	6
		(T)	8

3. Let A, B are two square matrices such that

	Column-I		Column-II
(A)	A is non singular and AB = 0, then	(P)	$A = 0$
(B)	A and B both are not null matrices satisfy AB = 0, then	(Q)	$B = 0$
(C)	$A^n = 0$ for some $n \ge 2$, $n \in N$, then	(R)	$\|A\| = 0$
(D)	B is non singular and AB = 0, then	(S)	$\|B\| = 0$
		(T)	$\|A\| + \|B\| \ne 0$

4.

	Column-I		Column-II
(A)	Let A be a square matrix which commutes with a square matrix B of same order (B is not null matrix). If $AA^T = I$ and $BA^T - A^TB = kI$, then k is equal to	(P)	0
(B)	If C is a 3 × 3 skew symmetric matrix and X is any column vector, then $X^TCX = kI$, where k is equal to	(Q)	1
(C)	Let A be an n × n square matrix given by $A = \begin{bmatrix} 0 & 1 & 1 & \ldots\ldots & 1 \\ 1 & 0 & 1 & \ldots\ldots & 1 \\ 1 & 1 & 0\ 1 & \ldots\ldots & 1 \\ \vdots & \vdots & \overset{1}{\vdots} & & \vdots \\ 1 & 1 & 1 & \ldots\ldots 1 & \overset{1}{0} \end{bmatrix}$, then in the inverse of A, each diagonal element is equal to $\frac{\lambda_1 - n}{n - \lambda_2}$, then $\lambda_1 =$	(R)	2
(D)	In part (C), $\lambda_2 =$	(S)	–1
		(T)	3

5. Let A, B, C, D be non singular matrices of order 3×3 such that $(AA^T)BC^2 = B^T$ and $C^2 = I$ and D is an orthogonal matrix such that A^T and D commute with each other and $D^TA^TD = A^{-1}$. Then

	Column-I		Column-II
(A)	If $B = B^{-1}$, then $\lvert B^{2018}C\rvert$ can be equal to	(P)	–1
(B)	If $(ACA^T)^{2019} = AC^kA^T$, then k can be equal to	(Q)	0
(C)	$\lvert ACA^T\rvert$ can be equal to	(R)	1
(D)	$\lvert BB^T(B^{-1})^2\rvert + \lvert(AA^T)^{2018}\rvert$ is equal to	(S)	2
		(T)	3

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. Let A and B are square matrices both of order 3 satisfying $A^2 + B^4 = (A^T)^2$, then |B| is equal to

(where A^T is transpose of matrix A)

2. Let $A_n = [a_{ij}]$ be a square matrix of order 3, where $a_{ij} = \frac{2i+j}{3^{2n}}$ for all i, j, $1 \le i, j \le 3$. Then $\lim_{n\to\infty} Tr(3A_1 + 3^2A_2 + 3^3A_3 + ... + 3^nA_n)$ is equal to

3. Let A and B be two non singular matrices such that $(AB)^2 = A^2B^2$, then $BA^2B^{-1} = A^n$, where n is equal to

4. Let $A = \begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix}$ and P be a 2 × 2 matrix such that $PP^T = I$ (where I is identity matrix of order 2).

If $Q = PAP^T$ and $R = [r_{ij}]_{2\times2} = P^TQ^8P$, then $\sqrt{r_{11}}$ is equal to

5. Let A be a non singular square matrix of order 2, such that |A + |A| adj(A)| = 0, then the value |A – |A|adj(A)| is equal to

(where adj(A) is the adjoint of matrix A)

6. If N be the total number of non singular matrices of order 3 × 3 whose four elements are 1 and remaining elements are zero, then $\left[\sqrt{N}\right]$ is equal to ([.] denote greatest integer function)

7. If A and B are two square matrices of order 3 such that AB = BA and $A^2 = AB + 2B^2$. If the matrices, A + B and B are non singular, then $|AB^{-1}|$ is equal to

8. Let $A = \begin{bmatrix} -3 & 0 & 2 \\ 1 & x & 5 \\ -2 & 0 & x^2 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ b \\ -1 \end{bmatrix}$ and $C = [3\ 5\ 1]$, then the number of integral values of 'b' for which $\text{Tr}(ABC) \le -18\ \forall\ x \in R$ is equal to

9. Given $A = [a_{ij}]_{3\times3}$ be a matrix, where $a_{ij} = \begin{cases} i - j & \text{if } i \ne j \\ i^2 & \text{if } i = j \end{cases}$. If C_{ij} be the cofactor of a_{ij} in the matrix A. If $B = [b_{ij}]_{3\times3}$ be a matrix such that $b_{ij} = \sum_{k=1}^{3} a_{ik}C_{jk}$, then the value of $\left[\dfrac{\sqrt[3]{|B|}}{8}\right]$ is equal to

([.] denote greatest integer function)

10. Consider a square matrix A of order 3 such that $|A| = 3$. If $\text{adj}(\text{adj } A) = \lambda A$, then λ is equal to

11. A and B are two non singular matrices such that $A^6 = I$ and $AB^2 = BA$, $B \ne I$. The smallest value of k so that $B^k = I$ is N, then N is equal to

12. Let $A = \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$ are two matrices such that $AB = (AB)^{-1}$ and $AB \ne I$ (where I is an identity matrix). Then $\text{Tr}(AB + (AB)^2 + (AB)^3 + (AB)^4 + (AB)^5 + (AB)^6)$ is equal to (Tr(A) denotes the trace of matrix A)

13. For any integer $n \ge 5$, let there are two $n \times n$ invertible matrices with real entries A, B satisfy the equation $A^{-1} + B^{-1} = (A + B)^{-1}$. If $|A| = 3$, then find the value of $|B|$.

14. If A is a non singular matrix of order 2 such that $A + \text{adj } A = A^{-1}$, then $|2A^{-1}|$ is equal to :

15. Let $S = [a_{ij}]$, $a_{ij} \in \{-1, 0, 1\}$ is a 3×3 symmetric matrix, such that $Tr(S) = 0$, then number of matrices S is equal to : (Tr(S) denotes the trace of matrix S)

16. If A and B are square matrices of the same order such that
$|A| = |B| = 1$ and $A(\text{adj } A + \text{adj } B) = B$.
Then the value of $|A + B|$ is equal to :

17. If $A_1 = [a_1]$, $A_2 = \begin{bmatrix} a_2 & a_3 \\ a_4 & a_5 \end{bmatrix}$, $A_3 = \begin{bmatrix} a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} \\ a_{12} & a_{13} & a_{14} \end{bmatrix}$........, where $a_r = [\log_2 r]$ (where [.] denotes greatest integer function). Then $Tr(A_{10})$ is equal to (Tr(A) denotes trace of matrix A)

18. A is a 2×2 matrix such that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find the sum of all elements of A.

19. Let $2P = \begin{bmatrix} \cos\theta + \sqrt{3}\sin\theta & \sin\theta - \sqrt{3}\cos\theta \\ \sqrt{3}\cos\theta - \sin\theta & \cos\theta + \sqrt{3}\sin\theta \end{bmatrix}$; $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$; $Q = PAP^T$ and $X = P^T Q^{50} P$, then find the sum of elements of X.

20. Let A and B be matrices of size 3×2 and 2×3 respectively. Suppose that their product $AB = \begin{bmatrix} 8 & 2 & -2 \\ 2 & 5 & 4 \\ -2 & 4 & 5 \end{bmatrix}$ and BA is non singular, then find $|BA|$.

21. Let A be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ whose entries are chosen from {0, 1, 2, 3,......,10} and satisfy the conditions $a + d$ leaves the remainder 1 when divided by 11 and $ad - bc$ is divisible by 11. Determine how many members A has.

22. Let $\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} \sqrt{3}+1 & 1-\sqrt{3} \\ \sqrt{3}-1 & 1+\sqrt{3} \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix}$, $n \in N$. If $a_1 = b_1 = 1$, then $a_{22} = 2^n$, where n =

23. Let A is a 3 × 3 matrices whose elements are –1 and 1 only, then number of possible distinct values |A| can take is equal to

24. The number of 2 × 2 matrices A with elements from the set {–1, 0, 1} such that $A^2 = I$ is

25. Let $P = \left\{ (a,b) \,\middle|\, A^3 = A, \text{ where } A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ a & b \end{bmatrix} \right\}$, then n(P) =

Answer Key

Single Choice Questions

1. D	**2.** A	**3.** B	**4.** B	**5.** B	**6.** A
7. A	**8.** B	**9.** A	**10.** C	**11.** A	**12.** B
13. C	**14.** C	**15.** B	**16.** A	**17.** D	**18.** D
19. B	**20.** D	**21.** D	**22.** A	**23.** A	**24.** C
25. C					

One or More than One Correct Questions

1. B,D	**2.** A,B,D	**3.** A,B,C,D	**4.** A,B,D	**5.** B,C,D	**6.** A,D
7. B,D	**8.** A,B,D	**9.** A,B,C	**10.** A,B,D	**11.** B,D	**12.** C,D
13. A,C,D	**14.** B,C,D	**15.** A,B,C	**16.** A,D	**17.** A,B,C	**18.** A,B
19. A,B,C,D	**20.** A,B,C,D	**21.** A,B,D	**22.** A,B,C,D	**23.** C,D	**24.** A,C,D
25. B,D	**26.** B,C,D	**27.** A,B,C	**28.** A,B,D	**29.** B,C,D	**30.** A,B,C
31. B,C,D	**32.** A,B,D	**33.** A,D	**34.** B,D		

Comprehension Type Questions

1. B	**2.** A	**3.** C	**4.** B	**5.** A	**6.** C
7. D	**8.** A	**9.** D	**10.** A	**11.** B	**12.** D
13. B	**14.** C	**15.** D	**16.** C	**17.** C	**18.** D
19. B	**20.** D	**21.** D	**22.** B	**23.** C	**24.** D
25. A	**26.** A	**27.** A	**28.** B	**29.** C	

Match The Column

1. A → P ; B → T ; C → Q ; D → R

2. A → R ; B → Q ; C → T ; D → P

3. A → Q, S, T ; B → R,S ; C → R ; D → P, R, T

4. A → P ; B → P ; C → R ; D → Q

5. A → P, R ; B → R, T ; C → P, R ; D → S

Subjective Type Questions

1. 0	**2.** 9	**3.** 2	**4.** 9	**5.** 4	**6.** 6
7. 8	**8.** 5	**9.** 7	**10.** 3	**11.** 63	**12.** 6
13. 3	**14.** 8	**15.** 189	**16.** 1	**17.** 80	**18.** 5
19. 52	**20.** 81	**21.** 132	**22.** 32	**23.** 3	**24.** 16
25. 3					

Previous Year Questions

1. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then **[AIEEE 2003]**

(A) $\alpha = a^2 + b^2, \beta = ab$
(B) $\alpha = a^2 + b^2, \beta = 2ab$
(C) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
(D) $\alpha = 2ab, \beta = a^2 + b^2$

2. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is : **[AIEEE 2004]**

(A) A is a zero matrix
(B) $A^2 = I$
(C) A^{-1} does not exist
(D) $A = -I$, where I is a unit matrix

3. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of A, then α is :

[AIEEE 2004]

(A) –2
(B) 5
(C) 2
(D) –1

4. If $A^2 - A + I = O$, then the inverse of A is : **[AIEEE 2005]**

(A) A + I
(B) A
(C) A – I
(D) I – A

5. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction **[AIEEE 2005]**

(A) $A^n = nA - (n-1)I$
(B) $A^n = 2^{n-1}A - (n-1)I$
(C) $A^n = nA + (n-1)I$
(D) $A^n = 2^{n-1}A + (n-1)I$

6. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true ? **[AIEEE 2006]**

(A) $A = B$ (B) $AB = BA$

(C) Either A or B is a zero matrix (D) Either A or B is an identity matrix

7. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, a, b $\in$ N. Then **[AIEEE 2006]**

(A) there cannot exists any B such that AB = BA.

(B) there exists more than one but finite numbe of B's such that AB = BA.

(C) there exists exactly one B such that AB = BA.

(D) there exists infinitely many B's such that AB = BA.

8. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals : **[AIEEE 2007]**

(A) 5^2 (B) 1

(C) 1/5 (D) 5

9. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entires of A. Assume that $A^2 = I$.

Statement 1 : If $A \neq I$ and $A \neq -I$, then det$A = -1$.

Statement 2 : If $A \neq I$ and $A \neq -I$, then tr(A) $\neq 0$. **[AIEEE 2008]**

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(D) Statement 1 is true, statement 2 is false.

10. Let A be a 2 × 2 matrix. **[AIEEE 2009]**

Statement 1 : adj.(adj A) = A

Statement 2 : |adj A| = |A|

(A) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

11. The number of 3 × 3 non-singular matrices with four entries as 1 and all other entries as 0 is : **[AIEEE 2010]**

(A) at least 7 (B) less than 4

(C) 5 (D) 6

12. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is a 2 × 2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A. **[AIEEE 2010]**

Statement 1 : Tr(A) = 0

Statement 2 : |A| = 1

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(D) Statement 1 is true, statement 2 is false.

13. Let A and B two symmetric matrices of order 3. **[AIEEE 2011]**

Statement 1 : A(BA) and (AB)A are symmetric matrices

Statement 2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

(A) Statement 1 is false, statement 2 is true.

(B) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

(D) Statement 1 is true, statement 2 is false.

14. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[AIEEE 2012]**

(A) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(B) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

(C) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

(D) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

15. Let P and Q be 3 × 3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to : **[AIEEE 2012]**

(A) –2

(B) 1

(C) 0

(D) –1

16. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to : **[JEE Main - 2013]**

(A) 0

(B) 4

(C) 11

(D) 5

17. If A is an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals **[JEE Main - 2014]**

(A) $(B^{-1})'$ (B) $I + B$

(C) I (D) B^{-1}

18. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to: **[JEE Main - 2015]**

(A) (–2, –1) (B) (2, –1)

(C) (–2, 1) (D) (2, 1)

19. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = A A^T$, then $5a + b$ is equal to : **[JEE Main- 2016]**

(A) –1 (B) 5

(C) 4 (D) 13

20. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to : **[JEE Main - 2017]**

(A) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (B) $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$ (D) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

21. Let A be a matrix such that $A \bullet \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals : **[JEE Main - 2018]**

(A) $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$ (B) $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$ (D) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

22. Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I^3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to : **[JEE Main - 2018]**

(A) 8 (B) 7

(C) 13 (D) 12

23. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is **[JEE Main - 2018]**

(A) 211 (B) 210

(C) 231 (D) 251

SECTION-2

1. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, abc = 1 and $A^TA = I$, then find the value of $a^3 + b^3 + c^3$.

[JEE 2003, Mains-2 out of 60]

2. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then $\alpha =$ **[JEE 2004(Scr)]**

(A) ± 3 (B) ± 2

(C) ± 5 (D) 0

3. If M is a 3 × 3 matrix, where $M^TM = I$ and det (M) = 1, then prove that det (M – I) = 0. **[JEE 2004, 2 out of 60]**

4. $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$, $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$, $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

If AX = U has infinitely many solution, then prove that BX = V cannot have a unique solution. If further afd ≠ 0, then prove that BX = V has no solution.

[JEE 2004, 4 out of 60]

5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \left[\frac{1}{6}(A^2 + cA + dI)\right]$, then the value of c and d are **[JEE 2005(Scr)]**

(A) –6, –11 (B) 6, 11

(C) –6, 11 (D) 6, – 11

6. If $P=\begin{bmatrix}\sqrt{3}/2 & 1/2\\ -1/2 & \sqrt{3}/2\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q = PAP^T$ and $x = P^TQ^{2005}P$, then x is equal to **[JEE 2005 (Screening)]**

(A) $\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$

(B) $\begin{bmatrix}4+2005\sqrt{3} & 6015\\ 2005 & 4-2005\sqrt{3}\end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix}2+\sqrt{3} & 1\\ -1 & 2-\sqrt{3}\end{bmatrix}$

(D) $\frac{1}{4}\begin{bmatrix}2005 & 2-\sqrt{3}\\ 2+\sqrt{3} & 2005\end{bmatrix}$

COMPREHENSION (3 QUESTIONS) **[JEE 2006, 5 MARKS EACH]**

7. $A=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2 & 1\end{bmatrix}$, U_1, U_2 and U_3 are columns matrices satisfying. $AU_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$; $AU_2=\begin{bmatrix}2\\3\\0\end{bmatrix}$, $AU_3=\begin{bmatrix}2\\3\\1\end{bmatrix}$ and U is 3 × 3 matrix whose columns are U_1, U_2, U_3 then answer the following questions

(a) The value of | U | is

(A) 3
(B) – 3
(C) 3/2
(D) 2

(b) The sum of elements of U^{-1} is

(A) – 1
(B) 0
(C) 1
(D) 3

(c) The value of $\begin{bmatrix}3 & 2 & 0\end{bmatrix}U\begin{bmatrix}3\\2\\0\end{bmatrix}$ is

(A) 5
(B) 5/2
(C) 4
(D) 3/2

8. Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in OMR.

	Column-I		Column-II
(A)	The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(P)	0
(B)	Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and $(A+B)(A-B)=(A-B)(A+B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(Q)	1
(C)	Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(R)	2
(D)	If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are	(S)	3

[JEE 2008, 6]

PARAGRAPH FOR QUESTION NOS. 9 TO 11 **[JEE-2009]**

Let A be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

9. The number of matrices in A is

(A) 12 (B) 6

(C) 9 (D) 3

10. The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is

(A) less than 4 (B) at least 4 but less than 7

(C) at least 7 but less than 10 (D) at least 10

11. The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

(A) 0 (B) more than 2

(C) 2 (D) 1

12. The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is **[JEE-2010]**

(A) 0 (B) $2^9 - 1$

(C) 168 (D) 2

PARAGRAPH FOR QUESTIONS 13 TO 15

Let P be an odd prime number and T_p be the following set of 2 × 2 matrices :

$$T_P = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}; a, b, c \in \{0, 1, 2...., P-1\} \right\}$$

13. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is

(A) $(p-1)^2$ (B) $2(p-1)$

(C) $(p-1)^2 + 1$ (D) $2p-1$

14. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[**Note :** The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$ (B) $p^3-(p-1)^2$

(C) $(p-1)^2$ (D) $(p-1)(p^2-2)$

15. The number of A in T_p such that det (A) is not divisible by p is **[JEE-2010]**

(A) $2p^2$ (B) p^3-5p

(C) p^3-3p (D) p^3-p^2

PARAGRAPH FOR QUESTION NOS. 16 TO 18 **[JEE-2011]**

Let a, b and c be three real numbers satisfying

$$[a\ b\ c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix} = [0\ 0\ 0] \qquad \text{.......(E)}$$

16. If the point P(a, b, c), with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12

(C) 7 (D) 6

17. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(A) –2 (B) 2

(C) 3 (D) –3

18. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is

(A) 6 (B) 7

(C) 6/7 (D) ∞

19. Let M and N be two 3 × 3 non-singular skew symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2 (M^TN)^{-1} (MN^{-1})^T$ is equal to

[JEE-2011]

(A) M^2 (B) $-N^2$

(C) $-M^2$ (D) MN

20. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b, and c is either ω or ω^2.

Then the number of distinct matrices in the set S is **[JEE-2011]**

(A) 2 (B) 6

(C) 4 (D) 8

21. Let M be a 3 × 3 matrix satisfying

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{ and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}.$$

Then the sum of the diagonal entries of M is : **[JEE-2011]**

22. Let $P = [a_{ij}]$ be a 3 × 3 matrix and let $Q = [b_{ij}]$ where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \leq i,j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is **[JEE-2012]**

(A) 2^{10} (B) 2^{11}

(C) 2^{12} (D) 2^{13}

23. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where PT is the transposes of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that **[JEE-2012]**

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = X$

(C) $PX = 2X$ (D) $PX = -X$

24. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are) **[JEE-2012]**

(A) – 2 (B) – 1

(C) 1 (D) 2

25. For 3 × 3 matrices M and N, which of the following statement(s) is (are) NOT correct? **[JEE Advanced - 2013]**

(A) $N^T M N$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(B) M N – N M is skew symmetric for all symmetric matrices M and N

(C) M N is symmeric for all symmetric matrices M and N

(D) (adj M) (adj N) = adj (M N) for all invertible matrices M and N

26. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if

[JEE Advanced 2014]

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with non-zero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not thesquare of an integer

27. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then **[JEE Advanced 2014]**

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3 × 3 non-zero matrix v such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ euqals the zero matrix then U is the zero matrix

28. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric? **[JEE Advanced 2015]**

(A) $Y^3Z^4 - Z^4Y^3$ (B) $X^{44} + Y^{44}$

(C) $X^4Z^3 - Z^3X^4$ (D) $X^{23} + Y^{23}$

29. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that PQ = kI, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then **[JEE Advanced 2016]**

(A) $\alpha = 0, k = 8$ (B) $4\alpha - k + 8 = 0$

(C) $\det(P\operatorname{adj}(Q)) = 29$ (D) $\det(Q \operatorname{adj}(P)) = 213$

30. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order ∈3. If $Q = [q_{ij}]$ is a matrix such that $P_{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals **[JEE Advanced 2016]**

(A) 52 (B) 103

(C) 201 (D) 205

31. For a real number α, if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha_2 =$ **[JEE Advanced 2017]**

32. Which of the following is (are) NOT the square of a 3 × 3 matrix with real entries ? **[JEE-Advanced-2017]**

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

33. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_2 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$? **[JEE Advanced 2018]**

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + 2y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

34. Let P be a matrix of order 3 × 3 such that all the entries in P are from the set {–1, 0, 1}. Then, the maximum possible value of the determinant of P is _________. **[JEE Advanced 2018]**

Answer Key

Section-1

1. B	**2.** B	**3.** B	**4.** D	**5.** A	**6.** B
7. D	**8.** C	**9.** D	**10.** B	**11.** A	**12.** D
13. C	**14.** D	**15.** C	**16.** C	**17.** C	**18.** A
19. B	**20.** D	**21.** D	**22.** A	**23.** C	

Section-2

1. 4	**2.** A	**5.** C	**6.** A	**7.** (a) A, (b) B, (c) A	
8. (A) R (B) Q,S (C) R,S (D) P,R			**9.** A	**10.** B	**11.** B
12. A	**13.** D	**14.** C	**15.** D	**16.** D	**17.** A
18. B	**19.** **Bonus**	**20.** A	**21.** 9	**22.** D	**23.** D
24. A, D	**25.** C, D	**26.** C, D	**27.** A,B	**28.** C,D	**29.** B, C
30. B	**31.** 1	**32.** A,C	**33.** A, D	**34.** 4	

7 CHAPTER

COMPLEX NUMBERS

SECTION-1

SINGLE CHOICE QUESTIONS

1. If $z_1, z_2, z_3, \ldots\ldots\ldots, z_n$ are the vertices of an n-sided regular polygon with z_0 as its centre. Then $\sum_{r=1}^{n} z_r^k =$

where $k \in N, k < n$

(A) 0 (B) $(n-1)z_0^k$

(C) nz_0^k (D) z_0^k

2. Let α, β be any two distinct complex numbers, then

$\left|\alpha - \sqrt{\alpha^2 - \beta^2}\right| + \left|\alpha + \sqrt{\alpha^2 - \beta^2}\right| =$

(A) $|\alpha + \beta| + |\alpha - \beta|$ (B) $2(|\alpha + \beta| + |\alpha - \beta|)$

(C) $|\alpha^2 - \beta^2|$ (D) $(|\alpha + \beta| + |\alpha - \beta|)^2$

3. Find the perimeter of the triangle whose vertices are the roots of the equation $(z + \alpha\beta)^3 = \alpha^3$, where α, β are given complex numbers

(A) $3\sqrt{3}|\beta|$ (B) $\sqrt{3}|\alpha|$

(C) $3|\alpha|$ (D) $3\sqrt{3}|\alpha|$

4. Let z_1, z_2, z_3 be three distinct complex numbers and α, β, γ are three positive real numbers such that

$$\frac{\alpha}{|z_2 - z_3|} = \frac{\beta}{|z_3 - z_1|} = \frac{\gamma}{|z_1 - z_2|}, \text{ then } \frac{\alpha^2}{(z_2 - z_3)} + \frac{\beta^2}{(z_3 - z_1)} + \frac{\gamma^2}{(z_1 - z_2)} \text{ is equal to}$$

(A) $z_1 + z_2 + z_3$ (B) $\bar{z}_1 + \bar{z}_2 + \bar{z}_3$

(C) $z_1 z_2 z_3$ (D) 0

5. If z_1, z_2 are two complex numbers such that $|z_1z_2| = \sqrt{2}$ and $\arg(z_1) - \arg(z_2) = \frac{\pi}{4}$, then $\overline{z}_1 z_2$ is equal to

(A) $1 + i$ (B) $-1 + i$

(C) $-1 - i$ (D) $1 - i$

6. If a, b, c, a_1, b_1, c_1 are non zero complex numbers satisfying $\frac{a}{a_1}+\frac{b}{b_1}+\frac{c}{c_1}=1+i$ and $\frac{a_1}{a}+\frac{b_1}{b}+\frac{c_1}{c}=0$, then $\frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}$ is equal to

(A) $2 + 2i$ (B) $2i$

(C) $2 - 2i$ (D) 2

7. If z lies in 4^{th} quadrant, then $\arg\left(\frac{\overline{z}-z}{2019}\right)$ is

(A) 0 (B) $\frac{\pi}{2}$

(C) $-\frac{\pi}{2}$ (D) π

8. It is given that complex numbers z_1 and z_2 satisfy $|z_1| = 2$ and $|z_2| = 3$. If the included angle of their corresponding vectors is 60º, then $\left|\frac{z_1+z_2}{z_1+z_2}\right| =$

(A) $\frac{\sqrt{133}}{7}$ (B) $\frac{\sqrt{131}}{7}$

(C) $\frac{\sqrt{19}}{7}$ (D) $\frac{\sqrt{134}}{7}$

9. Let $A = \{z : (1 + 2i)\overline{z} + (1 - 2i)z + 2 = 0\}$ and

$B = \{z : (3 + 2i)\overline{z} + (3 - 2i)z + 3 = 0\}$, then

(A) $n(A \cap B) = 0$ (B) $A \subseteq B$

(C) $n(A \cap B) = 1$ (D) $B \subseteq A$

10. If z_1, z_2, z_3 are complex numbers such that $|z_1| = 1, |z_2| = \sqrt{2}, |z_3| = \sqrt{3}$ and $|z_1 + z_2 + z_3| = 2$, then $|3z_1z_2 + z_2z_3 + 2z_3z_1| =$

(A) $2\sqrt{2}$ (B) $2\sqrt{3}$

(C) $\sqrt{6}$ (D) $2\sqrt{6}$

11. Let $z_1, z_2, z_3, \ldots\ldots\ldots, z_n$ be non zero complex numbers with $|z_1| = |z_2| = |z_3| = \ldots\ldots = |z_n|$, then the number

$$z = \frac{(z_1+z_2)(z_2+z_3)(z_3+z_4)\ldots.(z_{n-1}+z_n)(z_n+z_1)}{z_1z_2z_3\ldots..z_n} \text{ is}$$

(A) purely real (B) purely imaginary

(C) imaginary (D) nothing can be said

12. The number of solutions of the equation $z^{2017} = p\bar{z}$, where p is a non zero given complex number is equal to

(A) 2017 (B) 2018

(C) 2019 (D) 2020

13. Let $A(z_1), B(z_2), C(z_3), D(z_4)$ be vertices of a square, satisfy $|z_1 - 1| = |z_2 - 1| = |z_3 - 1| = |z_4 - 1|$. Then $z_1 + z_2 + z_3 + z_4 =$

(A) 0 (B) 1

(C) 2 (D) 4

14. If $\frac{7z_2}{5z_1}$ is purely imaginary, then $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$ is equal to

(A) 1 (B) $\frac{7}{5}$

(C) $\frac{2}{3}$ (D) $\frac{10}{21}$

15. If the imaginary part of $(1 - i)^n (1 + i)^{-n}$ be negative (where $n \in N, n < 100$), then the sum of all the possible values of n is

(A) 625 (B) 1025

(C) 1225 (D) 1238

16. If $|z - i| \le 2$ and $z_0 = 5 + 3i$, then the sum of maximum and minimum values of $|iz + z_0| =$

(A) 7 (B) 8

(C) 10 (D) 11

17. If $|z| = 1$, then the complex number $-2 + 4z$ lies on

(A) circle with centre 2 and radius 4 (B) circle with centre –2 and radius 4

(C) circle with centre 1 and radius 2 (D) circle with centre –1 and radius 2

18. If a, b, c, u, v, w are complex numbers representing the vertices of two triangles such that $c = (1 - r)a + rb$, $w = (1 - r)u + rv$ where r is a complex number, the two triangles

(A) have the same area (B) are similar

(C) are congruent (D) none of these

19. The complex number $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other, for

(A) $x = n\pi$ (B) $x = 0$

(C) $x = \left(n + \frac{1}{2}\right)\pi$ (D) no value of x (where $n \in I$)

20. If $\omega(\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$, then A and B are respectively

(A) 0, 1 (B) 1, 1

(C) 1, 0 (D) –1, 1

21. Let z and w be two non zero complex numbers such that $|z| = |w|$ and $\arg z + \arg w = \pi$, then z equals

(A) w
(B) –w
(C) $\overline{w}$
(D) $-\overline{w}$

22. Let z and w be two complex numbers such that $|z| \le 1$, $|w| \le 1$ and $|z + iw| = |z - i\overline{w}| = 2$, then z equals

(A) 1 or i
(B) i or –i
(C) 1 or –1
(D) i or –1

23. For positive integers n_1 and n_2 the value of the expression

$$(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$$

where $i = \sqrt{-1}$ is real if and only if

(A) $n_1 = n_2 + 1$
(B) $n_1 = n_2 - 1$
(C) $n_1 = n_2$
(D) $n_1 > 0, n_2 > 0$

24. If $|z| = 1$ and $w = \dfrac{z-1}{z+1}$ where $z \ne -1$, then Re(w) is

(A) 0
(B) $-\dfrac{1}{|z+1|^2}$
(C) $\left|\dfrac{2}{z+1}\right|\dfrac{1}{|z+1|^2}$
(D) $\dfrac{\sqrt{2}}{|z+1|^2}$

25. If $1, a_1, a_2, \ldots\ldots\ldots, a_{n-1}$ are the n^{th} roots of unity, then

$(1 - a_1)(1 - a_2)\ldots\ldots.(1 - a_{n-1}) =$

(A) 0
(B) n – 1
(C) n
(D) n + 1

26. It is given that n is an odd integer greater than three, but n is not a multiple of 3. Then $(x + 1)^n - x^n - 1$ is divisible by

(A) $x^3 - 1$
(B) $x^2 - x$
(C) $x^2 - x + 1$
(D) $x^3 + x^2 + x$

27. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C, then $(z_1 - z_3)(z_3 - z_2) =$

(A) $\frac{1}{2}(z_1 - z_2)^2$
(B) $(z_1 - z_2)^2$
(C) $2(z_1 - z_2)^2$
(D) $-(z_1 - z_2)^2$

28. If $x = a + b$, $y = a\alpha + b\beta$, $z = a\beta + b\alpha$ where α, β are imaginary cube roots of unity then $xyz =$

(A) 0
(B) $a + b$
(C) $(a + b)(a^2 + ab + b^2)$
(D) $a^3 + b^3$

29. Let x and y are real numbers satisfying the equation

$$\frac{(1+i)x-2i}{(3+i)}+\frac{(2-3i)y+i}{(3-i)}=i$$

then $x + y =$

(A) 2
(B) 3
(C) –1
(D) 4

30. The value of the expression $1.(2 - \omega)(2 - \omega^2) + 2(3 - \omega)(3 - \omega^2) + \ldots + (n - 1)(n - \omega)(n - \omega^2)$, where ω is an imaginary cube root of unity is

(A) $\frac{1}{2}n(n - 1)(n^2 + 3n + 4)$
(B) $\frac{1}{4}n(n - 1)(n^2 + 3n + 4)$
(C) $\frac{1}{4}n(n + 1)(n^2 + 3n + 4)$
(D) $\frac{1}{2}n(n - 1)(n - 2)(n - 3)$

31. If $z_1 = e^{i\alpha}$, $z_2 = e^{i\beta}$, $z_3 = e^{i\gamma}$ where $\alpha, \beta, \gamma \in R$ and $\frac{z_1}{z_2} + \frac{z_2}{z_3} + \frac{z_3}{z_1} = 1$, then $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) =$

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$

(C) 0 (D) 1

32. The locus of z satisfying the inequality $\log_{\frac{1}{3}}|z+1| > \log_{\frac{1}{3}}|z-1|$ is

(A) $Re(z) < 0$ (B) $Re(z) > 0$

(C) $Im(z) < 0$ (D) $Im(z) > 0$

33. Given that the equation $z^2 + (p + iq)z + r + is = 0$ where p, q, r, s are real and non zero has a real root, then

(A) $qrs = p^2 + qs^2$ (B) $pqr = r^2 + sp^2$

(C) $pqs = rq^2 + s^2$ (D) $prs = q^2 + pr^2$

34. For any two complex numbers z_1 and z_2 and any real numbers a and b, $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$

(A) $(a^2 + b^2)(|z_1|^2 + |z_2|^2)$ (B) $(a^2 + b^2)(|z_1|^2 - |z_2|^2)$

(C) $(a^2 - b^2)(|z_1|^2 + |z_2|^2)$ (D) $(a^2 - b^2)(|z_1|^2 - |z_2|^2)$

35. The sum of all real values of x in interval $[0, 2\pi]$ for which the expression $\dfrac{\sin\frac{x}{2} + \cos\frac{x}{2} + i\tan x}{1 + 2i\sin\frac{x}{2}}$ is real is

(A) $\frac{9\pi}{2}$ (B) $\frac{9\pi}{4}$

(C) $\frac{7\pi}{2}$ (D) $\frac{13\pi}{4}$

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. For a given real number 'a', a > 0, let z satisfy the equation $z|z| + az + i = 0$ then

(A) z is purely real
(B) z is purely imaginary
(C) number of values of z is 1
(D) number of values of z is 2

2. The equation $z^2 + \alpha z + \beta = 0$, where α, β are complex numbers has a

(A) real root, if $(\overline{\alpha}-\alpha)(\alpha\overline{\beta}-\overline{\alpha}\beta)=(\beta-\overline{\beta})^2$
(B) real root if $(\overline{\alpha}-\alpha)(\alpha\overline{\beta}+\overline{\alpha}\beta)=(\beta-\overline{\beta})^2$
(C) a purely imaginary root if $(\alpha+\overline{\alpha})(\alpha\overline{\beta}+\overline{\alpha}\beta)+(\beta-\overline{\beta})^2=0$
(D) a purely imaginary root if $(\alpha+\overline{\alpha})(\alpha\overline{\beta}-\overline{\alpha}\beta)+(\beta-\overline{\beta})^2=0$

3. C is the centre of a given circle $|z - c| = r$. A is a point represented by the complex number 'a' inside the circle and B is a point represented by the complex number 'b' outside the circle, such that C, A, B are collinear and $(CA)(CB) = r^2$. Then

(A) If C lies between A and B, then $b = c - \dfrac{r^2}{\overline{a}-\overline{c}}$
(B) If C lies between A and B, then $b = c + \dfrac{r^2}{\overline{a}-\overline{c}}$
(C) If A lies between C and B, then $b = c - \dfrac{r^2}{\overline{a}-\overline{c}}$
(D) If A lies between C and B, then $b = c + \dfrac{r^2}{\overline{a}-\overline{c}}$

4. Let z_1 be a point lying on curve $|z - 3 + 2i| = 4$ and z_2 be a point lying on curve $|z + 3| = 1$ such that $|z_1 - z_2|$ is minimum, then

(A) $z_1 = \left(-3-\dfrac{3}{\sqrt{10}}\right)+\dfrac{i}{\sqrt{10}}$
(B) $z_1 = \left(-3+\dfrac{3}{\sqrt{10}}\right)-\dfrac{i}{\sqrt{10}}$
(C) $z_2 = \left(3+\dfrac{12}{\sqrt{10}}\right)+i\left(-2-\dfrac{4}{\sqrt{10}}\right)$
(D) $z_2 = \left(3-\dfrac{12}{\sqrt{10}}\right)+i\left(-2+\dfrac{4}{\sqrt{10}}\right)$

5. If $z = x + iy$, $x, y \in R$ and $\operatorname{cosec}^{-1}\left(\frac{z}{1+i}\right)$ be defined, then z may lie on

(A) 1^{st} quadrant (B) 3^{rd} quadrant

(C) real axis (D) imaginary axis

6. Let 'b' be a complex number such that $|b| < 1$ and $z_1, z_2, z_3, ..., z_n$ be the vertices of a 'n' sided polygon, such that $z_r = 1 + b + b^2 + b^3 + ... + b^{r-1}$ $\forall$ $r \in \{1, 2, 3, ...n\}$. Then all the vertices of the polygon lie within or on the circle.

(A) $\left|z - \frac{1}{1-b}\right| = \left|\frac{1}{1-b}\right|$ (B) $\left|z - \frac{1}{1-b}\right| = \left|\frac{b}{1-b}\right|$

(C) $|z - b| = |b|$ (D) $\left|z - \frac{1}{1-b}\right| = \frac{1}{3}$

7. Let $z = (1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)....... (1+\omega^{2^{n-1}})$, $n \in N$ where $\omega = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$, then z is equal to

(A) 1 if n is odd (B) 1 if n is even

(C) $-\omega^2$ if n is odd (D) $-\omega^2$ if n is even

8. Let the equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ has ω and ω^2 as two of its roots, then

(A) the other two roots are those of quadratic equation $ax^2 + (b + a)x + e = 0$

(B) the other two roots are those of quadratic equation $ax^2 + (b - a)x + e = 0$

(C) $2(b + e) = 2a + c + d$

(D) $2(b + e) = a + c + d$

9. Let z_1, z_2 be complex numbers such that $\bar{z}_1 + i\bar{z}_2 = 0$ and $\arg(z_1z_2) = \pi$, then

(A) $\arg(z_1) = \frac{3\pi}{4}$ (B) $\arg(z_1) = \frac{3\pi}{2}$

(C) $\arg(z_2) = -\frac{\pi}{2}$ (D) $\arg(z_2) = \frac{\pi}{4}$

10. If the imaginary part of the expression $\frac{z-1}{e^{i\theta}} + \frac{e^{i\theta}}{z-1}$ be zero, $z \in C$, $\theta \in R$, then

(A) $|z|_{min} = 1$ (B) $|z|_{max} = 3$

(C) $|z|_{min} = 0$ (D) $|z|_{max} = 2$

11. If the lines $\alpha_1\bar{z}+\bar{\alpha}_1 z+\beta_1=0$ and $\alpha_2\bar{z}+\bar{\alpha}_2 z+\beta_2=0$, $\beta_1, \beta_2 \in R$ are mutually perpendicular, then

(A) $\alpha_1\bar{\alpha}_2+\bar{\alpha}_1\alpha_2=0$ (B) $\alpha_1\bar{\alpha}_2=\bar{\alpha}_1\alpha_2$

(C) $\left|\arg\left(\frac{\alpha_1}{\alpha_2}\right)\right|=\frac{\pi}{2}$ (D) $\arg\left(\frac{\alpha_1}{\alpha_2}\right)=0$ or π

12. Let $A(z_1)$, $B(z_2)$, $C(z_3)$, $D(z_4)$ are four distinct points in complex plane such that

$2|z_4-z_1|=|z_4-z_2|+|z_4-z_3|$, $2|z_4-z_2|=|z_4-z_1|+|z_4-z_3|$

$2|z_4-z_3|=|z_4-z_1|+|z_4-z_2|$ and $\frac{z_4-z_1}{z_3-z_2}$ is purely imaginary, then

(A) $\frac{z_4-z_2}{z_3-z_1}$ is purely real (B) $\frac{z_4-z_3}{z_2-z_1}$ is purely imaginary

(C) $3z_4=z_1+z_2+z_3$ (D) $|z_4-z_1|=|z_4-z_2|$

13. If $\alpha^3=5+i\sqrt{2}$ and $\beta^3=5-i\sqrt{2}$, then possible real values of $\alpha+\beta$ is/are

(A) 2 (B) –2

(C) $1-\sqrt{6}$ (D) $1+\sqrt{6}$

14. If a point $P(z_1)$ lying on curve $|z|=2$; a pair of tangents are drawn to curve $|z|=1$ meeting it at points $Q(z_2)$ and $R(z_3)$, then

(A) $\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)\left(\frac{4}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right)=9$

(B) $\left|\arg\left(\frac{z_2}{z_3}\right)\right|=\frac{2\pi}{3}$

(C) Point with complex representation $\left(\frac{z_1+z_2+z_3}{3}\right)$ will lie on $|z|=1$

(D) Point with complex representation $\left(\frac{z_1}{2}\right)$ will lie on $|z|=1$.

15. If $z=-2+2\sqrt{3}i$, then $z^{2n}+2^{2n}z^n+2^{4n}$, $n\in N$ may be equal to

(A) 0 (B) 3.2^{2n}

(C) 3.2^{4n} (D) 2^{4n}

16. Let 'z' satisfy the equation $iz^3 + z^2 - z + i = 0$, then which of the following may be correrct ?

(A) $\arg(z) = \frac{\pi}{4}$

(B) $z = \frac{1}{\sqrt{2}}(-1+i)$

(C) $\arg(z) = -\frac{\pi}{4}$

(D) $|z| = 1$

17. Let two distinct points $P(z_1)$ and $Q(z_2)$ lie on curve $C : z+\bar{z} = 2|z-1|$ such that $\arg(z_1 - z_2) = \frac{\pi}{4}$, then

(A) P, Q lie on ellipse

(B) $\text{Im}(z_1 + z_2) = 2$

(C) P, Q lie on parabola

(D) point of intersection of tangents to 'C' at P and Q lie on curve $\text{Im}(z) = 1$

18. Consider a curve 'C' given by equation $|z - 3 - 2i| = \left|z\cos\left(\frac{\pi}{4} - \arg z\right)\right|$, then

(A) represents a parabola

(B) 'C' represents a ellipse

(C) point $P\left(\frac{7+3i}{2}\right)$ lies on 'C'

(D) Point $P\left(\frac{7+3i}{4}\right)$ lies on 'C'

19. If 1, $z_1, z_2, z_3, \ldots\ldots, z_{n-1}$ are 'n' n^{th} roots of unity, then the value of $\frac{1}{3-z_1} + \frac{1}{3-z_2} + \frac{1}{3-z_3} + \ldots.. + \frac{1}{3-z_{n-1}}$ is equal to

(A) $\dfrac{\sum_{r=1}^{n} r\ 3^{r-1}}{\sum_{r=1}^{n} 3^{r-1}}$

(B) $\dfrac{\sum_{r=1}^{n-1} r\ 3^{r-1}}{\sum_{r=1}^{n} 3^{r-1}}$

(C) $\frac{n3^{n-1}}{3^n - 1} - \frac{1}{2}$

(D) $\frac{n3^n}{3^n - 1} - \frac{1}{2}$

20. Let points $P(z_1)$, $Q(z_2)$ and $R(z_3)$ be three points in argand plane such that $|z_1 + z_2| = |z_1| - |z_2|$ and $|(1 - i)z_1 + iz_3| = |z_1| + |z_3 - z_1|$, then

(A) $|z_2 - z_3| = |z_2 + z_3 - 2z_1|$

(B) P, Q, R are vertices of right triangle

(C) P, Q, R are vertices of equilateral triangle

(D) P, Q, R lie on circle with radius $\frac{1}{2}|z_3 - z_2|$

21. If $|2z + 5| = |6z - 9|$, then $|z|^2 = a\,\text{Re}(z) + b$, where a and b are real numbers, then

(A) $a = 4$
(B) $4(a + b) = 9$
(C) $2(a + b) = 9$
(D) $a + 4b + 3 = 0$

22. If the principle argument of complex number $(-1 + i)^{50}$ is $\frac{n\pi}{2}$ and the equation $z^2 - az + b + 2i = 0$ has a real root. If $a = (1 + i)^{-n}$, $b \in R$, then possible roots of equation are

(A) $2 + i$
(B) -1
(C) 2
(D) $-1 + i$

23. Let points $A(z_1)$, $B(z_2)$, $C(z_3)$ are 3 distinct collinear points such that B lies between A & C, then

(A) $\frac{|z_3| - |z_2|}{|z_3 - z_2|} \le \frac{|z_3| - |z_1|}{|z_3 - z_1|}$
(B) $\frac{|z_3| - |z_2|}{|z_3 - z_2|} \ge \frac{|z_3| - |z_1|}{|z_3 - z_1|}$
(C) $\frac{|z_3| - |z_1|}{|z_3 - z_1|} \ge \frac{|z_2| - |z_1|}{|z_2 - z_1|}$
(D) $\frac{|z_3| - |z_1|}{|z_3 - z_1|} \le \frac{|z_2| - |z_1|}{|z_2 - z_1|}$

24. Let a, b, c be distinct non zero complex numbers with $|a| = |b| = |c|$ and each of the equations $az^2 + bz + c = 0$ and $bz^2 + cz + a = 0$ has a root having modulus 1, then

(A) $b^2 = ac$
(B) $c^2 = ab$
(C) $|a - b| = |b - c| = |c - a|$
(D) $a^2 = bc$

25. Let z_1, z_2, z_3 be complex number such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then

(A) $z_1^2 + z_2^2 + z_3^2 = 1$
(B) $z_1^2 + z_2^2 + z_3^2 = 0$
(C) $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_3 z_1} + \frac{z_3^2}{z_1 z_2} = 3$
(D) $z_1 z_2 + z_2 z_3 + z_3 z_1 = 0$

26. The locus of point P(z) such that z, z^2 and z^3 are the vertices of a right angled triangle can be

(A) line, Re(z) = –1, $z \neq -1$

(B) line, Re(z) = 0, $z \neq 0$

(C) circle with centre (–1) and radius 1, $z \neq 0$

(D) circle with centre $\left(-\frac{1}{2}\right)$ and radius $= \frac{1}{2}$, $z \neq -1, 0$

27. If the equation $x^2 + (p + ip')x + (q + iq') = 0$ has two equal roots, then (p, p′, q, q′ are real)

(A) $p^2 + (p')^2 = 4q$

(B) $pp' = 4q'$

(C) $p^2 - (p')^2 = 4q$

(D) $pp' = 2q'$

28. Let $A(z_1)$, $B(z_2)$, $C(z_3)$ are points in complex plane such that $z_1|z_2 - z_3| - z_2|z_3 - z_1| - z_3|z_1 - z_2| = 0$, then which of the following may be correct?

(A) A, B, C are collinear such that A lies between B and C

(B) A, B, C are collinear such that B lies between A and C

(C) A, B, C are collinear such that C lies between A and B

(D) O(0) is the centre of circle which touches the sides of triangle ABC.

29. Let z_1, z_2, z_3 be three distinct non zero complex numbers, then

(A) there always exist real numbers p, q, r such that $pz_1 + qz_2 + rz_3 = 0$

(B) if $pz_1 + pz_2 + rz_3 = 0$ and $p + q + r = 0$, p, q, r, $\in$ R, then z_1, z_2, z_3 are collinear.

(C) there always exist complex numbers p, q, r such that $pz_1 + qz_2 + rz_3 = 0$ and $p + q + r = 0$

(D) there always exist real numbers p, q, r such that $pz_1 + qz_2 + rz_3 = 0$ and $p + q + r = 0$

30. Let $A(z_1)$, $B(z_2)$, $C(z_3)$ are the vertices of an equilateral triangle, then

(A) $z_1 + z_2\omega + z_3\omega^2 = 0$ or $z_1 + z_2\omega^2 + z_3\omega = 0$, where $\omega = e^{\frac{i2\pi}{3}}$

(B) $\frac{1}{z-z_1} + \frac{1}{z-z_2} + \frac{1}{z-z_3} = 0$, where $z = \frac{z_1+z_2+z_3}{3}$

(C) $\frac{1}{z_1-z_2} + \frac{1}{z_2-z_3} + \frac{1}{z_3-z_1} = 0$

(D) $\begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_2 & z_3 & z_1 \end{vmatrix} = 0$

31. Let $z_1 = 1 + i$, $z_2 = 2 - i$. If z_1, z_2 represent the points A and B. The possible Centroid rep. of equilateral triangle ABC is/are

(A) $\frac{3+2\sqrt{3}+i\sqrt{3}}{6}$ (B) $\frac{3-2\sqrt{3}-i\sqrt{3}}{6}$

(C) $\frac{9+2\sqrt{3}+i\sqrt{3}}{6}$ (D) $\frac{9-2\sqrt{3}-i\sqrt{3}}{6}$

32. Let $2 \le |z| \le 4$ m = minimum value of $\left|z+\frac{1}{z}\right|$. M = maximum value of $\left|z+\frac{1}{z}\right|$, then

(A) $m = \frac{3}{2}$ (B) $m = \frac{5}{2}$

(C) $M = \frac{15}{4}$ (D) $M = \frac{17}{4}$

33. If a, b, c are real numbers and z is a complex number such that $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)z$, then $\frac{1+iz}{1-iz} =$

(A) $\frac{a+ib}{1+c}$ (B) $\frac{(1-c)}{(a-ib)}$

(C) $\frac{c-1}{a-ib}$ (D) $-\frac{a+ib}{1+c}$

34. The centre of circle circumscribing the square ABCD is z_0. Let A is z_1, then the centroid of ΔABC can be

(A) $z_0\left(1-\frac{i}{3}\right)-\frac{i}{3}z_1$
(B) $z_0\left(1-\frac{i}{3}\right)+\frac{i}{3}z_1$
(C) $z_0\left(1+\frac{i}{3}\right)-\frac{i}{3}z_1$
(D) $z_0\left(1+\frac{i}{3}\right)+\frac{i}{3}z_1$

35. The sum of smallest positive values of arguments of all the roots of the equation $z^n = k$, $n \in N$, $k \in R$, $k \neq 0$ is odd multiple of π if

(A) $k < 0$, n is odd
(B) $k < 0$, n is even
(C) $k > 0$, n is odd
(D) $k > 0$, n is even

36. Let z_1, z_2 be non zero complex numbers, such that $z_1 = a + ib$, $z_2 = c + id$, a, b, c, d $\in R$ satisfy the equation $\frac{z_1+z_2}{z_1}=\frac{z_2}{z_1+z_2}$

(A) $a^2 + c^2 = b^2 + d^2$
(B) $a^2 + b^2 = c^2 + d^2$
(C) atleast one of a, c is non zero
(D) atleast one of b, d is non zero

37. If $z_1 = a + ib$ and $z_2 = c + id$ are complex number such that $|z_1| = |z_2| = 1$ and $\text{Re}(z_1\bar{z}_2)=0$, then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfies :

(A) $|\omega_1| = 1$
(B) $|\omega_2| = 1$
(C) $\text{Re}(\omega_1\bar{\omega}_2)=0$
(D) $\text{Im}(\omega_1\bar{\omega}_2)=0$

SECTION-3

COMPREHENSION TYPE QUESTIONS

Comprehension (Q.1 to Q.3) :

Given that $z_1 + z_2 + z_3 = p$, $z_1 + z_2\omega + z_3\omega^2 = q$ and $z_1 + z_2\omega^2 + z_3\omega = r$, where ω is usual cube root of unity and z_1, z_2, z_3, p, q, r may be complex numbers. Then

1. $z_2 =$

(A) $\frac{p - q\omega^2 + r\omega}{3}$ (B) $\frac{p + q\omega^2 + r\omega}{3}$ (C) $\frac{p + q\omega + r\omega^2}{3}$ (D) $\frac{p - q\omega + r\omega^2}{3}$

2. $z_3 =$

(A) $\frac{p - q\omega^2 + r\omega}{3}$ (B) $\frac{p + q\omega^2 + r\omega}{3}$ (C) $\frac{p + q\omega + r\omega^2}{3}$ (D) $\frac{p - q\omega + r\omega^2}{3}$

3. $\frac{|p|^2 + |q|^2 + |r|^2}{|z_1|^2 + |z_2|^2 + |z_3|^2} =$

(A) 1 (B) 3 (C) 6 (D) 9

Comprehension (Q.4 to Q.6) :

Let $A(z_1)$, $B(z_2)$, $C(z_3)$, $D(z_4)$ be the vertices of trapezium ABCD in argand plane. If $|z_1 - z_2| = 4$, $|z_3 - z_4| = 10$ and the diagonals AC, BD intersect at P. Also given that $\arg\left(\frac{z_4 - z_2}{z_3 - z_1}\right) = \frac{\pi}{2}$ and $\arg\left(\frac{z_4 - z_1}{z_3 - z_2}\right) = \frac{\pi}{4}$.

4. Area of trapezium ABCD is equal to

(A) $\frac{70}{3}$ (B) $\frac{100}{3}$ (C) $\frac{140}{3}$ (D) $\frac{160}{3}$

5. Area of triangle PCB is equal to

(A) $\frac{100}{21}$ (B) $\frac{200}{21}$ (C) $\frac{400}{21}$ (D) $\frac{500}{21}$

6. $|CP - DP|$ is equal to

(A) $\frac{10}{\sqrt{21}}$ (B) $\frac{12}{\sqrt{21}}$ (C) $\frac{15}{\sqrt{21}}$ (D) $\frac{16}{\sqrt{21}}$

COMPREHENSION (Q.7 TO Q.9) :

On the sides AB and BC of ΔABC, squares are drawn with centres D and E respectively such that points C and D lie on the same side of line AB and the points A and E lie on opposite sides of line BC. A, B, C are represented by complex numbers, 1, ω, ω^2 respectively $\left(\omega = \frac{-1+i\sqrt{3}}{2}\right)$.

7. Angle between AC and DE is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

8. The length of DE is

(A) $\sqrt{\frac{3}{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\sqrt{6}$

9. The length of AE is

(A) $\frac{3+\sqrt{3}}{4}$ (B) $\frac{3-\sqrt{3}}{2}$ (C) $\frac{3-\sqrt{3}}{4}$ (D) $\frac{3+\sqrt{3}}{2}$

COMPREHENSION (Q.10 TO Q.12) :

Let the point P(z) lies on the curve 'C' whose equation is given by $\left|z+\frac{1}{z}\right| = 2$ in the argand plane. If 'O' represents origin, then

10. The largest value of OP is equal to

(A) $\sqrt{5}$ (B) $\sqrt{2}$ (C) $1+\sqrt{5}$ (D) $1+\sqrt{2}$

11. If $\arg(z) = \frac{\pi}{6}$, then the sum of all possible values of $|z|$ is equal to

(A) $\sqrt{5}$ (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{3}$

12. 'C' in argand plane represents

(A) Two concentric circles (B) Two orthogonal circles

(C) Two circles non intersecting (D) Two externally touching circles

COMPREHENSION (Q.13 TO Q.15) :

Consider triangle ABC having vertices at points $A(2e^{i\frac{\pi}{4}})$, $B(2e^{i\frac{11\pi}{12}})$, $C(2e^{-i\frac{5\pi}{12}})$.

Let the incircle of ΔABC touches the sides BC, CA, AB at points D (z_D), E (z_E) and F(z_F) respectively. Then

13. Let P(z) be any point on the incircle, then $AP^2 + BP^2 + CP^2$ is equal to

(A) 3 (B) 9 (C) 15 (D) 18

14. $\text{Im}\left(\frac{1}{z_D}+\frac{1}{z_E}+\frac{1}{z_F}\right)$ is equal to

(A) 0 (B) 1 (C) –1 (D) $\frac{1}{3}$

15. If the altitude through vertex A cuts the circumcircle of ΔABC at Q, then complex number representing Q is

(A) $-1-i$ (B) $1-i$ (C) $-\sqrt{2}-\sqrt{2}i$ (D) $-\sqrt{2}+\sqrt{2}i$

COMPREHENSION (Q.16 TO Q.18) :

Suppose A, B, C are three collinear points corresponding complex numbers $z_1 = ai$, $z_2 = \frac{1}{2} + bi$, $z_3 = 1 + ci$ (a, b, c being real numbers), respectively. Consider a curve 'C' whose equation is given by $z = z_1\cos^4t + 2z_2\cos^2t\sin^2t + z_3\sin^4t$, $t \in R$.

16. 'C' in argand plane represents

(A) Straight line (B) Circle (C) Parabola (D) Ellipse

17. A line bisecting AB and parallel to AC meet 'C' in point P, then point P is given by

(A) $\left(-\frac{1}{2},\frac{a+c-2b}{2}\right)$ (B) $\left(\frac{1}{2},\frac{a+c-2b}{4}\right)$

(C) $\left(\frac{1}{2},\frac{a+c+2b}{2}\right)$ (D) $\left(\frac{1}{2},\frac{a+c+2b}{4}\right)$

18. Point P lies

(A) Inside ΔABC (B) Outside ΔABC

(C) On the side of ΔABC (D) Nothing can be said

COMPREHENSION (Q.19 TO Q.20) :

Let $p = x + y + z + a\,(y + z - 2x)$

$q = x + y + z + a\,(z + x - 2y)$

$r = x + y + z + a(x + y - 2z)$

19. $p^2 + q^2 + r^2 - pq - qr - rp$ is equal to

(A) $a^2(x^2 + y^2 + z^2 - xy - yz - zx)$
(B) $3a^2(x^2 + y^2 + z^2 - xy - yz - zx)$
(C) $9a^2(x^2 + y^2 + z^2 - xy - yz - zx)$
(D) 0

20. $p^3 + q^3 + r^3 - 3pqr$ is equal to

(A) $3a^2(x^3 + y^3 + z^3 - 3xyz)$
(B) $9a^2(x^3 + y^3 + z^3 - 3xyz)$
(C) $27a^2(x^3 + y^3 + z^3 - 3xyz)$
(D) 0

COMPREHENSION (Q.21 TO Q.23) :

If $\frac{1}{a+\omega} + \frac{1}{b+\omega} + \frac{1}{c+\omega} + \frac{1}{d+\omega} = \frac{2}{\omega}$ and $\frac{1}{a+\omega^2} + \frac{1}{b+\omega^2} + \frac{1}{c+\omega^2} + \frac{1}{d+\omega^2} = \frac{2}{\omega^2}$,

where ω and ω^2 are imaginary cube roots of unity, then

21. $a + b + c + d =$

(A) 0
(B) abcd
(C) –2abcd
(D) 2abcd

22. $abc + abd + acd + bcd =$

(A) 0
(B) 1
(C) 2
(D) 4

23. $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} + \frac{1}{d+1} =$

(A) 0
(B) 1
(C) 2
(D) 4

COMPREHENSION (Q.24 TO Q.26) :

Consider z_1, z_2 and z_3 are complex numbers such that $z_1z_2 = -80 - 320i$, $z_2z_3 = 60$ and $z_3z_1 = -96 + 24i$; where $i = \sqrt{-1}$; then

24. The possible value of $\arg(z_2)$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

25. The possible value of z_1 is

(A) $-10(1 + i)$ (B) $4(5 - 3i)$

(C) $4(5 + 3i)$ (D) $4(4 + 5i)$

26. $|z_1 + z_2 + z_3|^2$ is equal to

(A) 79 (B) 59 (C) 64 (D) 74

SECTION-4

MATCH THE COLUMN

1. If z_1, z_2, z_3, z_4 are the roots of the equation $z^4 + z^3 + z^2 + z + 1 = 0$, then

	Column-I		Column-II
(A)	$(z_1^2 - 1)(z_2^2 - 1)(z_3^2 - 1)(z_4^2 - 1) =$	(P)	-1
(B)	$(z_1^2 + 1)(z_2^2 + 1)(z_3^2 + 1)(z_4^2 + 1) =$	(Q)	0
(C)	$z_1^4 + z_2^4 + z_3^4 + z_4^4 =$	(R)	1
(D)	The least value of $[\|z_1 + z_2\|] =$ ([.] denote greatest integer function)	(S)	4
		(T)	5

2.

	Column-I		**Column-II**
(A)	The least value of $8\|z-7\|+6\|z-5\|$; $z \in C$	(P)	12
(B)	The least value of $\left\|\frac{z-7+11i}{\sqrt{2}}\right\|+\left\|\frac{z+5-i}{\sqrt{2}}\right\|+\left\|\frac{z+4}{\sqrt{2}}\right\|$; $z \in C$ is	(Q)	13
(C)	Find the least value of $\|z-3-4i\|+\|z-i\|+\|z\|+\|z-1\|$; $z \in C$	(R)	24
(D)	The least value of $\|z-3\|^2+\|z-5+2i\|^2+\|z-1+i\|^2$ is	(S)	10
		(T)	$5+\sqrt{2}$

3.

	Column-I		**Column-II**
(A)	If z_1 and z_2 are two non zero complex numbers such that $\|z_1z_2\|=2$ and $\arg z_1-\arg z_2=\frac{\pi}{2}$, then $3i\,\overline{z_1}z_2=$	(P)	1
(B)	If z_1 satisfies the equation $\|z-3\|=4$ and z_2 satisfies the equation $\|z+1\|+\|z-1\|=3$. If m, M be the minimum and maximum values of $\|z_1-z_2\|$ respectively. Then $m+2M=$	(Q)	2
(C)	If $z=(3-i)+\lambda(4-3i)$, then the minimum value of $\|z\|$ is equal to $(\lambda \in R)$	(R)	3
(D)	If z lies on the curve $\arg(z-1)=\frac{\pi}{4}$. Then the maximum value of $\|\|z+2-5i\|-\|z-5i\|\|$ is equal to	(S)	6
		(T)	17

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. Let ω be imaginary cube root of unity, then number of distinct complex numbers 'z' satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to

2. If α, β, γ are cube roots of unity, then the value of $\begin{vmatrix} e^{\alpha} & e^{2\alpha} & e^{3\alpha}-1 \\ e^{\beta} & e^{2\beta} & e^{3\beta}-1 \\ e^{\gamma} & e^{2\gamma} & e^{3\gamma}-1 \end{vmatrix}$ is equal to

3. If z is a complex number satisfying the equation $|z + i| + |z - i| = 8$ on the complex plane. Let m and M be the smallest and the largest value of $|z|$ respectively, then $[m + M] =$

([.] denote greatest integer function)

4. Let L be any line not passing through origin and $P(z_1)$ be the foot of perpendicular from origin to the line. Let $Q(z_2)$ be any point different from P on L. Then $\frac{z_2}{z_1} + \frac{\overline{z}_2}{\overline{z}_1}$ is equal to

5. Let a, b, c are given distinct numbers, then the number of distinct values of the expression $(\alpha + \beta\omega + \gamma\omega^2)^3$, where (α, β, γ) is a permutation of a, b, c is equal to $\left(\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$.

6. If z_1, z_2, z_3 be the vertices of ΔABC, taken in anticlockwise direction and z_0 its circumcentre, then $\left|\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \frac{(z_0 - z_3)}{(z_0 - z_2)}\frac{\sin 2C}{\sin 2B}\right|$ is equal to

7. Let α be a complex number such that $|\alpha| = 1$. If the equation $\alpha z^2 + z + 1 = 0$ has a purely imaginary root, then $\tan^2(\arg \alpha) = \frac{a+\sqrt{b}}{c}$, where a, b, c are coprime natural number, then $a + b + c =$

8. Let $a = \sum_{r=1}^{8} \tan^2\left(\frac{r\pi}{17}\right)$ and $b = \prod_{r=1}^{8} \tan^2\left(\frac{r\pi}{17}\right)$, then $\frac{a}{b}$ is equal to :

9. If z_1, z_2 are complex numbers such that $|z_1| = \frac{5}{3}$, then $\left|\frac{75 - 27 z_1 \bar{z}_2}{z_1 - z_2}\right|$ is equal to

10. Find the value of $|3k|$, $k \in R$ so that line joining $A(-5 + 7i)$ and $B(-2i)$ is perpendicular to the line joining $C(1 - 3i)$ and $D(4 + ik)$.

11. Consider the set, $A = \{z \in C | z = x - 1 + xi, x \in R\}$

Find the number of complex numbers z, $z \in A$ such that $|z| \le |\omega|$ for all $\omega \in A$.

12. Let $z \in C$ with $Re(z) > 1$, then $\left|\frac{1}{z} - \frac{1}{2}\right| < L$. Find the smallest value of L.

13. Let z_1, z_2, z_3 be complex numbers with $|z_1| = |z_2| = |z_3| = r$, $r > 0$. The maximum value of $|z_1 - z_2|\,|z_2 - z_3| + |z_2 - z_3|\,|z_3 - z_1| + |z_3 - z_1|\,|z_1 - z_2|$ is equal to λr^2, then λ is equal to

14. Let R be the region formed by the set of point P(x, y) in the complex plane such that $|z| \le \sqrt{10}$ where $Re(z) = \sqrt{x^2 + 4}$, $Im(z) = \sqrt{y - 4}$. Then the area of region R is equal to $\sqrt{N}$, where N =

15. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, then

$|z_1 + z_2 + z_3|^2 + |-z_1 + z_2 + z_3|^2 + |z_1 - z_2 + z_3|^2 + |z_1 + z_2 - z_3|^2$ is equal to

16. If $\cos\theta_1 + 2\cos\theta_2 + 3\cos\theta_3 = 0 = \sin\theta_1 + 2\sin\theta_2 + 3\sin\theta_3$, then $\sin 3\theta_1 + 8\sin 3\theta_2 + 27\sin 3\theta_3 = \lambda\sin(\theta_1 + \theta_2 + \theta_3)$, where $\lambda =$

17. Let α, β are real numbers, and

$((\cos\alpha + i\sin\alpha) + (\cos\beta + i\sin\beta))^7 + ((\cos\alpha - i\sin\alpha) + (\cos\beta - i\sin\beta)^7$

$= 2^m \cos^7\left(\dfrac{\alpha-\beta}{2}\right) \cos(n(\alpha + \beta))$, then $m + 2n =$

18. If $z + \dfrac{1}{z} = \sqrt{3}$, then $\displaystyle\sum_{r=1}^{5}\left(z^r + \frac{1}{z^r}\right)^2 =$

19. Given that f(z) = the real part of complex number z. For example f(–5 + 2i) = –5.

The value of $\displaystyle\sum_{n=1}^{600}\log_2\left|f\left((1+i\sqrt{3})^n\right)\right|$ is equal to

20. If z is the imaginary 5th root of unity, then $\dfrac{z^4}{1+z^4} + \dfrac{z}{1+z} + \dfrac{z^2}{1+z^2} + \dfrac{z^4}{z+z^4} =$

21. Let a, b, c are positive integers which satisfy $c = (a + ib)^3 - 107i$, then $a + b + c$ is equal to

22. Let $\omega_1, \omega_2, \omega_3, ..., \omega_n$ be complex numbers. A line L in the argand plane is called a mean line for the points $\omega_1, \omega_2, ..., \omega_n$ if L contains points (complex numbers) $z_1, z_2, ..., z_n$ such that $\sum_{i=1}^{n}(z_i - \omega_i) = 0$

For the numbers $\omega_1 = 32 + 170i$, $\omega_2 = -7 + 64i$, $\omega_3 = -9 + 200i$, $\omega_4 = 1 + 27i$ and $\omega_5 = -14 + 43i$, there is a unique mean line with y-intercept 3. Then the slope of this mean line is equal to

23. The sets $A = \{z | z^{18} = 1\}$ and $B = \{\omega | \omega^{48} = 1\}$ consider the set $C = \{z\omega | z \in A$ and $\omega \in B\}$. How many distinct elements are in C?

24. Let A be the area of region in argand plane that consists of all points z such that both $\frac{z}{40}$ and $\frac{40}{\overline{z}}$ have real and imaginary parts between 0 and 1. Then [A] is equal to ([.] denote greatest integer function).

25. The equation $z^{10} + (13z - 1)^{10} = 0$ has 10 complex roots $z_1, \overline{z}_1, z_2, \overline{z}_2, ..., z_5, \overline{z}_5$. Then the value of $\frac{1}{|z_1|^2} + \frac{1}{|z_2|^2} + \frac{1}{|z_3|^2} + \frac{1}{|z_4|^2} + \frac{1}{|z_5|^2}$ is equal to

26. Let P be the product of the roots of equation $z^6 + z^4 + z^3 + z^2 + 1 = 0$, that have positive imaginary part and if $P = r(\cos\theta^\circ + i\sin\theta^\circ)$, where $r > 0$ and $0 \le \theta < 360$. Then θ is equal to

27. A function f is defined on the complex numbers by $f(z) = (a + bi)z$, where a and b are real numbers. This function has the property that image of each point in the complex plane is equidistant from that point and the origin. Given that $|a + bi| = 8$ and that $b^2 = \frac{m}{n}$, where m and n are relatively prime positive integers, find m +n.

28. Let $F(z) = \frac{z+i}{z-i}$ for all complex numbers $z \neq i$ and let $z_n = F(z_{n-1})$ for all $n \in N$. Given that $z_0 = \frac{1}{137} + i$ and $z_{2020} = a + ib$, where $a, b \in R$, find a + b .

29. The polynomial $P(x) = (1 + x + x^2 + ... + x^{17})^2 - x^{17}$ has 34 complex roots of the form $z_k = r_k e^{i(2\pi a_k)}$, k = 1, 2, 3,....... 34 with $0 < a_1 < a_2 < a_3 < .... < a_{34} < 1$ and $r_k > 0$. Given that $a_1 + a_2 + a_3 + a_4 + a_5 = \frac{m}{n}$, where m and n are relatively prime natural numbers, then m + n =

30. There is a complex number z with imaginary part 164 and a positive integer n such that $\frac{z}{z+n} = 4i$. Find n.

31. For how many positive integers 'n' less than or equal to 1000 is $(\sin t + i\cos t)^n = \sin(nt) + i\cos(nt)$ true for all real t ?

32. The polynomial $f(z) = az^{2018} + bz^{2017} + cz^{2016}$ has real coefficients not exceeding 2019 and $f\left(\frac{1+\sqrt{3}i}{2}\right) = 2015 + 2019\sqrt{3}i$. Find the remainder when f(1) is divided by 1000.

Answer Key

Single Choice Questions

1. C	**2.** D	**3.** D	**4.** D	**5.** D	**6.** B
7. B	**8.** A	**9.** C	**10.** D	**11.** A	**12.** C
13. D	**14.** A	**15.** C	**16.** C	**17.** B	**18.** B
19. D	**20.** B	**21.** D	**22.** C	**23.** D	**24.** A
25. C	**26.** D	**27.** A	**28.** D	**29.** A	**30.** B
31. D	**32.** A	**33.** C	**34.** A	**35.** C	

One or More Than One Correct Questions

1. B,C	**2.** A,C	**3.** A,D	**4.** B,D	**5.** A,B	**6.** A,B
7. B,C	**8.** B,D	**9.** A,D	**10.** C,D	**11.** A,C	**12.** B,C,D
13. B,C,D	**14.** A,B,C,D	**15.** A,C	**16.** B,C,D	**17.** B,C,D	**18.** A,D
19. B,C	**20.** A,B,D	**21.** B,D	**22.** C,D	**23.** B,C	**24.** A,B,C,D
25. B,C,D	**26.** A,B,D	**27.** C,D	**28.** A,D	**29.** A,B	**30.** A,B,C,D
31. C,D	**32.** A,D	**33.** A,B	**34.** B,C	**35.** A,D	**36.** B,C,D
37. A,B,C					

Comprehension Type Questions

1. B	**2.** C	**3.** B	**4.** C	**5.** B	**6.** A
7. B	**8.** A	**9.** D	**10.** D	**11.** A	**12.** B
13. C	**14.** A	**15.** C	**16.** C	**17.** D	**18.** A
19. C	**20.** C	**21.** D	**22.** C	**23.** C	**24.** B
25. C	**26.** D				

Match The Column

1. A → T ; B → R ; C → P ; D → Q
2. A → P ; B → P ; C → T ; D → S
3. A → S ; B → T ; C → P ; D → Q

Subjective Type Questions

1. 1	**2.** 0	**3.** 7	**4.** 2	**5.** 2	**6.** 1
7. 8	**8.** 8	**9.** 45	**10.** 4	**11.** 1	**12.** 0.5
13. 9	**14.** 384	**15.** 56	**16.** 18	**17.** 15	**18.** 8
19. 179900	**20.** 2	**21.** 205	**22.** 163	**23.** 144	**24.** 571
25. 850	**26.** 276	**27.** 259	**28.** 275	**29.** 482	**30.** 697
31. 250	**32.** 53				

Previous Year Questions

SECTION-1

1. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals :

(A) $\pi - \theta$ (B) $-\theta$

(C) $\frac{\pi}{2} - \theta$ (D) θ

[IIT JEE Main 2013]

2. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$

(A) Is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(B) Is equal to $\frac{5}{2}$ **[IIT JEE Main 2014]**

(C) Lies in the interval (1, 2)

(D) Is strictly greater than $\frac{5}{2}$

3. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a : **[JEE Mains 2015]**

(A) circle of radius $\sqrt{2}$ (B) straight line parallel to x-axis

(C) straight line parallel to y-axis (D) circle of radius 2

4. A value of θ for which $\dfrac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is : **[JEE Mains 2016]**

(A) –
(B) $\dfrac{\pi}{6}$
(C) $\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$
(D) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

5. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$.

If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to : **[JEE Mains 2017]**

(A) –1
(B) 1
(C) – z
(D) z

6. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to : **[JEE Main 2018]**

(A) 2
(B) –1
(C) 0
(D) 1

SECTION-2

1. (a) If $i = \sqrt{-1}$, then $4 + 5\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{365}$ is equal to :

(A) $1 - i\sqrt{3}$
(B) $-1 + i\sqrt{3}$
(C) $i\sqrt{3}$
(D) $-i\sqrt{3}$

(b) For complex numbers z and ω, prove that, $|z|^2\,\omega - |\omega|^2\,z = z - \omega$ if and only if, $z = \omega$ or $z\overline{\omega} = 1$ **[JEE '99, 2 + 10 (out of 200)]**

2. (i) If $\alpha = e^{\frac{2\pi i}{7}}$ and $f(x) = A_0 + \sum\limits_{k=1}^{20} A_k x^k$, then find the value of,

$f(x) + f(\alpha x) + \ldots\ldots + f(\alpha^6 x)$ independent of α. **[REE '99, 6]**

(ii) Let $\alpha + i\beta$; $\alpha, \beta \in R$, be a root of the equation $x^3 + qx + r = 0$; $q, r \in R$. Find a real cubic equation, independent of α and β, whose one root is 2α. **[REE '99, 3]**

3. (a) If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is :

(A) equal to 1 (B) less than 1

(C) greater than 3 (D) equal to 3

(b) If $\arg(z) < 0$, then $\arg(-z) - \arg(z) =$

[JEE 2000 (Screening) 1 + 1 out of 35]

(A) π (B) $-\pi$

(C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

4. Given, $z = \cos\frac{2\pi}{2n+1} + i\sin\frac{2\pi}{2n+1}$, 'n' a positive integer, find the equation whose roots are, $\alpha = z + z^3 + ... + z^{2n-1}$ and $\beta = z^2 + z^4 + ... + z^{2n}$.

[REE 2000 (Mains) 3 out of 100]

5. Find all those roots of the equation $z^{12} - 56z^6 - 512 = 0$ whose imaginary part is positive. **[REE 2000, 3 out of 100]**

6. (a) The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is :

(A) of area zero (B) right-angled isosceles

(C) equilateral (D) obtuse – angled isosceles

(b) Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin. Then n must be of the form : **[JEE 2001 (Scr) 1 + 1 out of 35]**

(A) 4k + 1 (B) 4k + 2

(C) 4k + 3 (D) 4k

7. (a) Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is :

(A) 3ω (B) $3\omega(\omega - 1)$

(C) $3\omega^2$ (D) $3\omega(1 - \omega)$

(b) For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is : **[JEE 2002 (Scr) 3+3]**

(A) 0 (B) 2

(C) 7 (D) 17

(c) Let a complex number α , $\alpha \neq 1$, be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$ where p, q are distinct primes.

Show that either $1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + ... + \alpha^{q-1} = 0$, but not both together. **[JEE 2002, (5)]**

8. (a) If z_1 and z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$ then prove that

$$\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right| < 1.$$

(b) Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$

where $|a_r| < 2$. **[JEE-03, 2 + 2 out of 60]**

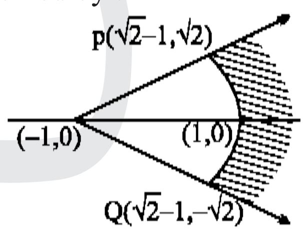
9. (a) ω is an imaginary cube root of unity. If $(1 + \omega^2)^m = (1 + \omega^4)^m$, then least positive integral value of m is : **[JEE 2004 (Scr)]**

(A) 6 (B) 5

(C) 4 (D) 3

(b) Find centre and radius of the circle determined by all complex numbers z = x + i y satisfying $\left|\frac{(z-\alpha)}{(z-\beta)}\right| = k$, where $\alpha = \alpha_1 + i\alpha_2, \beta = \beta_1 + i\beta_2$ are fixed complex and $k \neq 1$. **[JEE 2004, 2 out of 60]**

10. (a) The locus of z which lies in shaded region is best represented by :



(A) $z : |z + 1| > 2, |\arg(z + 1)| < \pi/4$

(B) $z : |z - 1| > 2, |\arg(z - 1)| < \pi/4$

(C) $z : |z + 1| < 2, |\arg(z + 1)| < \pi/2$

(D) $z : |z - 1| < 2, |\arg(z - 1)| < \pi/2$

(b) If a, b, c are integers not all equal and w is a cube root of unity ($w \neq 1$), then the minimum value of $|a + bw + cw^2|$ is : **[JEE 2005 (Scr), 3 + 3]**

(A) 0 (B) 1

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

(c) If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2+\sqrt{3}\,i$. Find the other vertices of square. **[JEE 2005 (Mains), 4]**

11. If $w = \alpha + i\beta$ where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\frac{w - \overline{w}z}{1-z}$ is purely real, then the set of values of z is : **[JEE 2006, 3]**

(A) $\{z : |z| = 1\}$ (B) $\{z : z = \overline{z})$

(C) $\{z : z \neq 1\}$ (D) $\{z : |z| = 1, z \neq 1\}$

12. (a) A man walks a distance of 3 units from the origin towards the North-East (N 45° E) direction. From there, he walks a distance of 4 units towards the North-West (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is :

(A) $3e^{i\pi/4} + 4i$ (B) $(3-4i)e^{i\pi/4}$

(C) $(4+3i)e^{i\pi/4}$ (D) $(3+4i)e^{i\pi/4}$

(b) If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on :

(A) a line not passing through the origin

(B) $|z| = \sqrt{2}$

(C) the x-axis

(D) the y-axis **[JEE 2007, 3+3]**

13. (a) A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by :

(A) $6 + 7i$ (B) $-7 + 6i$

(C) $7 + 6i$ (D) $-6 + 7i$

(b) Comprehension (3 questions together)

Let A, B, C be three sets of complex numbers as defined below :

[JEE 2008, 3 + 4 + 4 + 4]

$$A = \{z : \text{Im}\, z \geq 1\}$$

$$B = \{z : |z - 2 - i| = 3\}$$

$$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$$

(i) The number of elements in the set $A \cap B \cap C$ is :

(A) 0
(B) 1
(C) 2
(D) ∞

(ii) Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between:

(A) 25 and 29
(B) 30 and 34
(C) 35 and 39
(D) 40 and 44

(iii) Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$.

Then, $|z| - |w| + 3$ lies between :

(A) –6 and 3
(B) –3 and 6
(C) –6 and 6
(D) –3 and 9

14. Let $z = \cos\theta + i \sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is : **[JEE 2009]**

(A) $\frac{1}{\sin 2^\circ}$
(B) $\frac{1}{3\sin 2^\circ}$
(C) $\frac{1}{2\sin 2^\circ}$
(D) $\frac{1}{4\sin 2^\circ}$

15. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is :

[JEE 2009]

(A) 48
(B) 32
(C) 40
(D) 80

16. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)\, z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg (w) denotes the principal argument of a non-zero complex numbers w, then : **[JEE 2010]**

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$
(B) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z - z_2)$
(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$
(D) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z_2 - z_1)$

17. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to : **[JEE 2010]**

18. [**Note :** Here z takes the values in the complex plane and Im z and Rez denote, respectively, the imaginary part and the real part of z] **[JEE 2010]**

	Column-I		**Column-II**
(A)	The set of points z satisfying $\lvert z - i\lvert z\rvert\rvert = \lvert z + i\lvert z\rvert\rvert$	(P)	an ellipse with eccentricity $\frac{4}{5}$ is contained in or equal to
(B)	The set of points z satisfying $\lvert z + 4\rvert + \lvert z - 4\rvert = 10$	(Q)	the set of points z satisfying Im z = 0 is contained in or equal to
(C)	If $\lvert w\rvert = 2$, then the set of points $z = w - 1/w$ is	(R)	the set of points z satisfying $\lvert\text{Im } z\rvert \le 1$ contained in or equal to
(D)	If $\lvert w\rvert = 1$, then the set of points $z = w + 1/w$ is	(S)	the set of point z satisfying $\lvert\text{Re } z\rvert \le 2$ contained in or equal to
		(T)	the set of points z satisfying $\lvert z\rvert \le 3$

19. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is : **[JEE 2011]**

(A) 2
(B) 6
(C) 4
(D) 8

20. If z is any complex number satisfying $|z-3-2i| \le 2$, then the minimum value of $|2z-6+5i|$ is : **[JEE 2011]**

21. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that : **[JEE 2011]**

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\dfrac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is :

22. Match the statements given in Column I with the values given in Column II

[JEE 2011]

	Column-I		Column-II
(A)	If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(P)	$\dfrac{\pi}{6}$
(B)	If $\int_a^b (f(x)-3x)\,dx = a^2 - b^2$, then the value of $f\left(\dfrac{\pi}{6}\right)$ is	(Q)	$\dfrac{2\pi}{3}$
(C)	The value of $\dfrac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)\,dx$ is $z = w\ -$	(R)	$\dfrac{\pi}{3}$
(D)	The maximum value of $\left\|\text{Arg}\left(\dfrac{1}{1-z}\right)\right\|$ for $\|z\|=1$, $z^1 1$ is given by	(S)	π
		(T)	$\dfrac{\pi}{2}$

23. Match the statements given in Column I with the intervals/union of intervals given in Column II **[JEE 2011]**

(A)	The set $\left\{\text{Re}\left(\dfrac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z\rvert = 1,\ z^1 \pm 1\right\}$	(P)	$(-\infty, -1) \cup (1, \infty)$
(B)	The domain of the function $f(x) = \sin^{-1}\left(\dfrac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(Q)	$(-\infty, 0) \cup (0, \infty)$
(C)	If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $z = w - \left\{f(\theta) : 0 \le \theta < \dfrac{\pi}{2}\right\}$ is	(R)	$[2, \infty)$
(D)	If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then $f(x)$ is increasing in	(S)	$(-\infty, -1] \cup [1, \infty)$
		(T)	$(-\infty, 0] \cup [2, \infty)$

24. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a **cannot** take the value : **[JEE 2012]**

(A) -1 (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

25. Let complex numbers α and $\frac{1}{\alpha}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$ **[IIT JEE Advance 2013]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

26. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n = **[IIT JEE Advance 2013]**

(A) 57 (B) 55

(C) 58 (D) 56

27. Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, ...\}$. Further $H_1 = \left\{z \in C : \text{Re}z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \text{Re}z < \frac{-1}{2}\right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1Oz_2 =$ **[IIT JEE Advance 2013]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

COMPREHENSION (Q.28 TO Q.29) **[IIT JEE ADVANCE 2013]**

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left[z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right]$

and $S_3 = \{z \in C : \text{Re } z > 0\}$.

28. $\min_{z \in S} |1 - 3i - z| =$

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$

(C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

29. Area of S =

(A) $\frac{10\pi}{3}$

(B) $\frac{20\pi}{3}$

(C) $\frac{16\pi}{3}$

(D) $\frac{32\pi}{3}$

30. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; k = 1, 2, ...9. **[IIT JEE Advance 2014]**

	Column-I		Column-II
(P)	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	(1)	True
(Q)	There exists a $k \in \{1, 2, ..., 9\}$ such that $z_1 \cdot z = z_k$ has no solution z	(2)	False
(R)	in the set of complex numbers $\frac{\|1-z_1\|\|1-z_2\|.............\|1-z_9\|}{10}$ equals	(3)	1
(S)	$1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals	(4)	2

Codes :

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	1	2	3	4
(D)	2	1	4	3

31. For any integer k, let $a_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\dfrac{\sum_{k=1}^{12} |a_{k+1} - a_k|}{\sum_{k=1}^{3} |a_{4k-1} - a_{4k-2}|}$ is **[JEE Advance 2015]**

32. Let a, b $\in$ R and $a^2 + b^2 \neq 0$. Suppose $S = \left\{z \in C : z = \frac{1}{a + ibt}, t \in R, t \neq 0\right\}$, where $i = \sqrt{-1}$. If z = x + iy and $z \in S$, then (x, y) lies on **[JEE Advance 2016]**

(A) The circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$

(B) The circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

(C) The x-axis for $a \neq 0, b = 0$

(D) The y-axis for $a = 0, b \neq 0$

33. Let $z = \frac{-1 + \sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and r, s $\in$ {1, 2, 3}. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs(r, s) for which $P^2 = -I$ is : **[JEE Advance 2016]**

34. Let a, b, x and y be real numbers such that a – b = 1 and y ≠ 0. If the complex number z = x + iy satisfies $\text{Im}\left(\frac{az + b}{z + 1}\right) = y$, then which of the following is(are) possible value(s) of x? **[JEE Advanced 2017]**

(A) $-1 + \sqrt{1 - y^2}$

(B) $1 - \sqrt{1 + y^2}$

(C) $1 + \sqrt{1 + y^2}$

(D) $-1 - \sqrt{1 - y^2}$

35. For a non-zero complex number z, let arg(z) denote the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement(s) is(are) **FALSE**?

(A) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f : R \to (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in R$, is continuous at all points of R, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line

36. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ $(x, y \in \mathbb{R},\ i = \sqrt{-1})$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE?

[JEE Advance 2018]

(A) If L has exactly one element, then $|s| \ne |t|$

(B) If $|s| = |t|$, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) Let L has more than one element, then L has infinitely many elements

Answer Key

SECTION-1

1. D **2.** C **3.** D **4.** D **5.** C **6.** D

SECTION-2

1. **(a)** C

2. (i) $7A_0 + 7A_7x^7 + 7A_{14}x^{14}$; (ii) $x^3 + qx - r = 0$

3. **(a)** A **(b)** A

4. $z^2 + z + \dfrac{\sin^2 n\theta}{\sin^2\theta} = 0$, where $\theta = \dfrac{2\pi}{2n+1}$

5. $\pm 1 + i\sqrt{3}$, $\dfrac{\left(\pm\sqrt{3}+i\right)}{\sqrt{2}}$, $\sqrt{2}\,i$

6. **(a)** C, **(b)** D

7. **(a)** B; **(b)** B

9. **(a)** D ; **(b)** Centre $\equiv \dfrac{k^2\beta - \alpha}{k^2 - 1}$,

Radius $= \dfrac{1}{(k^2-1)}\sqrt{|\alpha - k^2\beta|^2 - \left(k^2.|\beta|^2 - |\alpha|^2\right).\left(k^2 - 1\right)}$

10. A, **(b)** B, **(c)** $z_2 = -\sqrt{3}\,i$; $z_3 = \left(1-\sqrt{3}\right)+i$; $z_4 = \left(1+\sqrt{3}\right)-i$

11. D **12.** **(a)** D ; **(b)** D

13. **(a)** D ; **(b)** (i) B; (ii) C; (iii) D

14. D **15.** A **16.** A, C, D **17.** 1

18. A – Q, R ; B – P ; C – P, S, T ; D – Q, R, S, T **19.** A **20.** 5

21. Bonus **22.** $A \to Q$; $B \to P$; $C \to S$; $D \to T$

23. $A \to S$; $B \to T$; $C \to R$; $D \to R$; **24.** D **25.** C **26.** B, C, D

27. C, D **28.** C **29.** B **30.** C **31.** 4

32. A,C,D **33.** 1 **34.** AD **35.** ABD **36.** ACD

8 CHAPTER

LOGARITHM

SECTION-1

SINGLE CHOICE QUESTIONS

1. If $\log_4 5 = a$ and $\log_5 6 = b$, then $\log_3 2$ is equal to :

(A) $2ab + 1$ (B) $\frac{1}{2ab-1}$

(C) $\frac{1}{2ab+1}$ (D) $\frac{1}{ab-1}$

2. Let (x, y) be the solution of following equation

$[5(x + 1)]^{\ln 5} = (2y)^{\ln 2}$ and $(x + 1)^{\ln 2} = 5^{\ln y}$, then x is equal to :

(A) $\frac{1}{5}$ (B) $-\frac{1}{5}$

(C) $\frac{4}{5}$ (D) $-\frac{4}{5}$

3. The value of ab if $\log_8 a + \log_4 b^2 = 5$ and $\log_8 b + \log_4 a^2 = 7$ is equal to :

(A) 64 (B) 256

(C) 512 (D) 1024

4. Let x,y,z and w be positive numbers such that $\log_x w = 24$, $\log_y w = 40$ and $\log_{xyz} w = 12$, then $\log_z w =$

(A) 120 (B) 40

(C) 80 (D) 60

5. The value of $\log_2 x$ if $\log_2(\log_8 x) = \log_8(\log_2 x)$ is equal to :

(A) 27 (B) $3\sqrt{3}$

(C) 3 (D) $\sqrt{3}$

6. The number $\frac{2}{\log_4(2000)^6}+\frac{3}{\log_5(2000)^6}$ is equal to

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 1

7. If $\log_6 a + \log_6 b + \log_6 c = 6$, where a,b and c are positive integers that form an increasing G.P. and b – a is the square of an integer. Then a + b + c =

(A) 97

(B) 101

(C) 109

(D) 111

8. Suppose $x \in \left[0, \frac{\pi}{2}\right]$ and $\log_{(24\sin x)}(24\cos x) = \frac{3}{2}$. Then $\cot^2 x =$

(A) 3

(B) 4

(C) 6

(D) 8

9. If $\log_b n = 2$ and $\log_n (2b) = 2$, then the value of $\log_{b^2}(2b) =$

(A) 0

(B) 1

(C) 2

(D) 4

10. The number of real values of x satisfying the equation

$\log_{10}(\log_{10} x)+\log_{10}[\log_{10}(x^3)-2]=0$ is :

(A) 0

(B) 1

(C) 2

(D) 3

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. Consider the equation E : $9^{-|x-2|} - 4\cdot 3^{-|x-2|} - a = 0$, a∈R, and S be the set of all real values of 'a' for which eqn. (E) has solutions. Then values of x satisfying (E) is/are

(A) $x = 2 + \log_3(2-\sqrt{4+a})$, $a \in S$ (B) $x = 2 - \log_3(2-\sqrt{4+a})$, $a \in S$

(C) $x = 2 + \log_3(2+\sqrt{4+a})$, $a \in S$ (D) $x = 2 - \log_3(2+\sqrt{4+a})$, $a \in S$

2. Let x, y are positive real numbers and satisfy the system of equations

$x^{x+y} = y^n$ & $y^{x+y} = x^{2n}y^n$ where $n > 0$, then

(A) $x = \dfrac{2n+2-\sqrt{4n+1}}{2}$ (B) $x = \dfrac{\sqrt{1+8n}-1}{2}$

(C) $y = \dfrac{\sqrt{4n+1}-1}{2}$ (D) $y = \dfrac{4n+1-\sqrt{1+8n}}{2}$

3. For each ordered pair of read numbers (x, y) satisfying

$$\log_2(2x+y) = \log_4(x^2+xy+7y^2)$$

There is a real number k such that

$$\log_3(3x+y) = \log_9(3x^2+4xy+ky^2)$$

Then possible values of k is/are :

(A) 9 (B) 12

(C) 14 (D) 21

4. Let a > 1, b > 1, satisfy

$\log_a(\log_a(\log_a 2) + \log_a 24 - 128) = 128$ and $\log_a(\log_a b) = 256$, then

(A) $a^2 = (128)^{\frac{1}{128}}$ (B) $b = 2^{194}$

(C) $b = 2^{192}$ (D) $a^2 = (64)^{\frac{1}{64}}$

5. The possible real values of k for which the equation $\log_{10}(kx) = 2\log_{10}(x+2)$ has exactly one solution is/are :

(A) –5 (B) –4

(C) 4 (D) 8

6. Positive integers a and b satisfy the condition

$$\log_2\left[\log_{2^a}\left(\log_{2^b}(2^{1000})\right)\right]=0$$

Then the possible values of a + b is/are :

(A) 501 (B) 252
(C) 128 (D) 66

7. If $\log_{10}5 = a$ and $\log_{10}3 = b$, then :

(A) $\log_{30}8=\dfrac{3(1+a)}{b+1}$ (B) $\log_{30}8=\dfrac{3(1-a)}{b+1}$

(C) $\log_{243}(32)=\dfrac{(1-a)}{b}$ (D) $\log_{40}(15)=\dfrac{a+b}{3-2a}$

8. The expression $2^{\left(\sqrt{\log_a\sqrt[4]{ab}+\log_b\sqrt[4]{ab}}-\sqrt{\log_a\sqrt[4]{\frac{b}{a}}+\log_b\sqrt[4]{\frac{a}{b}}}\right)\sqrt{\log_a b}}$ is equal to :

(A) $2^{\log_a b}$ if $1 < a < b$ (B) 2 if $1 < a < b$

(C) $2^{\log_a b}$ if $1 < b < a$ (D) 2 if $1 < b < a$

SECTİON-3

COMPRHENSION BASED QUESTIONS

COMPREHENSION (Q.1 TO Q.3):

Let $\log_3 N = \alpha_1 + \beta_1$

$\log_5 N = \alpha_2 + \beta_2$

$\log_7 N = \alpha_3 + \beta_3$

where α_1, α_2 and α_3 are integers and $\beta_1, \beta_2, \beta_3 \in [0, 1)$

1. Number of intergral values of N if $\alpha_1 = 4$ and $\alpha_2 = 2$:

(A) 46 (B) 45 (C) 44 (D) 47

2. Largest integral value of N if $\alpha_1 = 5$, $\alpha_2 = 3$ and $\alpha_3 = 2$.

(A) 342 (B) 343 (C) 243 (D) 242

3. Difference of largest and smallest integral values of N if $\alpha_1 = 5$, $\alpha_2 = 3$ and $\alpha_3 = 2$

(A) 97 (B) 100 (C) 98 (D) 99

COMPREHENSION-2 (Q.4 TO Q.6) :

Let x, y and z be real numbers satisfying the system of equations

$\log_2 (xyz - 3 + \log_5 x) = 5$

$\log_3 (xyz - 3 + \log_5 y) = 4$

$\log_4 (xyz - 3 + \log_5 z) = 4$ then

4. x =

(A) 5^{-100} (B) 5^{-90} (C) 5^{-80} (D) 5^{-81}

5. $|\log_5 x| + |\log_5 y| + |\log_5 z| =$

(A) 256 (B) 260 (C) 265 (D) 271

6. z =

(A) 5^{131} (B) 5^{125} (C) 5^{132} (D) 5^{134}

COMPREHENSION-2 (Q.7 TO Q.9) :

Let x, y and z be positive real numbers that satisfy

$2\log_x(2y) = 2\log_{2x}(4z) = \log_{2x^4}(8yz) \neq 0$, then

7. x =

(A) $2^{-\frac{1}{6}}$ (B) 1 (C) $2^{-\frac{1}{2}}$ (D) $2^{-\frac{1}{3}}$

8. $y^5 z =$

(A) $\frac{1}{32}$ (B) $\frac{1}{256}$ (C) $\frac{1}{64}$ (D) $\frac{1}{128}$

9. $xy^5z = \frac{1}{2^{m/n}}$, where m, n are relatively prime natural numbers, then m + n =

(A) 47 (B) 48 (C) 49 (D) 50

SECTION-4

SUBJECTIVE TYPE QUESTIONS

1. The number of non-negative integral values of 'a' for which the equation $\log_{10}(ax) - 2\log_{10}(x+1) =$ has exactly one root is

2. If $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ for $-1 < x < 1$ and if $f\left(\frac{3x+x^3}{1+3x^2}\right) = k\,f(x)$, then the value of k is

3. There are N positive integers 'b' such that $10 \le \log_{10}(\log_{10} b) \le 100$. Let P be the sum of digits of N and 'q' be the sum of digits of P, then $\left[\frac{q}{100}\right]$ is equal to (where [·] denote greatest integer function).

4. Find the sum of all possible greatest integer values of x, $x \in R$ and x satisfies the equation $x^2 - x - 1 = 2^x - \log_2(x^2 + 2^x)$

5. Let 'S' be the maximum value of $8.(27)^{\log_6 x} + 27(8)^{\log_6 x} - x^3$ for all positive values of x. Then $\sqrt[3]{S}$ is equal to

6. The number of negative integral values of x satisfying the inequality

$$\log_{\frac{1}{6}}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0 \text{ is}$$

7. There are N numbers of positive integers 'n' less than or equal to 2018 having the property that $[\log_2(n)]$ is odd. Find the largest digit of N. (where [·] denotes greatest integer function)

8. The maximum value of $\log_5(75x + 100y)$ if $x^2 + y^2 = 25$ is

9. If a, b, c are in G.P. and $\log_a c$, $\log_b a$, $\log_c b$ are in A.P., then the value of $7\log_c b - 2(\log_c b)^2$ is equal to

10. The number of real values of x satisfying the equation

$$(25)^{\log_{125}(2\sqrt{2})} + \log_{|x|}\left(\frac{|x|-\sqrt{3}}{\sqrt{3}|x|-2\sqrt{2}}\right) = \log_5\left(\frac{\sqrt{3}|x|+2\sqrt{2}}{|x|+\sqrt{3}}\right)\log_{|x|}5$$ is equal to

11. Let x, y, z are positive real numbers satisfy $xyz = 10^{81}$ and

$\log_2 x \log_2(yz) + (\log_2 y)(\log_2 z) = 468\,(\log_2 10)^2$, then the magnitude of the vector $\frac{1}{25}\left((\log_{10} x)\hat{i} + (\log_{10} y)\hat{j} + (\log_{10} z)\hat{k}\right)$ is equal to

12. $$\sum_{r=1}^{160} \frac{\left(\log_{(r+1)}3\right)\left(\log_{(r+2)}3\right)}{\left(\log_{\left(\frac{r+2}{r+1}\right)}3\right)}(\log_3 2)(\log_3 162)$$ is equal to

13. If $2^{x+y} = 6^y$ and $2^x = 3(2^{y+1})$, then $x = (\log_a 3)(\log_b 6)$, where ab is equal to

14. The number of positive roots of the equation $\log_{(x+\alpha-1)}\left(\frac{4}{x+1}\right) = \log_\alpha 2$, where $\alpha > 1$ is a real number is

15. Assume that a, b, c, d are positive integers such that $a^5 = b^4$, $c^3 = d^2$ and $c - a = 19$. Then $d - b$ is equal to

16. How many real numbers x satisfy the equation $\frac{1}{5}\log_2 x = \sin(5\pi x)$?

17. Find the positive integer n for which $[\log_2 1] + [\log_2 2] + [\log_2 3] + ... + [\log_2 n] =$ 2018, where $[\cdot]$ denote S greatest integer function.

18. Find the remainder if the product of positive roots of $\sqrt{2019x}^{\log_{2019} x} = x^2$ is divided by 1000.

19. How many positive integers n, n < 1000 are such that $[\log_2 n]$ is a positive even integer.

20. Determine the number of ordered pairs (a, b) of integers such that

$\log_a b + 6\log_b a = 5,\ 2 \le a \le 2019$ and $2 \le b \le 2019$

21. The sequence $a_1, a_2, a_3, ...$ is a G.P. of positive numbers. Given that

$\log_8 a_1 + \log_8 a_2 + \log_8 a_3 + ... + \log_8 a_{12} = 2006$, find the number of distinct values of a_1.

22. Find b, $b \ge 2$ satisfying the equations $3\log_b(\sqrt{x}\log_b x) = 56$ and

$3\log_{\log_b x}(x) = 54$, where x > 1.

Answer Key

Single Choice Questions

1. B	**2.** D	**3.** C	**4.** D	**5.** B	**6.** A
7. D	**8.** D	**9.** C	**10.** B		

One or More Than One Correct Questions

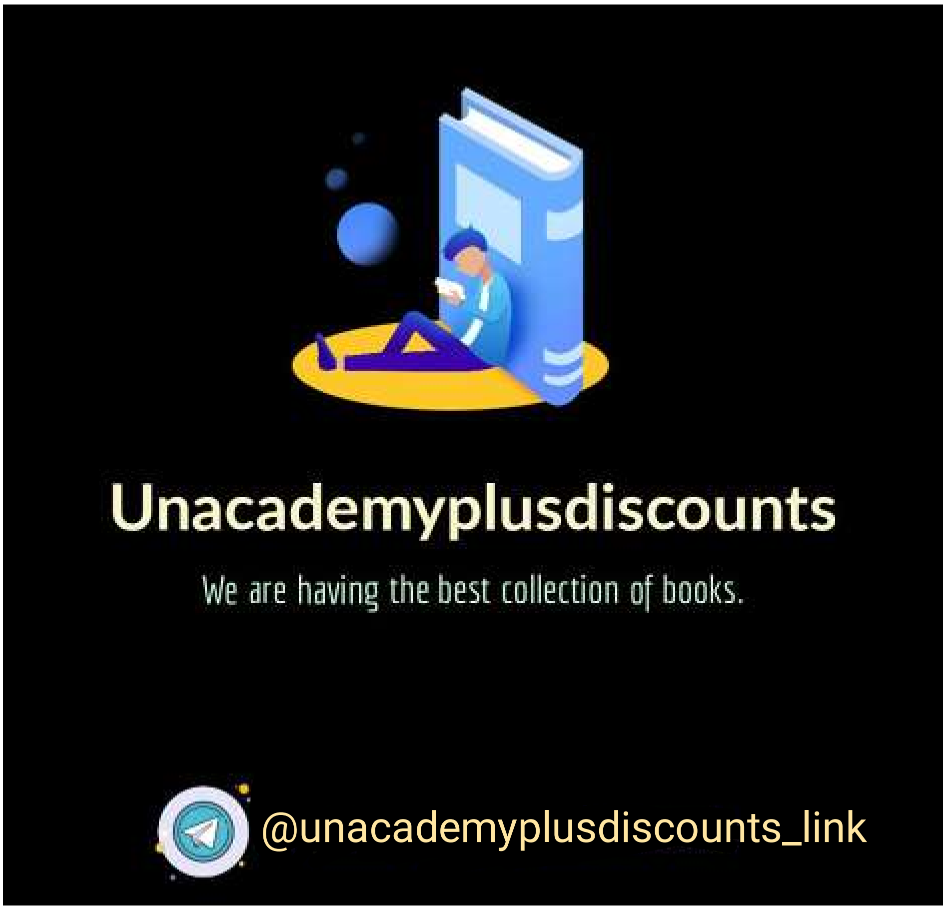
1. A, B	**2.** B, D	**3.** A, D	**4.** C, D	**5.** A, B, D	**6.** A, B, C
7. B, C, D	**8.** B, C				

Comprehension Based Question

1. C	**2.** A	**3.** D	**4.** B	**5.** C	**6.** D
7. A	**8.** D	**9.** C			

Subjective Type Questions

1. 1	**2.** 3	**3.** 8	**4.** 5	**5.** 6	**6.** 0
7. 8	**8.** 4	**9.** 2	**10.** 4	**11.** 3	**12.** 4
13. 3	**14.** 1	**15.** 757	**16.** 159	**17.** 315	**18.** 361
19. 340	**20.** 54	**21.** 46	**22.** 216		



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Solution

Solutions

CHAPTER 1

QUADRATIC EQUATIONS

SECTION-1

SINGLE CHOICE QUESTIONS

1. $\sin x + \cos x = y^2 - y + a$

$$y^2 - y + a > \sqrt{2} \quad \forall\, y \in R$$

$$\Rightarrow \quad \frac{1}{4} - \frac{1}{2} + a > \sqrt{2}$$

$$\Rightarrow \quad a > \sqrt{2} + \frac{1}{4}$$

2. $-2\cos 2x = a^2 + a$

For equation to have solution

$$-2 \le a^2 + a \le 2$$

$$\Rightarrow \quad a^2 + a - 2 \le 0$$

$$\Rightarrow \quad a \in [-2, 1]$$

3. Put $\sqrt{x} = t$

$$t - 3 - \frac{2}{t-2} \le 0$$

$$\Rightarrow \quad \frac{t^2 - 5t + 4}{t-2} \le 0$$

$$\Rightarrow \quad \sqrt{x} \in [0, 1] \cup (2, 4]$$

$$\Rightarrow \quad x \in [0, 1] \cup (4, 16]$$

4. Let α, β be roots of the given equation $\frac{4a+3b+2c}{a} > 0 \Rightarrow 4 - 3(\alpha+\beta) + 2\alpha\beta > 0$

$$\alpha\beta - 2\alpha - \beta + 2 + \alpha\beta - \alpha - 2\beta + 2 > 0$$

$\Rightarrow$ $$(\alpha-1)(\beta-2) + (\alpha-2)(\beta-1) > 0$$

If α, β both belong to (1, 2)

$\Rightarrow$ $$(\alpha-1)(\beta-2) < 0 \text{ and } (\alpha-2)(\beta-1) < 0$$

$\Rightarrow$ $$\frac{4a+3b+2c}{a} < 0$$

which is contradiction

5. $x = \frac{5+\sqrt{21}}{2}$, $y = \frac{5-\sqrt{21}}{2}$, $x + y = 5$, $xy = 1$

$$x^4 + y^4 + (x+y)^4 = (x^2+y^2)^2 - 2(xy)^2 + (x+y)^4$$

$$= [(x+y)^2 - 2xy]^2 - 2x^2y^2 + (x+y)^4$$

$$= (25-2)^2 - 2 + 5^4 = 1152$$

6. $$\frac{p^3+1}{(p^4-p^2)(p-1)} = \frac{(p+1)(p^2-p+1)}{p^2(p-1)^2(p+1)} = \frac{p^2-p+1}{(p^2-p)^2} = \frac{3+1}{(3)^2} = \frac{4}{9}$$

7. $(x-19)(x-97) - p = (x-\alpha)(x-\beta)$

$\Rightarrow$ $$(x-\alpha)(x-\beta) + p = (x-19)(x-97)$$

$\Rightarrow$ Roots of $(x-\alpha)(x-\beta) = -P$ are 19 and 97

8. $D = (a+b+c)^2 - 4(a^2+b^2+c^2) = -3(a^2+b^2+c^2) + 2(ab+bc+ca)$

$$= -(a^2+b^2+c^2) - 2(a^2+b^2+c^2 - ab - bc - ca)$$

$$= -(a^2+b^2+c^2) - ((a-b)^2 + (b-c)^2 + (c-a)^2)$$

$\Rightarrow$ $$D < 0$$

9. $(a-1)x^2 - (a^2-a)x - (a+2)x + a^2 + 2a = 0$

$$[(a-1)x - (a+2)](x-a) = 0$$

The roots of equations are $\frac{a+2}{a-1}$, a and $\frac{b+2}{b-1}$, b

For common root

$$\frac{a+2}{a-1} = b \Rightarrow ab - b - a = 2$$

$$\Rightarrow (a-1)(b-1) = 3$$

$$\Rightarrow (a, b) = (2, 4) \text{ or } (4, 2)$$

$$\therefore \frac{a^b + b^a}{\frac{1}{a^b} + \frac{1}{b^a}} = a^b b^a = 2^4 4^2 = 256$$

10. $\Delta_1 = 1 - 4q_1$

$\Delta_2 = p^2 - 4q_2$

$$\Delta_1 + \Delta_2 = p^2 + 1 - 4(q_1 + q_2)$$

$$= p^2 + 1 - 4(p - 1)$$

$$= (p - 2)^2 + 1 > 0$$

$\Rightarrow$ at least one of Δ_1, or $\Delta_2 > 0$

11. x satisfying inequality $ax^2 + bx + c > 0$ is (2, 3)

$$\Rightarrow a < 0$$

$$\therefore x^2 + \frac{b}{a}x + \frac{c}{a} < 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} < 0$$

$$(x-2)(x-3) = x^2 - 5x + 6 < 0$$

$$\Rightarrow \frac{b}{a} = -5, \frac{c}{a} = 6 \Rightarrow b = -5a, c = 6a$$

$$\therefore cx^2 + bx + a < 0 \Rightarrow a(6x^2 - 5x + 1) < 0$$

$$\Rightarrow 6x^2 - 5x + 1 > 0 \Rightarrow x \in \left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$$

12. $kx^2 - kx - 1 < 0 \ \forall \ x \in R$

k = 0 holds

OR

$k < 0$ and $D < 0$

$$k^2 + 4k < 0 \Rightarrow k \in (-4, 0)$$

Hence, $k \in (-4, 0]$

13. $\frac{1}{2}\left(\frac{1}{x}-\frac{1}{x+2}+\frac{1}{x+2}-\frac{1}{x+4}+\frac{1}{x+4}-\frac{1}{x+6}\right)=\frac{1}{5}-\frac{1}{2}\left(\frac{1}{x+6}-\frac{1}{x+8}\right)$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x}-\frac{1}{x+8}\right)=\frac{1}{5}$$

$$\Rightarrow \quad x^2+8x-20=0$$

$$\Rightarrow \quad x=-10, 2$$

14. $\left(x_1+\frac{1}{x_1}\right)\left(x_2+\frac{1}{x_2}\right)\left(x_3+\frac{1}{x_3}\right)$

$$=x_1x_2x_3+\frac{x_1x_2}{x_3}+\frac{x_2x_3}{x_1}+\frac{x_1x_3}{x_2}+\frac{x_1}{x_2x_3}+\frac{x_2}{x_3x_1}+\frac{x_3}{x_1x_2}+\frac{1}{x_1x_2x_3}$$

$$=x_1x_2x_3+\frac{x_1^2x_2^2+x_2^2x_3^2+x_1^2x_3^2}{x_1x_2x_3}+\frac{x_1^2+x_2^2+x_3^2}{x_1x_2x_3}+\frac{1}{x_1x_2x_3}$$

$$x_1^2+x_2^2+x_3^2=(x_1+x_2+x_3)^2-2(x_1x_2+x_2x_3+x_3x_1)=-2(3)=-6$$

$$x_1^2x_2^2+x_2^2x_3^2+x_3^2x_1^2=(x_1x_2+x_2x_3+x_3x_1)^2-2x_1x_2x_3(x_1+x_2+x_3)=9$$

$$\Rightarrow \quad \left(x_1+\frac{1}{x_1}\right)\left(x_2+\frac{1}{x_2}\right)\left(x_3+\frac{1}{x_3}\right)=-5+\frac{9}{-5}+\frac{-6}{-5}+\frac{1}{-5}=-\frac{29}{5}$$

15. Put $x^2-6=t$

$$(t+1)^4+(t-1)^4=16$$

$$\Rightarrow \quad t^4+6t^2-7=0=(t^2+7)(t^2-1)$$

$$\Rightarrow \quad x^2-6=\pm 1$$

$$\Rightarrow \quad x=\pm\sqrt{7}, \pm\sqrt{5}$$

16. Adding we get

$$x^2+6y+y^2+4z+z^2+2x=-14$$

$$\Rightarrow \quad (x+1)^2+(y+3)^2+(z+2)^2=0$$

$$\Rightarrow \quad x=-1, y=-3, z=-2$$

17. $P(x)=ax(x-1)(x-2)\ ......\ (x-14)$

18. Let α be the common root

$$\Rightarrow \quad \alpha^2 - 4\alpha + k = 0 \quad ...(1)$$

$$\alpha^2 + k\alpha - 4 = 0 \quad ...(2)$$

(2) and (1)

$$\Rightarrow \quad (k + 4)\alpha - (4 + k) = 0$$

$$\Rightarrow \quad \alpha = 1 \text{ or } k = -4$$

$$\alpha = 1 \Rightarrow k = 3$$

k = –4 gives both roots common

k = 3 satisfies the condition.

19. $x^4 + ax^2 + bx - c = 0$ with roots 1, 2, 3, α

$$\Rightarrow \quad 1 + 2 + 3 + \alpha = 0 \quad \Rightarrow \quad \alpha = -6$$

$$-c = 1\,(2)\,(3)\,(\alpha) = 6\alpha = -36$$

$$\Rightarrow \quad c = 36$$

20. $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 = ((a + b)^2 - 2ab)^2 - 2a^2b^2$

$$= \left(\lambda^2 + \frac{1}{\lambda^2}\right) - \frac{1}{2\lambda^4}$$

$$= \lambda^4 + \frac{1}{2\lambda^4} + 2$$

$$\geq 2 + \sqrt{2}$$

21. $x^3 - x^2 + x - 1 = 0$ with roots a, b, c

$$\Rightarrow \quad (x^2 + 1)(x - 1) = 0$$

$$\Rightarrow \quad x = 1, i, -i$$

22. If x is a root, then $\frac{1}{x}$ is also a root

$\therefore$ maximum number of distinct real roots = 5.

23. $ax^2 + bx + c = a(x-\alpha)(x-\beta)$

$$\therefore \quad a+b+c = a(1-\alpha)(1-\beta) = a\left(1-\frac{m}{m-1}\right)\left(1-\frac{m+1}{m}\right) = \frac{a}{m(m-1)}$$

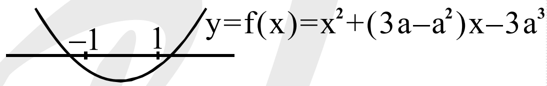
$$|\alpha-\beta| = \left|\frac{m}{m-1} - \frac{m+1}{m}\right| = \frac{1}{|m(m-1)|}$$

$$(a+b+c)^2 = \frac{a^2}{(m(m-1))^2} = a^2(\alpha-\beta)^2 = a^2((\alpha+\beta)^2 - 4\alpha\beta)$$

$$= a^2\left(\frac{b^2}{a^2} - \frac{4c}{a}\right) = b^2 - 4ac$$

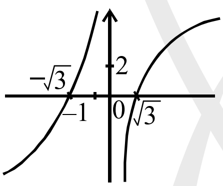
24. $f(-1) < 0 \Rightarrow \quad a \in \left(\frac{1}{3}, \infty\right)$

$f(1) < 0 \Rightarrow \quad a \in \left(-1, -\frac{1}{3}\right) \cup (1, \infty)$



Hence, $a \in (1, \infty)$

25.



$$x - \frac{3}{x} = 2a$$

from graph it is clear that

$$2a > 2 \Rightarrow a > 1$$

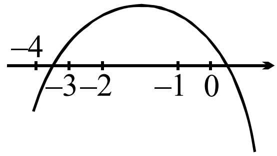
26. $ax^2 + 2bx + c = 0 <^{\alpha}_{\beta} \qquad px^2 + 2qx + r = 0 <^{\gamma}_{\delta}$

$$\frac{\alpha}{\beta} = \frac{\gamma}{\delta} \qquad \Rightarrow \qquad \frac{\alpha}{\gamma} = \frac{\beta}{\delta} = \frac{\alpha+\beta}{\gamma+\delta}$$

Also $\quad \dfrac{\alpha\beta}{\gamma\delta} = \left(\dfrac{\alpha+\beta}{\gamma+\delta}\right)^2$

$$\Rightarrow \quad \frac{c/a}{r/p} = \frac{4b^2(p^2)}{a^2(4q^2)} \qquad \Rightarrow \qquad \left(\frac{b^2}{q^2}\right)\left(\frac{p}{a}\right)\left(\frac{r}{c}\right) = 1$$

27. $f(x) = ax^2 - (3 + 2a)x + 6$



$f(-3) > 0 \Rightarrow 9a + 9 + 6a + 6 > 0 \Rightarrow a > -1$

and $f(-4) \le 0 \Rightarrow 16a + 12 + 8a + 6 \le 0$

$a \le -\frac{3}{4}$

$\Rightarrow a \in \left(-1, -\frac{3}{4}\right]$

28. Let $P(x) = ax^2 + bx + c$

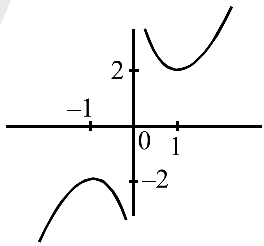
$P(x) = x$ has no real roots

$\Rightarrow P(x) > x \ \forall x \in R$ or $P(x) < x \ \forall x \in R$

$\Rightarrow P(P(x)) > P(x) > x \ \forall x \in R$ or $P(P(x)) < P(x) < x \ \forall x \in R$

Hence, $P(P(x)) = x$ has no real roots

29. $x + \frac{1}{x} = 4a$



$4a \ge 2 \Rightarrow a \ge \frac{1}{2}$

$\alpha, \beta = 2a \pm \sqrt{4a^2 - 1}$

$\alpha \ge 2a$

30. $f(x) = ax^2 - bx + 7$

$f(0) = 7 \Rightarrow f(x) \ge 0 \quad \forall x \in R$

$\therefore \quad f(-1) = a + b + 7 \ge 0 \Rightarrow a + b \ge -7$

31. $a\left(\frac{-x}{x+1}\right)^2 + b\left(\frac{-x}{x+1}\right) + c = 0$

$\Rightarrow \frac{-x}{x+1} = \alpha, \beta \Rightarrow x = \frac{-\alpha}{\alpha+1}, \frac{-\beta}{\beta+1}$

32. By observation put $x = -1$

$(6a + 3b + 4c) - (11a + 8b + 7c) + (3c + 5a + 5b) = 0$

$\therefore$ Equation has roots $-1, -1$

$\therefore$ $$(-1)(-1) = \frac{3c + 5a + 5b}{6a + 3b + 4c} = 1$$

$\Rightarrow$ $$a + c = 2b$$

33. $f(x) = x^2 - ax - b$

$$f(1) = 1 - a - b = -4$$

and $$x \to \pm\infty, f(x) \to \infty$$

$\Rightarrow$ $f(x) = 0$ has two distinct real roots.

34. $D \geq 0 \quad \Rightarrow a \in (-\infty, 0] \cup [4, \infty)$

$-\frac{b}{2a} > 0 \Rightarrow a - 1 > 0 \Rightarrow a \in (1, \infty)$

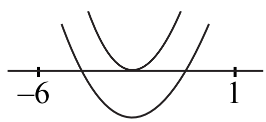
$f(0) > 0 \Rightarrow 2a + 1 > 0 \Rightarrow a \in \left(-\frac{1}{2}, \infty\right)$

Hence, $a \in [4, \infty)$

35. Let $f(x) = x^2 + 2(P - 3)x + 9$

$D \geq 0 \Rightarrow P \in (-\infty, 0] \cup [6, \infty)$

$-6 < -\frac{b}{2a} < 1 \quad \Rightarrow \quad P \in (2, 9)$



$f(-6) > 0 \quad \Rightarrow \quad P < \frac{27}{4}$

$f(1) > 0 \quad \Rightarrow \quad P > -2$

Hence, $$P \in \left[6, \frac{27}{4}\right)$$

37. $a^2\alpha^2 + b\alpha + c = 0 \quad \Rightarrow \quad b\alpha + c = -a^2\alpha^2$

$a^2\beta^2 - b\beta - c = 0 \quad \Rightarrow \quad b\beta + c = a^2\beta^2$

Let $f(x) = a^2x^2 + 2bx + 2c$

$$f(\alpha) = a^2\alpha^2 + 2(b\alpha + c) = -a^2\alpha^2 < 0$$

$$f(\beta) = a^2\beta^2 + 2(b\beta + c) = 3a^2\beta^2 > 0$$

$\Rightarrow \exists$ root γ, such that $\alpha < \gamma < \beta$

38. $y = \dfrac{x^2 - 3x + c}{x^2 + 3x + c}$

$\Rightarrow \quad (y-1)x^2 + 3(y+1)x + c(y-1) = 0$

$D \geq 0 \quad \Rightarrow \quad 9(y+1)^2 \geq 4c(y-1)^2$

If $c \leq 0$, then $y \in R \quad \Rightarrow \quad c > 0$

$\Rightarrow ((3 + 2\sqrt{c})\,y - (2\sqrt{c} - 3))\,((2\sqrt{c} - 3)\,y - (3 + 2\sqrt{c})) \leq 0$

$\Rightarrow \quad y \in \left[\dfrac{2\sqrt{c}-3}{3+2\sqrt{c}}, \dfrac{3+2\sqrt{c}}{2\sqrt{c}-3}\right]$

$\therefore \quad \dfrac{3+2\sqrt{c}}{2\sqrt{c}-3} = 7 \quad \Rightarrow \quad \sqrt{c} = 2 \quad \Rightarrow \quad c = 4$

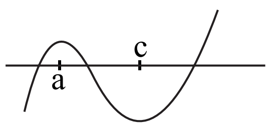
39. $x = a - y^2, \quad y = a - x^2$

$\Rightarrow \quad x - y = x^2 - y^2 \quad \Rightarrow \quad x + y = 1$

$\therefore \quad 1 - x = a - x^2 \quad \Rightarrow \quad x^2 - x + 1 - a = 0$

$D \geq 0 \quad \Rightarrow \quad 1 - 4 + 4a \geq 0 \quad \Rightarrow \quad a \geq \dfrac{3}{4}$

40. $f(a) = -q^2(a-b) - r^2(a-c) > 0$



$f(c) = -p^2(c-a) - q^2(c-b) < 0$

$f(x) = 0$ has 3 distinct real roots.

41. $f(x) = ax^2 + bx + 8 \geq 0 \;\; \forall\, x \in R$

$f(4) = 16a + 4b + 8 \geq 0 \Rightarrow 4a + b \geq -2$

SECTION-2

ONE OR MORE THAN ONE CORRECT

1. $a - b + c > -4$

$a + b + c < 0 \Rightarrow -a - b - c > 0$

$\Rightarrow -2b > -4 \Rightarrow b < 2$

$\Rightarrow 9a + 3b + c > 5$

$-a - b - c > 0$

$\Rightarrow 8a + 2b > 5$

$8a + 4 > 8a + 2b > 5$

$\Rightarrow a > \frac{1}{8}$

2. $f(x) = ax^2 + bx + c \in I \ \forall x \in I$

$f(0) = c \in I$

$f(1) = a + b + c \in I$

$f(-1) = a - b + c \in I$

$\Rightarrow f(1) - f(-1) = 2b \Rightarrow 2b \in I$

$f(1) + f(-1) = 2(a + c) \Rightarrow a = \frac{f(1) + f(-1)}{2} - f(0)$

$\Rightarrow 2a \in I$

$f(1) + f(-1) = 2k_1 + 1$ and $f(1) - f(-1) = 2k_2 + 1$

or $b = \frac{2k_2 + 1}{2}, a = \frac{2k_1 + 1}{2} - k = \frac{2k_1 - 2k + 1}{2}$

$f(1) + f(-1) = 2k_3$ and $f(1) - f(-1) = 2k_4, \ k_i \in I$

$f(2) = 4a + 2b + c$

3. $f(x) = ax^2 - bx + c$

$f(0) = c \in I$

$f(1) = a - b + c = k_1$ $\qquad k_1, k_2 \in I$

$f(2) = 4a - 2b + c = k_2$

$$a - b \in I$$

$$2a + 2(a - b) + c = k_2$$

$$\Rightarrow \qquad 2a \in I \quad \Rightarrow \quad 2b \in I$$

$$f(2k) = 4ak - 2bk + c \in I \qquad k \in I$$

$$f(2k + 1) = a(2k + 1)^2 - b(2k + 1) + c = 4ak^2 + 4ak + a - 2bk - b + c$$

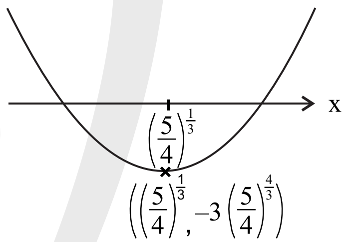
$$= 4ak^2 + 4ak - 2bk + (a - b + c) \in I$$

$$\Rightarrow \qquad f(x) \in I \ \forall \ x \in I$$

4. $f(x) = x^4 - 5x = 2a$

$$f'(x) = 4x^3 - 5$$

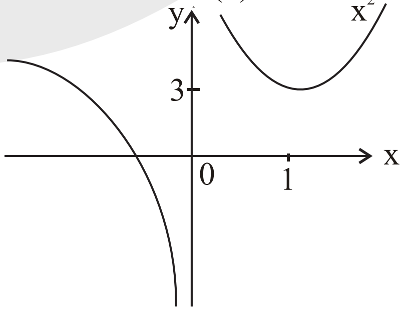
$$f\left(\left(\frac{5}{4}\right)^{\frac{1}{3}}\right) = \left(\frac{5}{4}\right)^{\frac{4}{3}} - 5\left(\frac{5}{4}\right)^{\frac{1}{3}} = (1-4)\left(\frac{5}{4}\right)^{\frac{4}{3}} = -3\left(\frac{5}{4}\right)^{4/3}$$



$$\text{Number of roots} = \begin{cases} 0 & \text{if} \quad 2a < -3\left(\frac{5}{4}\right)^{\frac{4}{3}} \Rightarrow a < -\frac{15}{16}\sqrt[3]{10} \\ 2 & \text{if} \quad 2a > -3\left(\frac{5}{4}\right)^{\frac{4}{3}} \Rightarrow a > -\frac{15}{16}\sqrt[3]{10} \\ 1 & \text{if} \quad 2a = -3\left(\frac{5}{4}\right)^{\frac{4}{3}} \Rightarrow a = -\frac{15}{16}\sqrt[3]{10} \end{cases}$$

5. $f(x) = \frac{x^3 + 2}{x} = x^2 + \frac{2}{x} = -a$

$$f'(x) = 2x - \frac{2}{x^2} = \frac{2(x^3 - 1)}{x^2}$$

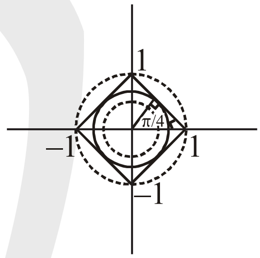


$$\text{Number of roots} = \begin{cases} 1 & \text{if} \quad -a < 3 \Rightarrow a \in (-3, \infty) \\ 2 & \text{if} \quad a = -3 \\ 3 & \text{if} \quad -a > 3 \Rightarrow a \in (-\infty, -3) \end{cases}$$

6. $|a| = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Number of solutions

$$= \begin{cases} 4 & \text{if} \quad a = \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -1, 1\right\} \\ 8 & \text{if} \quad a \in \left(-1, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right) \\ 0 & \text{if} \quad a \in (-\infty, -1) \cup \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \cup (1, \infty) \end{cases}$$



8. $f(x) = \frac{1}{2bx^2 - x^4 - 3b^2}$ $\quad x \in [-2, 1]$

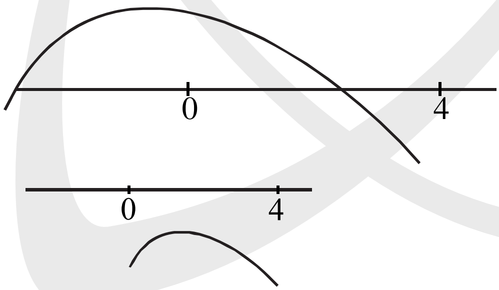
Let $g(t) = 2bt - t^2 - 3b^2 \quad t \in [0, 4]$

$$g(t) < 0 \ \forall \ t \in [0, 4]$$

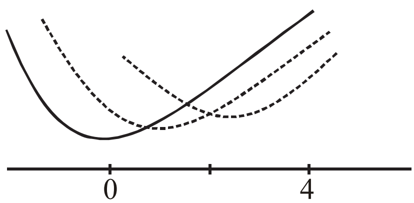
Case-I If $b \le 0$, then g(t) will decrease in [0, 4]

Maximum value of $f(t) = f(4) = \frac{1}{8b - 16 - 3b^2}$

Case



9. $f(t) = t^2 - 6bt + b^2, \qquad t \in [0, 4]$



maximum value of f(t)

$$= \begin{cases} f(4) = 16 - 24b + b^2, & 3b \le 2 \\ f(0) = b^2, & b \ge 2 \end{cases}$$

11. $\dfrac{\sqrt{a} + \sqrt{x-b}}{\sqrt{b} + \sqrt{x-a}} = \dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{\sqrt{x-b}}{\sqrt{x-a}}$

$\Rightarrow \quad ax - a^2 = bx - b^2$

$\Rightarrow \quad (a - b)(x - (a + b)) = 0$

$\therefore \quad x = a + b \quad \text{if} \quad a \ne b$

$x \in [a, \infty) \quad \text{if} \quad a = b$

12.

$a > 0,\ b^2 \le 4ac$

$b > 0,\ c^2 \le 4ab$

$c > 0,\ a^2 \le 4bc$

$\Rightarrow \quad a^2 + b^2 + c^2 < 4(ab + bc + ca)$ [$\because$ All equal is impossible]

Also $\quad a^2 + b^2 + c^2 > ab + bc + ca$ ($\because$ a, b, c are distinct)

$\Rightarrow \quad \dfrac{a^2 + b^2 + c^2}{ab + bc + ca} \in (1, 4)$

13. $mx^2 - 2(m + 2)x + m + 5 = 0$ has no real roots

$\Rightarrow \quad m \ne 0$ and $D < 0 \quad \Rightarrow \quad m > 4$

For the equation

$(m - 6)x^2 - 2(m + 2)x + m + 5 = 0$

$D = 4(10m + 4) > 0$ for $m > 4$

$\Rightarrow$ Equation has two distinct real roots if $m \in (4, \infty) - \{6\}$ and one real root if $m = 6$

14. Let α be the common root

$\Rightarrow \quad 2017\alpha^2 + b\alpha + 7102 = 0$

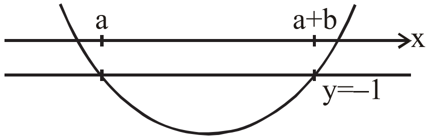
and $\quad 7102\alpha^2 + b\alpha + 2017 = 0$

Subtracting, $\quad 5085\alpha^2 - 5085 = 0$

$\Rightarrow \quad \alpha = \pm 1$

Put $\alpha = \pm 1,\ b = \pm 9119$

15. $f(x) = (x - a)(x - a - b) - 1$



$f(a) = f(a + b) = -1$

From graph it is clear that

root lie in internal $(-\infty, a) \cup (a + b, \infty)$

16. $p + q = \frac{p^2 + 11}{9}$ ...(1)

$$pq = \frac{15}{4}(p + q) + 16 \quad ...(2)$$

$$4pq - 15(p + q) = 64$$

$$(4q - 15)\left(p - \frac{15}{4}\right) = 64 + \frac{(15)^2}{4} = \frac{16^2 + 15^2}{4}$$

$\Rightarrow$ $(4q - 15)(4p - 15) = 481 = 13 \times 37$

$4p - 15 = 13,$	$4q - 15 = 37$	$\Rightarrow$	$(p, q) = (7, 13)$
$= 37$	$= 13$		$= (13, 7)$
$= 13 \times 37$	$= 1$		$= (124, 4)$
$= 1$	$= 13 \times 37$		$= (4, 124)$

Only $(p, q) = (13, 7)$ satisfy (1)

17. $M = 3x^2 - 8yx + 9y^2 - 4x + 6y + 15$

$= 2(x^2 - 4xy + 4y^2) + (y^2 + 6y + 9) + (x^2 - 4x + 4) + 2$

$= 2(x - 2y)^2 + (y + 3)^2 + (x - 2)^2 + 2$

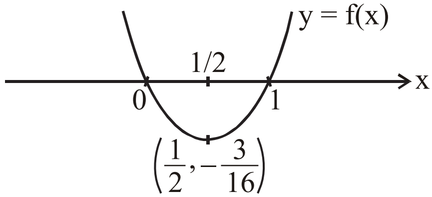
$\Rightarrow$ $M > 2$

18. $x^4 - 2x^3 + 2x^2 - x = -\frac{a}{8}$

Let $f(x) = x^4 - 2x^3 + 2x^2 - x = x(x - 1)(x^2 - x + 1)$

$f'(x) = 4x^3 - 6x^2 + 4x - 1$

$f''(x) = 12x^2 - 12x + 4 > 0 \quad \forall\ x \in R$



$$f'\left(\frac{1}{2}\right) = 0$$

Also $\quad f(1 - x) = f(x)$

If $\quad -\frac{a}{8} \geq -\frac{3}{16} \quad \Rightarrow \quad a \leq \frac{3}{2}$

then equation has two real roots α, $1 - \alpha$

$\Rightarrow$ Sum of all non real roots = 2 – 1 = 1

If $a > \frac{3}{2}$, then equation has 4 non real roots

$\Rightarrow$ Sum of all non real roots = 2.

19. $a + b + c = 3a \Rightarrow b + c = 2a$

$$abc = c^3 \quad \Rightarrow \quad ab = c^2 \quad \text{or } c = 0 \quad \Rightarrow \quad a = 0, b = 0 \quad \text{or } a = 3, b = 6$$

$$ab + bc + ca = 3b$$

$$(a,b, c) = (0,0,0), (3, 6, 0)$$

OR

$$c^2 + bc + ca = 3b = c(3a) \quad \Rightarrow \quad b = ac$$

$$b + c = 2a,\ b = ac,\ c^2 = ab$$

$$b + c^2 = a(b + c) = 2a^2$$

$$2a^2 - c^2 - ac = 0 = (2a + c)(a - c) = 0$$

$\therefore$

$$a = c = b \text{ or } c = -2a$$

$\Rightarrow$

$$b = 4a$$

$$4a = -2a^2 \quad \Rightarrow \quad a = 0, a = -2.$$

$$(a, b, c) = (-2, -8, 4)$$

$$x^3 + 6x^2 - 24x - 64 = (x + 2)(x + 8)(x - 4)$$

20. $$\frac{4x^2 + x + 4}{x^2 + 1} + \frac{x^2 + 1}{x^2 + x + 1} = \frac{31}{6}$$

Let $\quad \frac{4x^2 + x + 4}{x^2 + 1} = t \quad \Rightarrow \quad \frac{x^2 + x + 1}{x^2 + 1} = t - 3$

$$\Rightarrow \quad t + \left(\frac{1}{t-3}\right) = \frac{31}{6}$$

$$\Rightarrow \quad 6(t^2 - 3t + 1) = 31t - 93$$

$$\Rightarrow \quad 6t^2 - 49t + 99 = 0$$

$$\Rightarrow \quad 6t^2 - 27t - 22t + 99 = 0$$

$$\Rightarrow \quad (3t - 11)(2t - 9) = 0$$

$$\therefore \quad \frac{4x^2 + x + 4}{x^2 + 1} = \frac{11}{3}, \frac{9}{2}$$

$$\Rightarrow \quad x^2 + 3x + 1 = 0 \quad \text{or} \quad x^2 - 2x + 1 = 0$$

$$\boxed{x = \frac{-3 \pm \sqrt{5}}{2}, 1}$$

21. $x^2 + (x + 1)^2 - 4x - a = 0$

$$\Rightarrow \quad 2x^2 - 2x + 1 - a = 0$$

$D \geq 0 \quad \Rightarrow \quad 4 + 8(a - 1) > 0 \Rightarrow a - 1 \geq -\frac{1}{2} \quad \Rightarrow \quad a \geq \frac{1}{2}$

Put $x = 0, a = 1$

$$x = -1, a = 5$$

$$D = 0, a = \frac{1}{2}$$

Hence, $a = 1, 5, \frac{1}{2}$

22. Let x be the common root of equation.

$$x^3 - 3x + b = 0 \quad \text{...(1)}$$

$$x^2 + bx - 3 = 0 \quad \text{...(2)}$$

$(2) \times x - (1) \quad \Rightarrow \quad bx^2 - b = 0 \quad \Rightarrow \quad b = 0 \text{ or } x = \pm 1.$

Put $\quad x = 1, \Rightarrow b = 2$

$$x = -1, \Rightarrow b = -2$$

$$\therefore \quad b = 0, 2, -2$$

23. $3x^3(x^4 - 10x^2 + 1) - (x^4 - 10x^2 + 1) = 0$

$$(3x^3 - 1)(x^4 - 10x^2 + 1) = 0 \quad \Rightarrow \quad x = \frac{1}{\sqrt[3]{3}}$$

$$x^2 = 5 \pm 2\sqrt{6} = (\sqrt{3} \pm \sqrt{2})^2$$

$$x = \sqrt{3}+\sqrt{2}, -\sqrt{3}-\sqrt{2}, \sqrt{3}-\sqrt{2}, -\sqrt{3}+\sqrt{2}, \frac{1}{\sqrt[3]{3}}$$

24. $a^2 + (9x - 2x^2)a + (x^4 - 9x^3 + 20x^2) = 0$

$$\Rightarrow a = \frac{(2x^2 - 9x) \pm \sqrt{4x^4 + 81x^2 - 36x^3 - 4(x^4 - 9x^3 + 20x^2)}}{2}$$

$a = x^2 - 4x, \quad x^2 - 5x$

for real roots of $x^2 - 4x - a = 0$, $D \geq 0 \Rightarrow 16 + 4a \geq 0 \quad a \geq -4$

for real roots of $x^2 - 5x - a = 0$, $D \geq 0 \Rightarrow 25 + 4a \geq 0 \quad a \geq -\frac{25}{4}$

for $a > -4$

$$x = 2 - \sqrt{4+a} \in (-\infty, 2), \quad x = 2 + \sqrt{4+a} \in (2, \infty)$$

$$x = \frac{5-\sqrt{25+4a}}{2} \in (-\infty, 1), \quad x = \frac{5+\sqrt{25+4a}}{2} \in (4, \infty)$$

$$2 - \sqrt{4+a} = \frac{5-\sqrt{25+4a}}{2} \Rightarrow a = 0$$

$$2 + \sqrt{4+a} = \frac{5+\sqrt{25+4a}}{2} \text{ is impossible.}$$

For 4 distinct real roots.

$$a \in (-4, 0) \cup (0, \infty)$$

For 3 distinct rea roots.

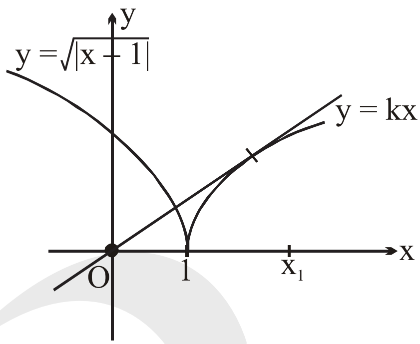
$$a = \{-4, 0\}$$

For 2 distinct real roots.

$$a \in \left(-\frac{25}{4}, -4\right)$$

For no real roots $\quad a \in \left(-\infty, -\frac{25}{4}\right)$

25. $\sqrt{|1-x|} = kx$



For tangency of two curves

$$\sqrt{x_1 - 1} = kx_1$$

and $$\frac{1}{2\sqrt{x_1-1}} = k.$$

$\Rightarrow$ $$\sqrt{x_1 - 1} = \frac{x_1}{2\sqrt{x_1-1}}$$

$\Rightarrow$ $$2(x_1 - 1) = x_1 \Rightarrow x_1 = 2, k = \frac{1}{2}$$

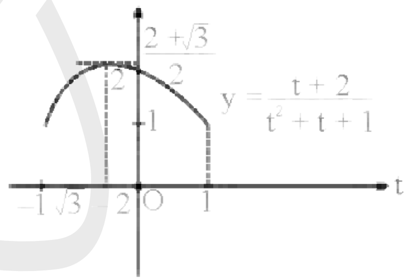
From graph it is clear that equation has

one solution if $k \in (-\infty, 0] \cup \left(\frac{1}{2}, \infty\right)$

two solutions if $k = \frac{1}{2}$

three solutions if $k \in \left(0, \frac{1}{2}\right)$

26. Put sinx = t



$\Rightarrow$ $$m = \frac{t+2}{t^2+t+1}, \; t \in [-1, 1]$$

28. $x^2 - ax - bx + ab = 0 \quad \Rightarrow \quad x = a, \; b$

Put $x = a$ in $ax^2 - px + ab = 0 \quad \Rightarrow \quad p = a^2 + b$

Put $x = b$ in $ax^2 - px + ab = 0 \quad \Rightarrow \quad p = a + ab$

29. $\frac{b}{a} = 3x - 4x^3$

Put $x = \sin\theta \Rightarrow \sin 3\theta = \frac{b}{a}$

$\sin 10^\circ$ is a root $\Rightarrow \sin 30^\circ = \frac{b}{a} = \frac{1}{2}$

$\therefore \quad \sin 3\theta = \sin(360^\circ + 30^\circ) = \sin(720 + 30^\circ)$

$\Rightarrow \quad 3\theta = 390^\circ, 750^\circ$

$\Rightarrow \quad \theta = 130^\circ, 250^\circ$

30. $x^3 - ax + b = 0$ with roots α, β, γ: $\alpha + \beta + \gamma = 0$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -a$$

$$\alpha\beta\gamma = -b$$

Equation will have 2 positive and 1 negative root.

Let $\gamma < 0 < \alpha \le \beta$ and $|\alpha| \le |\beta| \le |\gamma|$

$$b - a\alpha = -\alpha\beta\gamma + \alpha(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$= \alpha^2\beta + \alpha^2\gamma$$

$$= -\alpha^3 < 0$$

$\Rightarrow \quad \frac{b}{a} < \alpha$

$$3b - 2a\alpha = -3\alpha\beta\gamma + 2\alpha(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$= -\alpha\beta\gamma + 2(\alpha^2\beta + \alpha^2\gamma)$$

$$= \alpha\left(-2(\beta+\gamma)^2 - \beta\gamma\right)$$

$$= -\alpha(2\beta^2 + 2\gamma^2 + 5\beta\gamma)$$

$$= -\alpha(2\beta + \gamma)(\beta + 2\gamma)$$

$$= -\alpha(\beta - \alpha)(\gamma - \alpha) \le 0$$

$\Rightarrow \quad \frac{3b}{2a} \le \alpha$

31. $D_1 = b^2 - 4ac$ and $D_2 = b^2 - 4ac$

$\Rightarrow$ $D_1 < 0$ and $D_2 < 0$

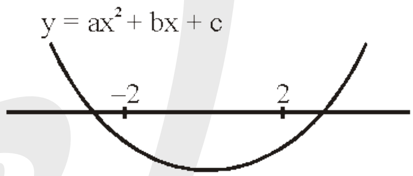
$\Rightarrow$ Equation have both roots common

$\Rightarrow$ $\frac{a}{c} = \frac{b}{b} = \frac{c}{a}$

$\Rightarrow$ $a = c$ and $b^2 < 4ac = 4a^2$

$-2|a| < |b| < 2|a|$

32. Let $f(x) = ax^2 + bx + c$



$f(-2) < 0 \Rightarrow 4a - 2b + c < 0$

$\Rightarrow 4a + c < 2b$

and $f(2) < 0 \Rightarrow 4a + 2b + c < 0$

$\Rightarrow 4a + c < -2b$

$\therefore$ $4a + c < -2|b|$.

33. $y = \dfrac{(ax - b)(dx - c)}{(bx - a)(cx - d)}$

$\Rightarrow$ $(ybc - ad)\, x^2 - (bd + ac)\,(y - 1)x + (ady - bc) = 0$

$\Rightarrow$ $(bd + ac)^2\,(y - 1)^2 - 4(ybc - ad)\,(ady - bc) \geq 0$

$\Rightarrow$ $y^2(bd - ac)^2 + 2y(2a^2d^2 + 2b^2c^2 - (bd + ac)^2) + (bd - ac)^2 \geq 0 \ \forall\, y \in R$

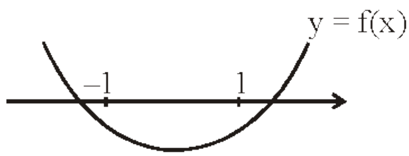
$\Rightarrow$ $(2a^2d^2 + 2b^2c^2 - (bd + ac)^2) - (bd - ac)^4 \leq 0$

$\Rightarrow$ $4(a^2d^2 + b^2c^2 - b^2d^2 - a^2c^2)\,(a^2d^2 + b^2c^2 - 2abcd) \leq 0$

$\Rightarrow$ $(a^2 - b^2)\,(c^2 - d^2)\,(ad - bc)^2 \geq 0$

$\Rightarrow$ $(a^2 - b^2)\,(c^2 - d^2) > 0$

34. $(x - (1 - a))\left(x - \dfrac{a^2 - 1}{a}\right) < 0 \quad \forall\, x \in [-1, 1]$



Let $f(x) = (x - (1 - a))\left(x - \dfrac{a^2 - 1}{a}\right)$

$$f(-1) < 0 \Rightarrow \frac{(a-2)(a^2+a-1)}{a} > 0$$

$$\Rightarrow a \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(0, \frac{-1+\sqrt{5}}{2}\right) \cup (2, \infty)$$

and $f(1) < 0 \Rightarrow a\left(\frac{a^2-a-1}{a}\right) > 0$

Sign chart: $+$ before $\frac{1-\sqrt{5}}{2}$, $-$ between $\frac{1-\sqrt{5}}{2}$ and $\frac{1+\sqrt{5}}{2}$, $+$ after $\frac{1+\sqrt{5}}{2}$

$$\Rightarrow a \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, \infty\right)$$

Hence, $a \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup (2, \infty)$.

35. $x^2 + 4x + 3 - \alpha \leq 0$

$$\Rightarrow x \in [-2 - \sqrt{\alpha+1}, -2 + \sqrt{\alpha+1}]$$

and $x^2 - 2x - (3 - 6\alpha) \leq 0$

$$\Rightarrow x \in \left[1-\sqrt{4-6\alpha}, 1+\sqrt{4-6\alpha}\right]$$

$\alpha = -1$ satisfy the condition

$\alpha = \frac{2}{3}$ doesn't satisfy

If $\alpha \in \left(-1, \frac{2}{3}\right)$

$$-2 + \sqrt{\alpha+1} = 1 - \sqrt{4-6\alpha}$$

$$\Rightarrow 7\alpha + 6 = 6\sqrt{1+\alpha} \Rightarrow \alpha \geq -\frac{6}{7}$$

$$\Rightarrow 49\alpha^2 + 48\alpha = 0$$

$$\Rightarrow \alpha = 0, -\frac{48}{49}$$

$\alpha = -\frac{48}{49}$ is rejected $\left[\because \alpha \geq -\frac{6}{7}\right]$

Hence, $\alpha = 0, -1$

36. $f(x) = b^2x^2 - Dx - 4ac$

$$f(1) = b^2 - 4ac - D = 0$$

$$\text{other root} = \frac{c}{a} \text{ which is also rational}$$

$$\therefore \quad \text{roots of } f(x) = 0 \text{ are 1 and } \frac{c}{a}$$

37. $x_1x_2x_3x_4 = 81$

$$x_1 + x_2 + x_3 + x_4 = 12$$

$$\Rightarrow \quad \frac{x_1 + x_2 + x_3 + x_4}{4} = (x_1x_2x_3x_4)^{1/4}$$

$$\Rightarrow \quad x_1 = x_2 = x_3 = x_4 = 3$$

$$\Rightarrow \quad (x - 3)^4 = 0 = x^4 - 12x^3 + 54x^2 - 108x + 81 = 0$$

$$a = 54, \; b = -108$$

38. $x^2 + ax + 2 = 0 \; \langle \alpha, \beta \qquad \alpha + \beta = -a \qquad \alpha\beta = 2$

$x^2 + bx + 6 = 0 \; \langle \beta, \gamma \qquad \beta + \gamma = -b \qquad \beta\gamma = 6$

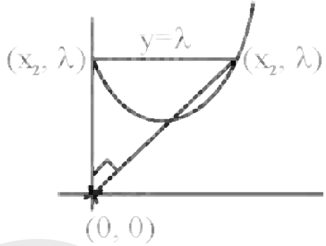
$x^2 + cx + 3 = 0 \; \langle \gamma, \alpha \qquad \gamma + \alpha = -c \qquad \gamma\alpha = 3$

$$\Rightarrow \quad (\alpha\beta\gamma)^2 = 36 \quad \Rightarrow \quad \alpha\beta\gamma = \pm 6$$

If $\alpha\beta\gamma = 6$, $\alpha = 1$, $\beta = 2$, $\gamma = 3$, $a + b + c = -2(3 + 3) = -12$

If $\alpha\beta\gamma = -6$, $\alpha = -1$, $\beta = -2$, $\gamma = -3$, $\quad a + b + c = 12$

39. $D < 0 \quad \Rightarrow \quad p^2 - q < 0$



$$\frac{\lambda}{x_1}\frac{\lambda}{x_2} = -1 \quad \Rightarrow \quad x_1x_2 = -\lambda^2$$

$$\lambda = x^2 - 2px + q \Rightarrow x^2 - 2px + q - \lambda = 0 \begin{cases} x_1 \\ x_2 \end{cases}$$

$$\Rightarrow \quad q - \lambda = -\lambda^2 \quad \Rightarrow \quad q = \lambda - \lambda^2 \le \frac{1}{4}$$

40. $$S_2 = \sum_{i=1}^{n}\alpha_i^2 = \left(\sum_{i=1}^{n}\alpha_i\right)^2 - 2\sum_{1\le i<j\le n}\sum\alpha_i\alpha_j = 0 - 2(0) = 0$$

$\Rightarrow$ All roots can not be real

$$\alpha_i^{\,n} = -p\alpha_i^{\,2} - q\alpha_i - r$$

Put i = 1, 2, ..., n and add

$$\Rightarrow \quad S_n = -pS_2 - qS_1 - nr$$

$$S_n = -p(0) - q(0) - nr$$

$$\Rightarrow \quad S_n = -nr$$

41. Let tanD be the fourth root

$$\Rightarrow \quad \tan(A + B + C + D) = \tan(\pi + D) = \tan D = \frac{S_1 - S_3}{1 - S_2 + S_4}$$

$$\Rightarrow \quad \tan D = \frac{p - r}{1 - q + s}$$

$$\tan A \tan B \tan C = \tan A + \tan B + \tan C$$

$$\Rightarrow \quad \frac{s}{\tan D} = p - \tan D$$

$$\Rightarrow \quad \tan^2 D - p\tan D + s = 0$$

$$\Rightarrow \quad \tan D = \frac{p \pm \sqrt{p^2 - 4s}}{2}$$

SECTION-3

COMPRHENSION BASED QUESTIONS

Comprehension (Q.1 To Q.3):

Let $f(x) = ax^2 + bx + c$

$$f(0) = c,\ f(1) = a + b + c,\ f(-1) = a - b + c$$

$$b = \frac{f(1) - f(-1)}{2},\ a = \frac{f(1) + f(-1)}{2} - f(0)$$

Maximum value of $ax + b = a + b = 2$

$\Rightarrow \quad f(1) - f(0) = 2$

$f(1) - f(0) \le 1 - (-1)$

$\Rightarrow \quad f(1) = 1$ and $f(0) = -1 \Rightarrow c = -1$

Least value of $f(x) = f(0) = -1$

$\Rightarrow \quad -\frac{b}{2a} = 0 \quad \Rightarrow \quad b = 0$

$\Rightarrow \quad f(1) = f(-1) = 1$

$\therefore \quad a = 2$

Comprhension (Q.4 to Q.6) :

$$f(x) - f(-x) = 2x^3 + 2bx$$

$$|f(x) - f(-x)| \le |f(x)| + |f(-x)| \le 4$$

$\therefore \quad |x^3 + bx| \le 2$

Put $x = 1$, $\quad |1 + b| \le 2 \quad \Rightarrow \quad -3 \le b \le 1$

Put $x = 2$, $\quad |8 + 2b| \le 2 \quad \Rightarrow \quad -5 \le b \le -3$

$\therefore \quad b = -3 \quad \Rightarrow \quad f(x) = x^3 + ax^2 - 3x + c$

$f(2) = 4a + c + 2 \quad \Rightarrow \quad 4a + c = f(2) - 2 \le 0$

$f(-2) = 4a + c - 2 \quad \Rightarrow \quad 4a + c = f(-2) + 2 \geq 0$

$\Rightarrow \quad 4a + c = 0$ ...(1)

$f(1) = a + c - 2 \quad \Rightarrow \quad a + c = f(1) + 2 \geq 0$

$f(-1) = a + c + 2 \quad \Rightarrow \quad a + c = f(-1) - 2 \leq 0$

$\Rightarrow \quad a + c = 0$ ...(2)

From (1) and (2), $a = 0, c = 0$

$\therefore \quad f(x) = x^3 - 3x$

Comprhension (Q.7 to Q.8) :

$f(1) = 1 + a + b + c = g(1) = 0$

$\therefore$ Let $x^3 + ax^2 + bx + c = 0$ (roots 1, α, β)

$\therefore \quad 1 + \alpha + \beta = -a \qquad \alpha\beta = -c$

$1 + \alpha^2 + \beta^2 = -b \qquad (\alpha\beta)^2 = -a$

$-a = c^2$

$a^2 = (1 + \alpha + \beta)^2 = (1 + \alpha^2 + \beta^2) + 2(\alpha + \beta + \alpha\beta)$

$a^2 = -b + 2(-a - 1 - c)$

$= -b + 2b = b$

$-a = c^2, b = a^2 = c^4$

Also $\quad 1 + a + b + c = 0$

$\Rightarrow \quad 1 - c^2 + c^4 + c = 0$

$\Rightarrow \quad (1 + c) + c^2(c - 1)(c + 1) = 0$

$\Rightarrow \quad (1 + c)(1 + c^3 - c^2) = 0$

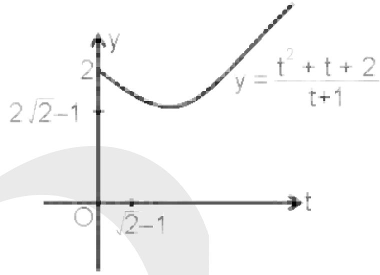
$c = -1, b = 1, a = -1$

Comprhension (Q.9 to Q.11) :

Put $x^2 = t$

$$t^2 - (k-1)t + 2 - k = 0$$

$$k = \frac{t^2 + t + 2}{t + 1} \quad t \geq 0$$



From graph it is clear that

$k \in \{2\sqrt{2} - 1\} \cup (2, \infty)$, 2 real and distinct roots

$k \in \{2\}$, 3 real and distinct roots

$k \in (2\sqrt{2} - 1, 2)$, 4 real and distinct roots.

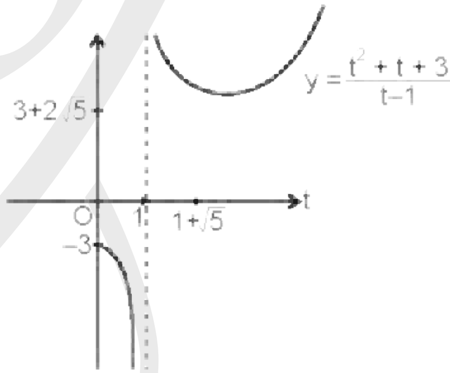
Comprhension (Q.14 to Q.15) :

$\tan\theta = t > 0 \quad \forall\ \theta \in \left(0, \frac{\pi}{2}\right)$

$f(t) = t^2 + t + 3 + a(t-1),\ t > 0$

14. $-a < \frac{t^2 + t + 3}{t - 1} \quad \forall\ t > 1$

$$-a > \frac{t^2 + t + 3}{t - 1} \quad \forall\ t \in (0, 1)$$



$\therefore \quad -a < 3 + 2\sqrt{5}$ and $-a \geq -3$

$\Rightarrow \quad a \in (-3 - 2\sqrt{5}, 3]$

15. $f(1) = 5$

$\Rightarrow \quad a \in \phi$

Comprhension (Q.19 to Q.21) :

$$\frac{x^4 - x^3 + x^2 - x + 1}{x^3 + x} = -\alpha$$

From graph it is clear that

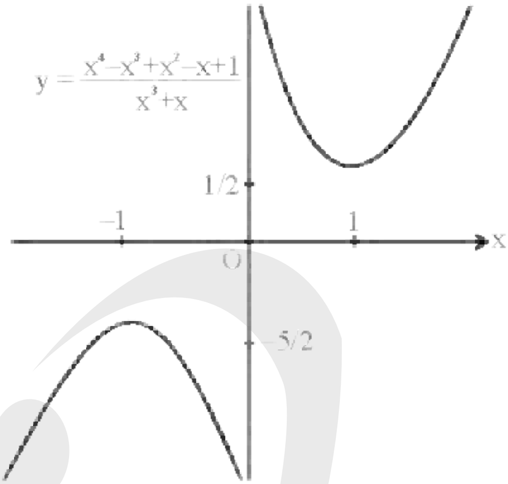
19. $\alpha \in \phi$

20. $-\alpha < -\frac{5}{2}$

$\Rightarrow \quad \alpha > \frac{5}{2}$

21. $-\frac{5}{2} < -\alpha < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < \alpha < \frac{5}{2}$



Comprhension (Q.22 to Q.23) :

22. $x^2 - ax + b = 0 \left\langle \begin{matrix}\alpha \\ \beta\end{matrix}\right.$ $\qquad$ $x^2 - px + q = 0 \left\langle \begin{matrix}\alpha \\ \frac{1}{\beta}\end{matrix}\right.$

$\alpha^2 - a\alpha + b = 0$

$\alpha^2 - p\alpha + q = 0$

substracting

$(p - a)\alpha + b - q = 0 \quad \Rightarrow \quad \alpha = \frac{q - b}{p - a}$

$\alpha + \beta = a, \qquad \alpha\beta = b$

$\alpha + \frac{1}{\beta} = p, \qquad \frac{\alpha}{\beta} = q$

$\Rightarrow \quad (\alpha\beta)\left(\frac{\alpha}{\beta}\right) = bq = \alpha^2$

$\therefore \quad \left(\frac{q - b}{p - a}\right)^2 = bq$

$\Rightarrow \quad (q - b)^2 = bq\,(p - a)^2$

23. $x^2 - ax + b = 0 \left\langle \begin{matrix}\alpha \\ \beta\end{matrix}\right.$ $\qquad$ $x^2 - px + q = 0 \left\langle \begin{matrix}\alpha \\ \alpha\end{matrix}\right.$

$\alpha + \beta = a \qquad \alpha\beta = b$

$2\alpha = p \qquad \alpha^2 = q$

$b + q = \alpha(\beta + \alpha) = \frac{p}{2}(a) \quad \Rightarrow \quad b + q = \frac{ap}{2}$

Comprhension (Q.24 to Q.26) :

$$x^4 - 5x^3 + ax^2 + bx + c = 0 \;\; \begin{matrix} 3+\sqrt{2} \\ 3-\sqrt{2} \\ \alpha \\ \beta \end{matrix}$$

$$\alpha + \beta + 6 = 5 \quad \Rightarrow \quad \alpha + \beta = -1$$

$$(3 + \sqrt{2})(3 - \sqrt{2}) + \alpha\beta + (\alpha + \beta)[(3 + \sqrt{2}) + (3 - \sqrt{2})] = a$$

$$\Rightarrow \quad 7 + \alpha\beta - 6 = a \quad \Rightarrow \quad \alpha\beta = a - 1$$

$$\alpha\beta(3 + \sqrt{2}) + (3 - \sqrt{2})) + (3 + \sqrt{2})(3 - \sqrt{2})(\alpha + \beta) = -b$$

$$6\alpha\beta + 7(-1) = -b$$

$$\Rightarrow \quad \alpha\beta = \frac{7-b}{6}$$

$$\alpha\beta(3 + \sqrt{2})(3 - \sqrt{2}) = c$$

$$\Rightarrow \quad \alpha\beta = \frac{c}{7}$$

$$\therefore \quad x^2 + x + (a - 1) = 0 \;\; \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$D \geq 0 \quad \Rightarrow \quad 1 - 4(a - 1) \geq 0 \quad \Rightarrow \quad a \leq \frac{5}{4}$$

$$\text{Also} \quad x^2 + x + \frac{7-b}{6} = 0 \;\; \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$D \geq 0 \quad \Rightarrow \quad 1 - \frac{2(7-b)}{3} \geq 0 \quad \Rightarrow \quad b \geq \frac{11}{2}$$

$$\text{Also} \quad x^2 + x + \frac{c}{7} = 0 \;\; \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$D \geq 0 \quad \Rightarrow \quad 1 - \frac{4c}{7} \geq 0 \quad \Rightarrow \quad c \leq \frac{7}{4}$$

Put $a = \frac{5}{4}$, $b = \frac{11}{2}$, $c = \frac{7}{4}$

$$x^4 - 5x^3 + \frac{5}{4}x^2 + \frac{11}{2}x + \frac{7}{4} = 0$$

Roots are $3 + \sqrt{2}$, $3 - \sqrt{2}$, $-\frac{1}{2}, \frac{-1}{2}$

Comprhension (Q.27 to Q.29) :

$$-\frac{a}{2} = x_1 + x_2$$

(1) $b = x_1x_2 + x_3x_4 + (x_1 + x_2)(x_3 + x_4) = x_1x_2 + x_3x_4 + \frac{a^2}{4}$

(2) $-c = x_1x_2(x_3 + x_4) + x_3x_4(x_1 + x_2) = -\frac{9}{2}(x_1x_2 + x_3x_4)$

$(1) \times \frac{9}{2} + (2)$

$\Rightarrow \quad \frac{ab}{2} - c = \frac{a^3}{8}$

27. Put a = 2

$$b - c = 1$$

28. $f(a) = \frac{a^3}{8} - \frac{ab}{2} + c$

$$f'(a) = \frac{3a^2}{8} - \frac{b}{2} > 0 \quad \forall\, a \in R \qquad [\because b < 0]$$

$\therefore$ f(a) = 0 holds for only one real value of 'a'.

29. $\frac{a^3}{8} - \frac{a}{2}(1 - c) + c = 0$

$\Rightarrow \quad a^3 - 4a + 4ac + 8c = 0$

$\Rightarrow \quad a(a + 2)(a - 2) + 4c(a + 2) = 0$

$\Rightarrow \quad a^2 - 2a + 4c = 0$

$\Rightarrow \quad c = \frac{2a - a^2}{4}$

$\Rightarrow \quad c \in \left(-\infty, \frac{1}{4}\right]$

Comprhension (Q.30 to Q.31) :

30. $x^4 + ax^3 + bx^2 + ax + 1 = (x^2 + kx + 1)^2$

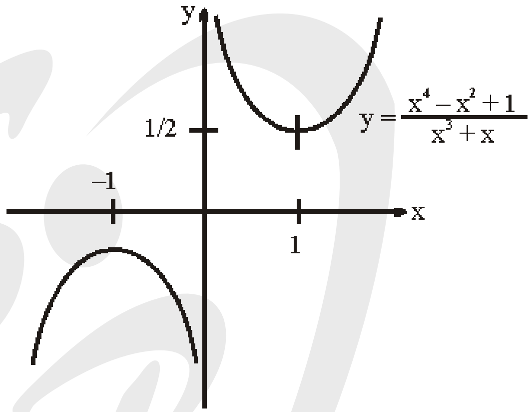
$\Rightarrow \quad a = 2k,\ b = k^2 + 2$

$\Rightarrow \quad a + b = (k + 1)^2 + 1 \geq 1$

31. $x^4 + ax^3 - x^2 + ax + 1 = 0$

$\Rightarrow \quad -a = \dfrac{x^4 - x^2 + 1}{x^3 + x}$

Hence, $\quad a \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$



Comprhension (Q.32 to Q.33) :

32. $(x_2 + x_2)^2 - 2x_1x_2 = 6$

$\Rightarrow \quad (m - 4)^2 - 2(m^2 - 3m + 3) = 6$

$\Rightarrow \quad m^2 + 2m - 4 = 0$

$\Rightarrow \quad m = -1 \pm \sqrt{5}$

Also $D \geq 0 \Rightarrow (m - 4)^2 - 4(m^2 - 3m + 3) \geq 0$

$\Rightarrow \quad 3m^2 - 4m - 4 \leq 0 \quad \Rightarrow \quad m \in \left[-\dfrac{2}{3}, 2\right]$

Hence $m = \sqrt{5} - 1$

33. $\dfrac{mx_1^2}{1 - x_1} + \dfrac{mx_2^2}{1 - x_2} = \dfrac{m\left(x_1^2 + x_2^2 - x_1x_2(x_1 + x_2)\right)}{(1 - x_1)(1 - x_2)}$

$$= \frac{m\left((m-4)^2 - 2(m^2 - 3m + 3) + (m - 4)(m^2 - 3m + 3)\right)}{1 + (m - 4) + m^2 - 3m + 3}$$

$$= \frac{m^3 - 8m^2 + 13m - 2}{m - 2}$$

$= m^2 - 6m + 1 \qquad m \in \left[-\dfrac{2}{3}, 2\right)$

$$2^2 - 6(2) + 9 < m^2 - 6m + 9 \le \left(-\frac{2}{3}\right)^2 - 6\left(-\frac{2}{3}\right) + 9$$

$$1 < m^2 - 6m + 1 \le \frac{121}{9}$$

Comprhension (Q.34 to Q.35) :

$$y = \frac{x^2 + 4x + 3}{x^2 + 7x + 14} \quad \Rightarrow \quad (y-1)x^2 + (7y-4)x + (14y-3) = 0$$

$$D \ge 49y^2 + 16 - 56y - 4(14y^2 - 17y + 3) \ge 0$$

$$\Rightarrow \quad 7y^2 - 12y - 4 \le 0$$

$$\Rightarrow \quad (7y+2)(y-2) \le 0$$

$$y \in \left[-\frac{2}{7}, 2\right]$$

Maximum value of f(x) is 2 at x = –5

Now let $y = \frac{x^2 - 5x + 10}{x^2 + 5x + 20} \Rightarrow (y-1)x^2 + 5(y+1)x + 10(2y-1) = 0$

$$D \ge 0 \quad \Rightarrow 5(y+1)^2 - 8(2y^2 - 3y + 1) \ge 0$$

$$\Rightarrow \quad 11y^2 - 34y + 3 \le 0$$

$$\Rightarrow \quad (11y-1)(y-3) \le 0$$

$$\Rightarrow \quad y \in \left[\frac{1}{11}, 3\right]$$

$\therefore$ Maximum value of g(x) is 3 at x = –5

$\Rightarrow$ Maximum value of $(g(x))^{f(x)} = 3^2 = 9$.

SECTION-4

MATCH THE COLUMN :

1. (A) $D = (2abc\cos C)^2 - 4a^2b^2 = -4a^2b^2\sin^2 C < 0$

(B) $D = 4(b^2 - ac) > 0$

$\left[\because \dfrac{a+c}{2} = b > \sqrt{ac} \Rightarrow b^2 > ac\right]$

and $f(x) = ax^2 + 2bx + c > 0 \quad \forall\ x \geq 0$

Hence f(x) = 0 has both negative roots.

(C) $f(x) = x^2 - (a+1)x - (a^2+4)$

$f(0) < 0$

(D) $\sqrt{ac} > b \Rightarrow b^2 - ac < 0$

$D = 4(b^2 - ac) < 0$

2. $\Sigma\alpha\beta = \dfrac{2^2 - (6)}{2} = -1$

$11 - 3\alpha\beta\gamma = 2(6 - (-1)) = 14 \Rightarrow \alpha\beta\gamma = -1$

$\therefore \quad x^3 - 2x^2 - x + 1 = 0$ (roots α, β, γ)

$\therefore \quad x^3 = 2x^2 + x - 1$ for $x = \alpha, \beta, \gamma$

$\Rightarrow \quad x^4 = 2x^3 + x^2 - x = 5x^2 + x - 2$

$\Rightarrow \quad \alpha^4 + \beta^4 + \gamma^4 = 5(6) + 2 - 6 = 26$

$x^5 = 5x^3 + x^2 - 2x = 11x^2 + 3x - 5$ for $x = \alpha, \beta, \gamma$

$\Rightarrow \quad \alpha^5 + \beta^5 + \gamma^5 = 11(6) + 3(2) - 15 = 57$

$x^6 = 11x^3 + 3x^2 - 5x = 25x^2 + 6x - 11$ for $x = \alpha, \beta, \gamma$

$\Rightarrow \quad \alpha^6 + \beta^6 + \gamma^6 = 25(6) + 6(2) - 33 = 129$

$-(4-\alpha^2)(4-\beta^2)(4-\gamma^2) = -(2-\alpha)(2-\beta)(2-\gamma)(2+\alpha)(2+\beta)(2+\gamma)$

$= (8 - 2(4) - 2 + 1)((-8 - 8 + 2 + 1))$

$= (-1)(-13) = 13$

3. (A) $\lambda > 2$ and $D < 0 \Rightarrow 64 - 4(\lambda^2 + 2\lambda - 8) < 0$

$\Rightarrow \quad \lambda^2 + 2\lambda - 24 > 0$

$\Rightarrow \quad \lambda \in (-\infty, -6) \cup (4, \infty)$

$\therefore \quad \boxed{\lambda \in (4, \infty)}$

(B) $(a^2 - 14a + 48) < 0 \Rightarrow \boxed{a \in (6, 8)}$

(C) $f(x) = ax^2 + 2bx + 4c - 16$

$f(-2) = 4(a - b + c - 4) > 0$

$\Rightarrow \quad f(x) > 0 \quad \forall\, x \in R$

$\therefore \quad f(0) > 0 \Rightarrow \boxed{c > 4}$

(D) $|(x - 3)(x + 2)| = x + 2$

$\Rightarrow \quad x = -2$ or $x - 3 = \pm (1) \Rightarrow x = 4, 2.$

$\therefore \quad \boxed{x = -2, 2, 4}$

4. (A) $f(t) = t^2 - 2at + a + 3, \qquad t \in [-1, 1]$

$$f_{max} = \begin{cases} f(1) = 4 - a & a \in (-\infty, 0] \\ f(-1) = 3a + 4 & a \in [0, \infty) \end{cases} \Rightarrow a = -3, 1$$

(B) $f(t) = t^2 - at + 2a - 1, \qquad t \in [0, 4]$

$$f_{max} = \begin{cases} f(4) = 15 - 2a & \frac{a}{2} \le 2 \\ f(0) = 2a - 1 & \frac{a}{2} \ge 2 \end{cases} \Rightarrow a = 3, 5$$

(C) $f(t) = t^2 - 3t + a - 1 = 0 \qquad t \in [0, \infty)$

$a = 1 + 3t - t^2$

$a \in \left(1, \frac{13}{4}\right)$

(D) $f(t) = t^2 - at + 2a - 1 \qquad t \in [0, 4]$

$$f_{min} = \begin{cases} f(0) = 2a - 1 & \frac{a}{2} \le 0 \\ f\left(\frac{a}{2}\right) = \frac{8a - 4 - a^2}{4} & 0 < \frac{a}{2} < 4 \\ f(4) = 15 - 2a & \frac{a}{2} \ge 4 \end{cases} \Rightarrow a = -3, 11$$

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. $\sqrt{1-\sqrt{x^4-x^2}} = x-1$

$$x \geq 1$$

$$1-\sqrt{x^4-x^2} = (x-1)^2 = x^2-2x+1$$

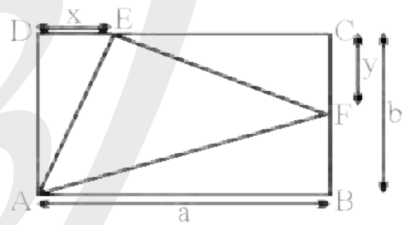
$\Rightarrow \quad \sqrt{x^2-1} = 2-x, \qquad x = 0$ (rejected)

$$1 \leq x \leq 2$$

$$x^2-1 = 4+x^2-4x$$

$\Rightarrow \quad x = \frac{5}{4}$

2. $\frac{1}{2}bx = 16 \qquad ....(1)$



$\frac{1}{2}(a-x)y = 9 \qquad ....(2)$

$\frac{1}{2}a(b-y) = 25 \qquad ....(3)$

From (2) and (1), $\quad y = \frac{18}{a-x} = \frac{18}{a-\frac{32}{b}} = \frac{18b}{ab-32}$

Put in (3) $\quad a\left(b-\frac{18b}{ab-32}\right) = 50$

$$ab(ab-50) = 50(ab-32)$$

$$(ab)^2 - 100(ab) + 1600 = 0 = (ab-80)(ab-20)$$

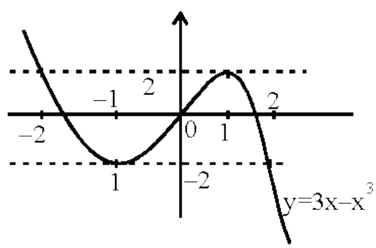
$\Rightarrow \quad ab = 80,\ ab = 20$ rejected

$\Rightarrow \quad$ area of $\Delta AEF = 80 - (16+9+25) = 30$

3. $a = 3x - x^3$

Clearly from graph

$a = -2$ and $a = 2$



4. $D \geq 0 \Rightarrow 4a^2 + 3b^2 - 4ab - 4b + 2 \leq 0$

$\Rightarrow (2a - b)^2 + 2(b - 1)^2 \leq 0$

$\Rightarrow 2a - b = 0$ and $b - 1 = 0$

$\Rightarrow b = 1, a = \frac{1}{2}$

5. $6 = -\frac{b}{a'}, 8 = \frac{c}{a'} \Rightarrow -\frac{b}{c} = \frac{3}{4}$

$b^2 - 4ac < 0$

$\Rightarrow b^2 < 4ac$

$\Rightarrow ac > 0$

$3 = \left|\frac{b}{a}\right|, \frac{c}{a} = 4$

$\Rightarrow -\frac{b}{a} = 3$

$b = -3a, \; c = 4a$

$$\frac{2b + 3c}{a} = \frac{-6a + 12a}{a} = 6$$

6. $\Delta > 0 \Rightarrow (m - 2)^2 - (m^2 - 3m + 3) > 0$

$\Rightarrow m < 1$

$\Rightarrow m \in [-1, 1)$

$$m\left(\frac{x_1^2}{1 - x_1} + \frac{x_2^2}{1 - x_2}\right) = m\left[\frac{x_1^2 + x_2^2 - x_1x_2(x_1 + x_2)}{(1 - x_1)(1 - x_2)}\right]$$

$$= m\left[\frac{(x_1 + x_2)^2 - 2x_1x_2 - x_1x_2(x_1 + x_2)}{(1 - x_1)(1 - x_2)}\right]$$

$$= m\left[\frac{4(m - 2)^2 - (2 - 2(m - 2))(m^2 - 3m + 3)}{1 + 2(m - 2) + m^2 - 3m + 3}\right]$$

$$= \left[\frac{2m[2(m^2 - 4m + 4) + (m - 3)(m^2 - 3m + 3)]}{m^2 - m}\right]$$

$= 2(m^2 - 3m + 1) \in (-2, 10]$

7. Put $x = \sqrt[3]{y}$ in equation $x^2 - px + q = 0$

$\Rightarrow \quad y^{2/3} - py^{1/3} = -q$

$\Rightarrow \quad y^2 - p^3y - 3py(-q) = -q^3$

$\Rightarrow \quad x^2 + (3pq - p^3)x + q^3 = 0$ has roots α^3, β^3

$\Rightarrow \quad q = q^3$

$\Rightarrow \quad q = 0, 1, -1$

and $\quad 3pq - p^3 = -p$

If $q = 0$, $\quad p^3 = p \quad \Rightarrow \quad p = 0, 1, -1$

If $q = 1$, $\quad p^3 = 4p \quad \Rightarrow \quad p = 0, 2, -2 \quad$ [p = 0, q = 1 rejected]

If $q = -1$, $\quad p^3 = -2p \quad \Rightarrow \quad p = 0$

$$(p, q) = (0, 0)\ (1, 0), (-1, 0), (2, 1), (-2, 1), (0, -1)$$

8. $\alpha^3 = (\alpha^2 + \alpha - 3)(\alpha - 1) + 4\alpha - 3 = 4\alpha - 3$

$$\alpha^3 - 4\beta^2 + 19 = 4\alpha - 3 + 4(\beta - 3) + 19$$

$$= 4(\alpha + \beta) + 4 = 4(-1) + 4 = 0$$

9. By observation, if is clear that equation

$19x^2 + 99x + 1 = 0$ has roots $a, \frac{1}{b}$

$$\therefore \quad \left(a + \frac{1}{b}\right) + 4a\left(\frac{1}{b}\right) = -\frac{99}{19} + \frac{4}{19} = -5$$

10. $a, b, c = b - \beta, b, b + \beta$

$$(b - \beta)^2 + b^2 + (b + \beta)^2 = 84$$

$\Rightarrow \quad 3b^2 + 2\beta^2 = 84$

$\Rightarrow \quad 3b^2 < 84$

$\Rightarrow \quad b^2 < 28$

Also $\quad (a + c)^2 - 2ac + b^2 = 84$

$\Rightarrow \quad 5b^2 - 84 = 2ac$

$ac > 0$

$\Rightarrow \quad b^2 > \frac{84}{5}$

$\Rightarrow \quad \frac{84}{5} < b^2 < 28$

$\Rightarrow \quad b = 5$

11. $x^2 - pqx + p + q = 0 <^{\alpha}_{\beta}$

$\alpha + \beta = pq$

$\alpha\beta = p + q$

Both roots α, β must be positive integers

$\Rightarrow \quad \alpha + \beta - \alpha\beta - 1 = pq - p - q - 1$

$\Rightarrow \quad (\alpha - 1)(\beta - 1) + (p - 1)(q - 1) = 2$

Case-I : If $(\alpha - 1)(\beta - 1) = 0$ and $(p - 1)(q - 1) = 2$

$\Rightarrow \quad (p, q) = (2, 3), (3, 2)$

$x^2 - 6x + 5 = 0 = (x - 5)(x - 1) = 0$

$\therefore \quad (p, q) = (2, 3), (3, 2)$ satisfy

Case-II : If $(\alpha - 1)(\beta - 1) = 1$ and $(p - 1)(q - 1) = 1$

$\Rightarrow \quad (p, q) = (2, 2)$

$\Rightarrow \quad x^2 - 4x + 4 = (x - 2)^2$

$(p, q) = (2, 2)$ satisfy

Case-III : If $(\alpha - 1)(\beta - 1) = 2$ and $(p - 1)(q - 1) = 0$

$\Rightarrow \quad (\alpha, \beta) = (3, 2), (2, 3)$

$x^2 - 5x + 6 = (x - 2)(x - 3) = 0$

$\Rightarrow \quad pq = 5$

$p + q = 6 \qquad x^2 - 6x + 5 = (x - 5)(x - 1) = 0$

$(p, q) = (1, 5), (5, 1)$ satisfy

Hence, $(p, q) = (1, 5), (5, 1), (2, 3), (3, 2), (2, 2)$

12. 5 $\qquad p^2 - 20(66p - 1) = k^2$

$\Rightarrow \qquad (5p - 132)^2 - k^2 = 17404$

$\Rightarrow \quad (5p - 132 - k)(5p - 132 + k) = 2^2 \times 19 \times 229$

$$5p - 132 - k = 2 \times 19$$

$$5p - 132 + k = 2 \times 229$$

$\Rightarrow \qquad p = 76$

and $\qquad 5p - 132 - k = 2$

$$5p - 132 + k = 2 \times 19 \times 229$$

Gives no integer solution for p

hence, $\qquad p = 76$

13. Put $\frac{1}{a} = x, \frac{1}{b} = y$

$$\left(\frac{x}{x-y} - \frac{y}{x+y}\right)(x - y)\left(\frac{1}{x^2+y^2}\right) = \frac{2}{3}$$

$\Rightarrow \qquad \frac{1}{x+y} = \frac{2}{3} \quad \Rightarrow \quad \frac{ab}{a+b} = \frac{2}{3}$

$\Rightarrow \qquad 3ab - 2a - 2b = 0 \qquad \Rightarrow (3b - 2)\left(a - \frac{2}{3}\right) = \frac{4}{3}$

$\Rightarrow \quad (3b - 2)(3a - 2) = 4$

$3b - 2 = 4$ and $3a - 2 = 1 \quad \Rightarrow \quad (a, b) = (1, 2)$

$3b - 2 = 1$ and $3a - 2 = 4 \quad \Rightarrow \quad (a, b) = (2, 1)$

$3b - 2 = 2$ and $3a - 2 = 2 \quad \Rightarrow \quad (a, b)$ are not integers

$3b - 2 = -4$ and $3a - 2 = -1$ also not satisfy

$\therefore \qquad (a, b) = (1, 2), (2, 1)$

14. $\frac{1}{8}(2a - a^2) \le x^2 - 3x + 2 \quad \forall\, x \in [0, 2]$

$\Rightarrow \quad \frac{1}{8}(2a - a^2) \le -\frac{1}{4} \quad \Rightarrow \quad a \in (-\infty, 1 - \sqrt{3}\,] \cup [1 + \sqrt{3}, \infty)$

and $\quad 3 - a^2 \ge x^2 - 3x + 2 \quad \forall\, x \in [0, 2]$

$3 - a^2 \ge 2 \quad \Rightarrow \quad a \in [-1, 1]$

Hence, $a \in [-1, 1 - \sqrt{3}\,]$

15. $\alpha^2 + \beta^2 - \alpha\beta - 1 = 0$

$\Rightarrow \quad (\alpha + \beta)^2 - 3\alpha\beta - 1 = 0$

$\Rightarrow \quad (a + b)^2 - 4ab = 1$

$\Rightarrow \quad (a - b)^2 = 1$

$\Rightarrow \quad a - b = \pm 1$

$D \ge 0 \quad \Rightarrow \quad 9(a + b)^2 - 12(4ab) \ge 0$

$\Rightarrow \quad 9(a + b)^2 - 12((a + b)^2 - (a - b)^2) \ge 0$

$\Rightarrow \quad (a + b)^2 \le 4$

$\Rightarrow \quad -2 \le a + b \le 2$

If $a = b + 1$, $\quad -2 \le 2b + 1 \le 2$

$\Rightarrow \quad -\frac{3}{2} \le b \le \frac{1}{2}$

$\Rightarrow \quad b = -1, 0$

$\Rightarrow \quad (a, b) = (0, -1), (1, 0)$

If $a = b - 1 \quad \Rightarrow \quad -2 \le 2b - 1 \le 2$

$\Rightarrow \quad -\frac{1}{2} \le b \le \frac{3}{2}$

$\Rightarrow \quad b = 0, 1$

$\Rightarrow \quad (a, b) = (-1, 0), (0, 1)$

16. $c^2 = b(8 - b) - 16$

$$\Rightarrow \quad b^2 - 8b + 16 + c^2 = 0$$

$$\Rightarrow \quad (b - 4)^2 + c^2 = 0$$

$$\Rightarrow \quad b = 4, c = 0, a = 4$$

17. $a + b + c = -a,\ ab + bc + ca = b, abc = -c$

Case-I $c = 0, ab = b$

If $b = 0, a = 0$

If $a = 1, b = -2$

$$\Rightarrow \quad (a, b, c) = (0, 0, 0), (1, -2, 0)$$

Case-II $ab = -1$

$$\Rightarrow \quad (a + b)\, c = b + 1, \quad b + c = -2a = \frac{2}{b} \quad \Rightarrow \quad c = \frac{2}{b} - b$$

$$\Rightarrow \quad \left(-\frac{1}{b} + b\right)\left(\frac{2}{b} - b\right) = b + 1$$

$$\Rightarrow \quad b^4 + b^3 - 2b^2 + 2 = 0$$

$$\Rightarrow \quad (b + 1)\,(b^3 - 2b + 2) = 0$$

$$\Rightarrow \quad (a, b, c) = (1, -1, -1) \text{ or } b^3 - 2b + 2 = 0$$

Let $b = \frac{p}{q}$ $\quad p, q \in I$ satisfy

$$p^3 - 2pq^2 + 2q^3 = 0$$

$$\Rightarrow \quad (p^2 - 2q^2)p + 2q^3 = 0$$

$$\Rightarrow \quad p = \pm 1, \pm 2$$

$$p^3 + 2(q - p)q^2 = 0$$

$$\Rightarrow \quad q = \pm 1$$

But roots $b = \pm 1, \pm 2$ do not satisfy $b^3 - 2b + 2 = 0$

Hence, $(a, b, c) = (0, 0, 0), (1, -2, 0), (1, -1, -1)$.

18. $\alpha^5 - \alpha^3 + \alpha - 2 = 0$

$$\alpha(\alpha^4 - \alpha^2 + 1) = 2$$

$$\Rightarrow \quad \alpha > 0$$

If $0 \le \alpha \le 1$, $(\alpha^5 - \alpha^3) + (\alpha - 2) < 0$

If $\alpha \ge 2$, $\alpha(\alpha^4 - \alpha^2 + 1) \ge 2(16 - 4 + 1) = 26$

$\Rightarrow$ $1 < \alpha < 2$

$$\alpha^6 = \alpha^4 - \alpha^2 + 2\alpha = \frac{2}{\alpha} - 1 + 2\alpha = 2\left(\alpha + \frac{1}{\alpha}\right) - 1$$

$\Rightarrow$ $3 < \alpha^6 < 4$

$\therefore$ $[\alpha^6] = 3$

19. $x^2 - (y + 1)x + y^2 - y = 0$

$D \ge 0 \Rightarrow 3y^2 - 6y - 1 \le 0$

$\Rightarrow y \in \left[1 - \frac{2}{\sqrt{3}}, 1 + \frac{2}{\sqrt{3}}\right]$

Put $y = 0, x = 0, 1$

$y = 1, x = 0, 2$
$y = 2, x = 1, 2$

$(x, y) = (0, 0), (1, 0), (0, 1), (2, 1), (1, 2), (2, 2)$

20. $x^2 + 4y^2 - 2xy - 2x - 4y - 8 = 0$

$x^2 - (2y + 2)x + (4y^2 - 4y - 8) = 0$

$D \ge 0 \Rightarrow (y + 1)^2 - (4y^2 - 4y - 8) \ge 0$

$\Rightarrow 3y^2 - 6y - 9 \le 0$

$\Rightarrow y^2 - 2y - 3 \le 0$

$\Rightarrow y \in [-1, 3]$

Put $y = -1, x = 0$

$y = 0,$ $x = -2, 4$

$y = 1,$ $x = 2 \pm 2\sqrt{3}$

$y = 2,$ $x = 0, 6$

$y = 3,$ $x = 4$

Hence, $(x, y) = (0, -1), (-2, 0), (4, 0), (0, 2), (6, 2), (4, 3)$

21. Put $y = \frac{x-1}{x+1} \Rightarrow x = \frac{1+y}{1-y}$

$$\therefore \quad \frac{(1+y)^3}{(1-y)^3} - 2007\frac{(1+y)}{(1-y)} + 2002 = 0$$

$$\Rightarrow \quad 4008y^3 - 8016y^2 + 3996y + 4 = 0$$

$$\Rightarrow \quad \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1} + \frac{\gamma-1}{\gamma+1} = \frac{8016}{4008} = 2$$

22. $a^2 + 3xa + 2x^2 - x^3 - x^4 = 0$

$$a = \frac{-3x \pm \sqrt{9x^2 - 4(2x^2 - x^3 - x^4)}}{2} = \frac{-3x \pm \sqrt{x^2 + 4x^3 + 4x^4}}{2}$$

$$a = \frac{-3x \pm (2x^2 + x)}{2} = x^2 - x, -2x - x^2$$

$x^2 - x - a = 0$, $D > 0 \Rightarrow 1 + 4a > 0 \Rightarrow a > -\frac{1}{4}$

$x^2 + 2x + a = 0$, $D > 0 \Rightarrow 4 - 4a > 0 \Rightarrow a < 1$

Hence, $a \in \left(-\frac{1}{4}, 1\right)$

Also
$$x = -1 - \sqrt{1-a} \in \left(\frac{-2-\sqrt{5}}{2}, -1\right)$$

$$x = -1 + \sqrt{1-a} \in \left(-1, \frac{-2+\sqrt{5}}{2}\right)$$

$$x = \frac{1-\sqrt{1+4a}}{2} \in \left(\frac{1-\sqrt{5}}{2}, \frac{1}{2}\right)$$

$$x = \frac{1+\sqrt{1+4a}}{2} \in \left(\frac{1}{2}, \frac{1+\sqrt{5}}{2}\right)$$

$$-1 + \sqrt{1-a} = \frac{1-\sqrt{1+4a}}{2}$$

$$\Rightarrow \quad 2\sqrt{1-a} = 3 - \sqrt{1+4a}$$

$$\Rightarrow \quad 4 - 4a = 9 + 1 + 4a - 6\sqrt{1+4a}$$

$$3\sqrt{1+4a} = 4a + 3$$

$$9 + 36a = 16a^2 + 24a + 9$$

$$a = \frac{3}{4}, 0$$

Hence, for 4 distinct real roots

$$a \in \left(-\frac{1}{4}, 0\right) \cup \left(0, \frac{3}{4}\right) \cup \left(\frac{3}{4}, 1\right)$$

23. $(a-2)^3 + 3(a-2) = 7$

$$(b-2)^3 + 3(b-2) = -7$$

$\Rightarrow \quad (a-2)^3 + 3(a-2) = (2-b)^3 + 3(2-b)$

$\because \; f(x) = x^3 + 3x$ is a one-one function

$\Rightarrow \quad a - 2 = 2 - b$

$\Rightarrow \quad a + b = 4$

24. $x^2 - kx + \ell = 0 \begin{matrix} \nearrow a \\ \searrow b \end{matrix} \quad x^2 - px + q = 0 \begin{matrix} \nearrow a + \frac{1}{b} \\ \searrow b + \frac{1}{a} \end{matrix}$

$$q = \left(a + \frac{1}{b}\right)\left(b + \frac{1}{a}\right) = ab + \frac{1}{ab} + 2 = \ell + \frac{1}{\ell} + 2$$

$q, \ell \in N \quad \Rightarrow \quad \ell = 1$

$\Rightarrow \quad q = 4$

25. $ab + bc + ca = \frac{9-5}{2} = 2$

$$-a^3 + b^3 + c^3 - 3abc = 3(5-2) = 9$$

$\Rightarrow \quad abc = -\frac{2}{3}$

$$x^3 - 3x^2 + 2x + \frac{2}{3} = 0$$

$\Rightarrow \quad x^4 = 3x^3 - 2x^2 - \frac{2}{3}x$

$\Rightarrow \quad a^4 + b^4 + c^4 = 3(a^3 + b^3 + c^3) - 2(a^2 + b^2 + c^2) - \frac{2}{3}(a + b + c)$

$$= 3(7) - 2(5) - \frac{2}{3}(3)$$

$$= 9$$

26. $(x^2 - r)(x^2 - 1) = 0$

$$x = \pm\sqrt{r}, \pm 1 ; \; r > 0, r \neq 1$$

$-\sqrt{r}, -1, 1, \sqrt{r}$ or $-1, -\sqrt{r}, \sqrt{r}, 1$ will form A.P.

$\Rightarrow \quad 2\sqrt{r} = 3(2)$

$\Rightarrow \quad r = 9$

or $\quad 2 = 2\sqrt{r} \quad \Rightarrow \quad r = \frac{1}{9}$

27. Let $f(x) = x^2 - 7x + 6, \; x \in (-\infty, -5] \cup [6, \infty)$

$$f_{min} = f(6) = 0$$

28. $x^2 + y^2 - 8x - 6y - 11 = 0$

$\Rightarrow \quad (x - 4)^2 + (y - 3)^2 = 36$

$$x - 4 = 0, y - 3 = \pm 6$$

or $\quad x - 4 = \pm 6, y - 3 = 0$

$$(x, y) = (4, 9), (4, -3), (10, 3), (-2, 3)$$

29. Subtracting, we get

$$y^2 - x^2 + x - y = 24$$

$\Rightarrow \quad (y - x)(y + x + 1) = 24$

Different possibilities are

	$y - x = 3$	and	$y + x + 1 = 8$
or	$y - x = 8$		$y + x + 1 = 3$
or	$y - x = -3$		$y + x + 1 = -8$
or	$y - x = -8$		$y + x + 1 = -3$
or	$y - x = 24$		$y + x + 1 = 1$
or	$y - x = 1$		$y + x + 1 = 24$
or	$y - x = -24$		$y + x + 1 = -1$

or $\quad y - x = -1 \qquad y + x + 1 = -24$

$\Rightarrow \quad (x, y, z) = (3, 6, -85), (-2, 6, -90), (-2, -5, -101), (3, -5, -96), (-11, 13, 34),$

$(12, 13, 57), (12 - 12, 32), (-11, -12, 9)$

30. $x^4 - 2x^2 + ax + b = 0$ with roots $\alpha, \beta, \gamma, \delta$

$$\alpha + \beta + \gamma + \delta = 0, \Sigma\alpha\beta = -2$$

$$\Rightarrow \quad \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 4$$

$$\frac{\beta^2 + \gamma^2 + \delta^2}{3} \geq \left(\frac{\beta + \gamma + \delta}{3}\right)^2$$

$$\Rightarrow \quad \frac{4 - \alpha^2}{3} \geq \frac{\alpha^2}{9}$$

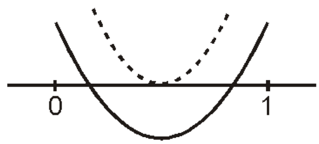
$$\Rightarrow \quad \alpha^2 \leq 3$$

Similarly for others

$$\Rightarrow \quad \alpha, \beta, \gamma, \delta \in (-\sqrt{3}, \sqrt{3})$$

$\because$ Equality not hold true.

31. $f(x) = ax^2 - bx + c = a(x - \alpha)(x - \beta)$



$f(0) > 0, f(1) > 0 \Rightarrow f(0) f(1) > 0$

$\therefore \quad a^2 \alpha(1 - \alpha) \beta(1 - \beta) \geq 1 \qquad f(0), f(1) \in I$

$$\Rightarrow \quad \frac{a^2}{16} \geq a^2(\alpha(1 - \alpha)\beta(1 - \beta)) \geq 1$$

$$\Rightarrow \quad \frac{a^2}{16} \geq 1$$

$$\Rightarrow \quad a^2 > 16 \quad \because \quad \alpha \neq \beta$$

$$\Rightarrow \quad a \geq 5$$

and $\quad 0 < \frac{b}{2a} < 1 \qquad$ and $\qquad b^2 \geq 4ac$

If $a = 5, c = 1, b^2 \geq 20 \Rightarrow b \geq 5$

$f(x) = 5x^2 - 5x + 1$ satisfies the conditions

32. $\alpha + \beta = -a$

$\alpha\beta = b$

$N(b) = 20 \Rightarrow$ Number of divisons of b $= 40 = 2^3 5^1$ or $39 = 13 \times 3$

Smallest value of $b = 2^4 3^1 5^1 7^1 = 1680$

$n = 15$

33. $\frac{1}{2}\left(\frac{(\alpha+\beta)^2 - 4\alpha\beta - 2}{(\alpha+\beta)^2 + 2}\right) = \frac{1}{2}\left(\frac{4p^2 - 4(p^2 - 2p - 1) - 2}{4p^2 + 2}\right) = k \quad k \in I$

$\Rightarrow \quad \frac{4p+1}{4p^2+2} = k$

$\Rightarrow \quad 4kp^2 - 4p + (2k - 1) = 0$

If $k \neq 0$, $D \geq 0 \quad \Rightarrow \quad 2k^2 - k - 1 \leq 0 \quad \Rightarrow \quad k \in \left[-\frac{1}{2}, 1\right]$

Put $k = 1$, $\quad 4p^2 - 4p + 1 = 0 \quad \Rightarrow \quad p = \frac{1}{2}$

If $k = 0$, $-4p - 1 = 0 \quad \Rightarrow \quad p = -\frac{1}{4}$

$\therefore \quad \frac{1}{p_1^2} + \frac{1}{p_2^2} = 16 + 4 = 20$

34. $x^2 - (y + 4)x + (y^2 - 4y + 16) = 0$

$D \geq 0 \quad \Rightarrow \quad y^2 - 8y + 16 \leq 0 \quad \Rightarrow \quad y = 4$

Put $y = 4 \quad \Rightarrow \quad x^2 - 8x + 16 = 0 \quad \Rightarrow \quad x = 4$

$\therefore \quad (x, y) = (4, 4)$

35. $(x - 2)^4 = 30$

$\Rightarrow \quad x_1, x_2 = 2 + \sqrt[4]{30}, 2 - \sqrt[4]{30}$

$x_3, x_4 = 2 + i\sqrt[4]{30}, 2 - i\sqrt[4]{30}$

$x_1x_2 + x_3x_4 = \left(4 - \sqrt{30}\right) + \left(4 + \sqrt{30}\right) = 8$

36. $x^2 - px - (P + C) = (x - \alpha)(x - \beta)$

Put $x = -1$

$\Rightarrow \quad 1 + P - P - C = (1 + \alpha)(1 + \beta) = 1 - C$

$$\therefore \frac{(\alpha+1)^2}{(\alpha+1)^2(c-1)} + \frac{(\beta+1)^2}{(\beta+1)^2+(c-1)} = \frac{(\alpha+1)^2}{(\alpha+1)^2-(\alpha+1)(\beta+1)} + \frac{(\beta+1)^2}{(\beta+1)^2-(1+\alpha)(1+\beta)}$$

$$= \frac{\alpha+1}{\alpha-\beta} - \frac{\beta+1}{\alpha-\beta}$$

$$= 1$$

37. $a > 0,\ x^2 + a^2 \geq a(4 + x)\ \forall\ x \in (-1, 1)$

$$x^2 - ax + (a^2 - 4a) \geq 0 \qquad \forall\ x \in (-1, 1)$$

Let $\quad f(x) = x^2 - ax + (a^2 - 4a)$

If $\quad 0 < \frac{a}{2} < 1, \quad 0 < a < 2$

$\Rightarrow \quad f\left(\frac{a}{2}\right) \geq 0$

$\Rightarrow \quad -\frac{a^2}{4} + a^2 - 4a \geq 0 \quad \Rightarrow \quad 3a^2 - 16a \geq 0$

$\Rightarrow \quad a \geq \frac{16}{3} \quad \Rightarrow \quad a \in \phi$

If $\quad \frac{a}{2} \geq 1,\ a \geq 2$

$\Rightarrow \quad f(1) \geq 0$

$\Rightarrow \quad a^2 - 5a + 1 \geq 0$

$\Rightarrow \quad a \geq \frac{5+\sqrt{21}}{2}$

$\therefore \quad a \in \left[\frac{5+\sqrt{21}}{2}, \infty\right)$

38. $\log_x(x^2 - 4x + a) > 0$

$\Rightarrow \quad 0 < x^2 - 4x + a < 1 \quad \forall\ x \in (0, 1)$

$a < 4x - x^2 + 1$ and $a > 4x - x^2 \quad \forall\ x \in (0, 1)$

$\Rightarrow \quad a \in (-\infty, 1)$ and $a \in (3, \infty)$

$\Rightarrow \quad a \in \phi$

39. $0 < x^2 \le x + 2 \quad \Rightarrow \quad x \in [-1, 0) \cup (0, 2]$

$-\frac{2a^2}{7} \le -1 \quad \Rightarrow \quad a^2 \ge \frac{7}{2}$

and $\quad 2 \le \frac{2a^2}{7} \quad \Rightarrow \quad a^2 \ge 7$

Hence, $a \in (-\infty, -\sqrt{7}\,] \cup [\,\sqrt{7}, \infty)$

40. Let $f(x) = mx^3 - 9x^2 + 12x - 5$

Let α be repeated root

$\Rightarrow \quad f(\alpha) = f'(\alpha) = 0$

$\Rightarrow \quad m\alpha^3 - 9\alpha^2 + 12\alpha - 5 = 0 \quad$ ...(1)

and $\quad 3m\alpha^2 - 18\alpha + 12 = 0$

$\Rightarrow \quad m\alpha^2 - 6\alpha + 4 = 0 \quad$ ...(2)

(1) – (2) × α

$\Rightarrow \quad -3\alpha^2 + 8\alpha - 5 = 0$

$\Rightarrow \quad \alpha = \frac{5}{3}, 1$

$\alpha = \frac{5}{3}, m = \frac{54}{25}$

$\alpha = 1, m = 2$

$\Rightarrow \quad S = 2 + \frac{54}{25} = \frac{104}{25}$

41. Let $f(x) = (ax + 1)(x - a) > 0, |x| \le 2$

Clearly $a < 0$, otherwise $f(x) \to \infty$ as $x \to \infty$

which is not required

$\therefore \quad x \in \left(a, -\frac{1}{a}\right)$

$\therefore \quad |a| \le 2$ and $\frac{1}{|a|} \le 2$

$\Rightarrow \quad \frac{1}{2} \le |a| \le 2$

$\Rightarrow \quad a \in \left[-2, -\frac{1}{2}\right]$

42. $8(x - 1)\,p(x) = (x - 8)\,p(2x)$

$\Rightarrow$ p(x) contain $x - 8$ as factor

$\Rightarrow$ p(2x) contain $2(x - 4)$ as factor

$\Rightarrow$ p(x) contain $(x - 4)$ as factor

$\Rightarrow$ p(2x) contain $2(x - 2)$ as factor

p(x) contain $(x - 2)$ as factor

p(2x) contain $2(x - 1)$ as factor

$\therefore \quad p(x) = (x - 2)(x - 4)(x - 8)$

$\Rightarrow \quad p(10) = 96$

43. $p(x) = (x - \alpha)\,g(x)$;

Put $x = 2$, then $x = 10$, we get g(x) integral will have integral coefficients

$\Rightarrow \quad 13 = (2 - \alpha)\,g(2)$

$\Rightarrow \quad 2 - \alpha = 13, 1, -13, -1$

and $\quad 5 = (10 - \alpha)\,g(5)$

$\Rightarrow \quad 10 - \alpha = 5, 1, -5, -1$

$\Rightarrow \quad \alpha = 15$

44. $1 - 2\sin^2 x + a\sin x = 2a - 7$

$\Rightarrow \quad 2\sin^2 x - a\sin x + 2a - 8 = 0$

$\Rightarrow \quad (\sin x - 2)\,[2(\sin x + 2) - a] = 0$

$\Rightarrow \quad \sin x = 2$ rejected and $\sin x = \frac{a-4}{2}$

$\therefore \quad -1 \le \frac{a-4}{2} \le 1$

$\Rightarrow \quad a \in [2, 6]$

45. $4^{|x^2 - 8x + 12|} = 7^{2y} \quad \Rightarrow \quad y \ge 0$

If $0 \le y \le 3$

$\Rightarrow \quad 3 - y - 3y - 2y^2 - 2 - 4y \ge 1$

$\Rightarrow \quad 2y^2 + 8y \le 0$

$\Rightarrow \quad -4 \le y \le 0$

$\Rightarrow \quad y = 0, \; x = 6, 2$

If $y > 3$

$\Rightarrow \quad y - 3 - 3y - 2y^2 - 2 - 4y \ge 1$

$\Rightarrow \quad 2y^2 + 6y + 6 \le 0$

$\Rightarrow \quad y^2 + 3y + 3 \le 0$

$\Rightarrow \quad y \in \phi$

Hence, $\quad (x, y) = (6, 0), (2, 0)$

46. $D = (n+1)^2 - 4n(n+2) = -3n^2 - 6n + 1 = 4 - 3(n+1)^2$

$D \ge 0 \quad \Rightarrow -\frac{2}{\sqrt{3}} \le n + 1 \le \frac{2}{\sqrt{3}} \quad \Rightarrow \quad n = -2, -1, 0$

For all values of n, D become perfect square

Hence, $n = -2, -1, 0$

47. $ax^2 + bx + c = 0 \begin{cases} \alpha - 3d \\ \alpha + d \end{cases}$

$$\Rightarrow \qquad 2(\alpha - d) = -\frac{b}{a}$$

$$\alpha^2 - 3d^2 - 2\alpha d = \frac{c}{a}$$

$$cx^2 + bx + a = 0 \begin{cases} \alpha - d \\ \alpha + 3d \end{cases}$$

$$\Rightarrow \qquad 2(\alpha + d) = -\frac{b}{c}$$

$$\alpha^2 - 3d^2 + 2\alpha d = \frac{a}{c}$$

$$4\alpha = -b\left(\frac{1}{a} + \frac{1}{c}\right), \; 4d = b\left(\frac{1}{a} - \frac{1}{c}\right)$$

$$4\alpha d = \frac{a}{c} - \frac{c}{a} = \frac{a^2 - c^2}{ac}$$

Also $\qquad 16\alpha d = \dfrac{b^2(a^2 - c^2)}{a^2c^2}$

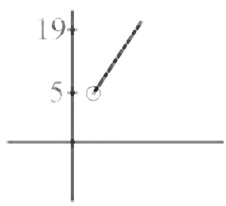
$$\therefore \qquad \frac{a^2 - c^2}{ac} = \frac{b^2(a^2 - c^2)}{4a^2c^2}$$

$\Rightarrow \qquad a - c = 0 \quad \text{or} \quad a + c = 0 \quad \text{or} \quad b^2 = 4ac$

$b^2 = 4ac$ and $a - c = 0$ are rejected as p_1, q_1, p_2, q_2 are distinct.

Hence, $\qquad a + c = 0$

48. $\left(\dfrac{x^2 + x + 2}{x^2 + x + 1}\right)^2 - (a - 3)\dfrac{x^2 + x + 2}{x^2 + x + 1} + (a - 4) = 0$



Put $\dfrac{x^2 + x + 2}{x^2 + x + 1} = t = 1 + \dfrac{1}{x^2 + x + 1}, \; t \in \left(1, \dfrac{7}{3}\right]$

$$t^2 + 3t - 4 = a(t - 1)$$

$$\Rightarrow \qquad a = \frac{t^2 + 3t - 4}{t - 1} = \frac{(t + 4)(t - 1)}{t - 1} = t + 4$$

For real roots $a \in \left(5, \frac{19}{3}\right]$

51. Put (a, b)

$$m^2(a^2 + a) - 4(a + 1)m + (4a^2 + 4a + 2 - b) = 0 \ \forall \ m \in R$$

$$\Rightarrow \qquad a^2 + a = 0,\ a + 1 = 0 \text{ and } 4a^2 + 4a + 2 - b = 0$$

$$\Rightarrow \qquad a = -1,\ b = 2$$

52. $1 + 2 + 3 + \delta = 0$

$$\Rightarrow \qquad \delta = -6$$

$$1 \cdot 2 \cdot 3 \cdot \delta = -36 = -c$$

$$\Rightarrow \qquad c = 36$$

53. Let $ax^4 + bx^3 + x^2 + x + 1 = 0$ with roots $\alpha_1, \alpha_2, \alpha_3, \alpha_4$

$\alpha_1\alpha_2\alpha_3\alpha_4 = \frac{1}{a} \neq 0 \quad \Rightarrow \quad$ None of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ is '0'

$$\left(\sum \frac{1}{\alpha_i}\right)^2 = \sum \frac{1}{\alpha_i^2} + 2\sum \frac{1}{\alpha_i}\frac{1}{\alpha_j}$$

$$\left(-\frac{1/a}{1/a}\right)^2 = \sum \frac{1}{\alpha_i^2} + \frac{2\left(+\frac{1}{a}\right)}{\frac{1}{a}}$$

$$\Rightarrow \qquad -1 = \sum_{i=1}^{4} \frac{1}{\alpha_i^2}$$

$\Rightarrow$ $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ all real is impossible.

54. $t^3 - 3t + \lambda = 0$ with roots x, y, z, $\lambda = -xyz$

$x^3 = 3x - \lambda \quad \Rightarrow \quad x^3y = 3xy - \lambda y$

$y^3 = 3y - \lambda \quad \Rightarrow \quad y^3z = 3yz - \lambda z$

$z^3 = 3z - \lambda \quad \Rightarrow \quad z^3x = 3zx - \lambda x$

$\Rightarrow \quad x^3y + y^3z + z^3x = 3(xy + yz + zx) - \lambda(x + y + z) = -9$

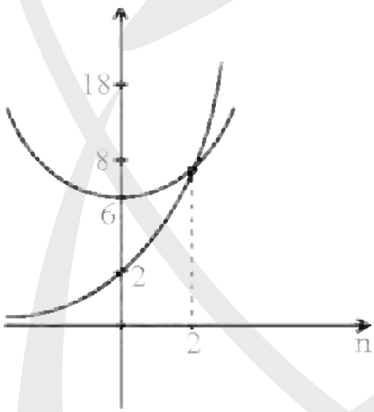
55. $3^{2n} + 3n^2 + 7 > 3^{2n}$

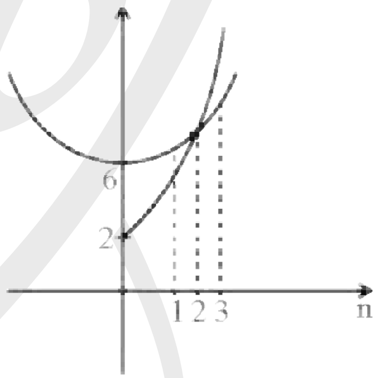
$\Rightarrow \quad 3^{2n} + 3n^2 + 7 \geq (3^n + 1)^2$

$\Rightarrow \quad 2 \cdot 3^n \leq 3n^2 + 6$

$y = 2 \cdot 3^n$

$y = 3n^2 + 6$





From graph is clear that $n \leq 2$

$n = 1$ doesn't satisfy

$n = 2$ satisfy

$\Rightarrow \quad n = 2$ only

Solutions

CHAPTER 2

SEQUENCE AND PROGRESSION

SECTION-1

SINGLE CHOICE QUESTIONS

1. $$c = a + (n-1)(c-b)$$

$$\Rightarrow \quad n = \frac{2c-a-b}{c-b}$$

$$\text{sum} = \frac{n}{2}(a+c)$$

$$= \frac{(2c-a-b)(a+c)}{2(c-b)}$$

2. $x\ln a = y \ln b = z\ln c$

$$\Rightarrow \quad \frac{\ln a}{\ln b} = \frac{y}{x} = \frac{z}{y} = \frac{\ln b}{\ln c}$$

$$\Rightarrow \quad \log_b a = \log_c b$$

3. $(a-d) + a + (a+d) = 1 \quad \Rightarrow \quad a = \frac{1}{3}$

$$\frac{1}{a-d} + \frac{1}{a} + \frac{1}{a+d} = 11$$

$$\Rightarrow \quad 2a = 8(a^2 - d^2)$$

$$\Rightarrow \quad \frac{1}{3} = 4\left(\frac{1}{9} - d^2\right)$$

$$\Rightarrow \quad d^2 = \frac{1}{36}$$

$$\Rightarrow \quad d = \pm\frac{1}{6}$$ numbers are 6, 3, 2

4. $\frac{1}{x} = a + (p-1)d \quad \frac{1}{y} = a + (q-1)d \quad \frac{1}{z} = a + (r-1)d$

$$\frac{1}{x} - \frac{1}{y} = (p-q)d$$

$\Rightarrow \quad xy(p-q) = \frac{y-x}{d}$; $yz(q-r) = \frac{z-y}{d}$; $zx(r-p) = \frac{x-z}{d}$

$\Rightarrow \quad (p-q)xy + yz(q-r) + zx(r-p) = \frac{1}{d}(y - x + z - y + x - z) = 0$

5. $A_1 + A_2 = a + b$; $G_1G_2 = ab$

$$\frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

$\Rightarrow \quad \frac{H_1 + H_2}{H_1H_2} = \frac{A_1 + A_2}{G_1G_2}$

6. $\frac{1}{c} = \frac{2}{b} - \frac{1}{a}$

$$\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{c}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right) = \left(\frac{1}{b} - \frac{1}{a} + \frac{2}{b} - \frac{1}{a}\right)\left(\frac{2}{b} - \frac{1}{a} + \frac{1}{a} - \frac{1}{b}\right)$$

$$= \left(\frac{3}{b} - \frac{2}{a}\right)\left(\frac{1}{b}\right) = \frac{3}{b^2} - \frac{2}{ab}$$

7. $xA = yG = zH = \lambda$

$$G^2 = AH$$

$\Rightarrow \quad \left(\frac{\lambda}{y}\right)^2 = \left(\frac{\lambda}{x}\right)\left(\frac{\lambda}{z}\right)$

$\Rightarrow \quad y^2 = xz$

8. $(a + nd)^2 = (a + md)(a + rd)$

$\Rightarrow \quad (n^2 - mr)d^2 = ad(r + m - 2n)$

$\Rightarrow \quad \frac{d}{a} = \frac{\left(\frac{2mr}{n} - 2n\right)}{n^2 - mr} \qquad \left[\because n = \frac{2mr}{m+r}\right]$

$\Rightarrow \quad \frac{d}{a} = \frac{2}{n}$

9. $$a + (m-1)d = \frac{2(a+(\ell-1)d)(a+(n-1)d)}{(a+(\ell-1)d+a+(n-1)d)}$$

$\Rightarrow$ $(2a + (\ell + n - 2)d)\,(a + (m - 1)\,d) = 2\,(a + (\ell - 1)d)\,(a + (n - 1)d)$

$\Rightarrow$ $ad[(2m - 2 + \ell + n - 2) - (2(n - 1) + 2(\ell - 1))]$

$= d^2\,(2(\ell - 1)\,(n - 1) - (\ell + n - 2)\,(m - 1))$

$\Rightarrow$ $$\frac{a}{d} = \frac{2\ell n - \ell - n - \ell m - mn + 2m}{2m - n - \ell}$$

$$= \frac{(2m - \ell - n)(m+1)}{(2m - n - \ell)}$$

$= (m + 1)$

10. $$\frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{A_1 + A_2}{G_1G_2}$$

$\Rightarrow$ $$\frac{H_1 + H_2}{H_1H_2} = \frac{A_1 + A_2}{G_1G_2}$$

11. $$\frac{6}{2}(2a + 5d) = \frac{9}{2}(2a + 8d)$$

$\Rightarrow 4a + 10d = 6a + 24d \;\Rightarrow\; a + 7d = 0$

$$\frac{a_3}{a_5} = \frac{a+2d}{a+4d} = \frac{-5d}{-3d} = \frac{5}{3}$$

12. $3(a_1 + 7d) = 5(a_1 + 12d) \;\Rightarrow\; 2a_1 = -39d$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d) = \frac{n}{2}(n - 40),\ d < 0$$

S_n is maximum at $n = 20$

13. $\alpha(1 + d) = -\frac{2b}{a}$ $\qquad \alpha^2 d = \frac{c}{a}$

$\alpha d^2(1 + d) = -\frac{2q}{p}$ $\qquad \alpha^2 d^5 = \frac{r}{p}$

$\Rightarrow$ $d^2 = \frac{qa}{bp}$ $\qquad d^4 = \frac{ra}{cp}$

$\Rightarrow$ $$\frac{q^2a^2}{b^2p^2} = \frac{ra}{cp} \Rightarrow \frac{q^2ac}{b^2} = pr \;\Rightarrow\; \frac{pr}{q^2} = \frac{ac}{b^2}$$

14. $\sum_{r=1}^{99} r!((r+1)^2 - r) = \sum_{r=1}^{99} ((r+1)(r+1)! - rr!) = 100(100!) - 1$

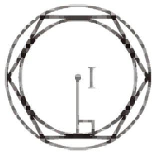
15. Area $= \frac{1}{2}\begin{vmatrix} a & x & 1 \\ a+d & xr & 1 \\ a+2d & xr^2 & 1 \end{vmatrix} = \frac{1}{2}xd(r-1)^2$

16. Let a, ar, ar^2,......ar^{2n-1}

$$\frac{a(1-r^{2n})}{1-r} = \frac{5a(1-(r^2)^n)}{1-r^2}$$

$\Rightarrow$ $r = 4$

17. Let r $\rightarrow$ inradius, R $\rightarrow$ circumradius, P $\rightarrow$ perimeter of polygon



$P < 2\pi R$

$$B = \frac{1}{2}rP < \frac{1}{2}r(2\pi R) = \pi rR$$

$\Rightarrow$ $B < \pi\, rR < \pi\left(\frac{r^2 + R^2}{2}\right) = \frac{A + C}{2}$

18. $\sum_{k=1}^{n} \frac{k^2 - \frac{1}{2}}{\left(k^2 - k + \frac{1}{2}\right)\left(k^2 + k + \frac{1}{2}\right)} = \sum_{k=1}^{n}\left(\frac{k - \frac{1}{2}}{k^2 - k + \frac{1}{2}} - \frac{k + \frac{1}{2}}{k^2 + k + \frac{1}{2}}\right)$

$$= 1 - \frac{n + \frac{1}{2}}{n^2 + n + \frac{1}{2}} = 1 - \frac{2n+1}{2n^2 + 2n + 1}$$

19. $k^4 + \frac{1}{4} = \left(k^2 - k + \frac{1}{2}\right)\left(k^2 + k + \frac{1}{2}\right)$

$$\prod_{k=1}^{n} \frac{\left((2k-1)^2 - (2k-1) + \frac{1}{2}\right)\left((2k-1)^2 + (2k-1) + \frac{1}{2}\right)}{\left((2k)^2 - 2k + \frac{1}{2}\right)\left((2k)^2 + 2k + \frac{1}{2}\right)}$$

$$= \prod_{k=1}^{n} \frac{(2k-1)^2 - (2k-1) + \frac{1}{2}}{(2k)^2 + 2k + \frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{4n^2 + 2n + \frac{1}{2}} = \frac{1}{8n^2 + 4n + 1}$$

20. $\sum_{k=1}^{n} \frac{1}{\sqrt{k}\sqrt{k+1}\left(\sqrt{k+1}+\sqrt{k}\right)} = \sum_{k=1}^{n} \frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k}\sqrt{k+1}}$

$$= \sum_{k=1}^{n}\left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right) = 1 - \frac{1}{\sqrt{n+1}}$$

21. $\sum_{k=1}^{n} k!\left((k+1)^2 - k\right) = \sum_{k=1}^{n} \left((k+1)(k+1)! - k(k!)\right)$

$$= (n + 1)\ (n + 1)! - 1$$

22. $\sum_{k=1}^{n-1} \frac{1}{a_k a_{k+1}} = \frac{1}{d}\sum_{k=1}^{n-1} \frac{a_{k+1} - a_k}{a_k a_{k+1}} = \frac{1}{d}\sum_{k=1}^{n-1}\left(\frac{1}{a_k} - \frac{1}{a_{k+1}}\right)$

$$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_n}\right) = \frac{(n-1)}{a_1 a_n} = \frac{(n-1)}{a_1(a_1 + (n-1)d)}$$

23. $S_{2n} - S_n = \frac{2n}{2}(2a + (2n - 1)d) - \frac{n}{2}(2a + (n - 1)d) = \frac{n}{2}(2a + (3n - 1)d)$

$$= \frac{1}{3} S_{3n}$$

24. $\sum_{r=1}^{10} \frac{2r+5}{(r+1)(r+2)}\frac{1}{3^r} = \sum_{r=1}^{10} \frac{3(r+2) - (r+1)}{(r+1)(r+2)}\frac{1}{3^r}$

$$= \sum_{r=1}^{10}\left(\frac{1}{(r+1)3^{r-1}} - \frac{1}{(r+2)3^r}\right)$$

$$= \frac{1}{2} - \frac{1}{12 \cdot 3^{10}}$$

25. $i \to 10-i,\ j \to 10-j$

$$S = \sum_{0\le i<j\le 10}\sum (10-i+10-j) = \sum_{0\le i<j\le 10}\sum (20-(i+j))$$

$$S = \sum_{0\le i<j\le 10}\sum 20 - S = 20\times 55 - S$$

$\Rightarrow$ $S = 550$

26. $A = \dfrac{a(1+r+r^2+....r^{n-1})}{n}$

$$H = \frac{n}{\frac{1}{a}\left(1+\frac{1}{r}+\frac{1}{r^2}+...\frac{1}{r^{n-1}}\right)} = \frac{nar^{n-1}}{1+r+r^2+...+r^{n-1}}$$

$AH = a^2r^{n-1}$

27. $\dfrac{x+y}{2} - \sqrt{xy} = 2 \quad \Rightarrow \sqrt{x} - \sqrt{y} = -2$

$(x, y) = (1, 9), (4, 16), (9, 25), ... (42^2, 44^2)$

28. $S_n = \displaystyle\sum_{r=1}^{n} \frac{(r^2+6r+12)}{(r)(r+1)(r+2)}\frac{1}{2^{r+1}} = \sum_{r=1}^{n} \frac{2(r+2)(r+3)-r(r+4)}{(r)(r+1)(r+2)^{r+1}}$

$$= \sum_{r=1}^{n}\left(\frac{r+3}{r(r+1)2^r} - \frac{r+4}{(r+1)(r+2)2^{r+1}}\right)$$

$$= 1 - \frac{n+4}{(n+1)(n+2)2^{n+1}}$$

29. $$S = \sum_{r=1}^{\infty} r^2\left(-\frac{1}{5}\right)^{r-1}$$

$$\sum_{r=0}^{\infty} x^r = \frac{1}{1-x}$$

$\Rightarrow$ $$\sum_{r=1}^{\infty} rx^r = \frac{x}{(1-x)^2} = \frac{1}{x-1} + \frac{1}{(x-1)^2}$$

$\Rightarrow$ $$\sum_{r=1}^{\infty} r^2x^{r-1} = -\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}$$

$\Rightarrow$ $$\sum_{r=1}^{\infty} r^2\left(-\frac{1}{5}\right)^{r-1} = \frac{25}{54}$$

30. $2(x+2y) = x + (2x+y) \Rightarrow x = 3y$

$(y+1)^2 (x+1)^2 = (xy+5)^2 \quad \Rightarrow (3y+1)(y+1) = \pm(3y^2+5)$

$\Rightarrow \quad y = 1$ or $3y^2 + 2y + 3 = 0$

$\therefore \quad y = 1, x = 3$

31. $\Rightarrow \quad \sin^2\theta = 6\cos^3\theta \Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

$\Rightarrow \quad (3\cos^2\theta + 2\cos\theta + 1)(2\cos\theta - 1) = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \quad \theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}, 4\pi - \frac{\pi}{3}$

Sum $= 8\pi$

32. $$\sum_{r=1}^{n} \frac{n-(r-1)}{r(r+1)(r+2)} = \frac{(n+1)}{2}\sum_{r=1}^{n} \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} - \sum_{r=1}^{n}\left(\frac{1}{r+1} - \frac{1}{r+2}\right)$$

$$= \frac{(n+1)}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right) - \left(\frac{1}{2} - \frac{1}{n+2}\right)$$

$$= \frac{1}{2(n+2)} + \frac{n+1}{4} - \frac{1}{2}$$

33. $$-\frac{1}{1-x} + \left(\frac{1}{1+x} + \frac{1}{1-x}\right) + \frac{2}{1+x^2} + \frac{4}{1+x^4} + \dots + \frac{2^{n-1}}{1+x^{2^{n-1}}}$$

$$= -\frac{1}{1-x} + \left(\frac{2}{1-x^2} + \frac{2}{1+x^2}\right) + \frac{4}{1+x^4} + \dots + \frac{2^{n-1}}{1+x^{2^{n-1}}}$$

$$= -\frac{1}{1-x} + \frac{2^n}{1-x^{2^n}}$$

34. $$\frac{(1-x)(1-y)(1-z)}{xyz} = \frac{(y+z)(z+x)(x+y)}{xyz} \geq \frac{(2\sqrt{yz})(2\sqrt{zx})(2\sqrt{xy})}{xyz} = 8$$

35. $$\left(1 - \frac{1}{1+a_1}\right) + \left(\frac{1}{1+a_1} - \frac{1}{(1+a_1)(1+a_2)}\right)$$

$$+\left(\frac{1}{(1+a_1)(1+a_2)} - \frac{1}{(1+a_1)(1+a_2)(1+a_3)}\right) + \dots +$$

$$\left(\frac{1}{(1+a_1)(1+a_2)\dots(1+a_{n-1})} - \frac{1}{(1+a_1)(1+a_2)\dots(1+a_n)}\right)$$

$$= 1 - \frac{1}{(1+a_1)(1+a_2)(1+a_3)\dots(1+a_n)}$$

36. $\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}=\sum_{r=1}^{n}\frac{(r+1)^2-r^2}{r^2(r+1)^2}=\sum_{r=1}^{n}\left(\frac{1}{r^2}-\frac{1}{(r+1)^2}\right)=1-\frac{1}{(n+1)^2}$

37.
$$S_n = 1 + (1+2)x + (1+2+3)x^2 + \dots + (1+2+3+\dots n)x^{n-1}$$
$$x S_n = x + (1+2)x^2 + \dots + (1+2+\dots n-1)x^{n-1} + (1+2+\dots n)x^n$$
$$(1-x)S_n = 1 + 2x + 3x^2 + 4x^3 + \dots + nx^{n-1} - (1+2+3+\dots+n)x^n$$
$$x(1-x)S_n = x + 2x^2 + 3x^3 + \dots + (n-1)x^{n-1} + nx^n - (1+2+3+\dots n)x^{n+1}$$
$$(1-x)^2S_n = 1 + x + x^2 + \dots x^{n-1} - nx^n - (1+2+3+\dots n)\,x^n(1-x)$$
$$= \frac{1-x^n}{1-x} - nx^n - \frac{n(n+1)}{2}x^n(1-x)$$
$$\Rightarrow \quad S_n = \frac{1-x^n}{(1-x)^3} - \frac{nx^n}{(1-x)^2} - \frac{n(n+1)x^n}{2(1-x)}$$

38. $S_n = \sum_{r=1}^{n}(r+1)(r^2+2) = \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6} + n(n+1) + 2n$

$$= \frac{n}{12}(3n^3 + 10n^2 + 21n + 38)$$

39. $\frac{(x^{1/2}+x^{-1/2})^2(x+x^{-1})^2(x^2+x^{-2})^2}{(x+x^{-1})(x^2+x^{-2})(x^4+x^{-4})}\dots\frac{(x^{2^{n-2}}+x^{-2^{n-2}})^2}{x^{2^{n-1}}+x^{-2^{n-1}}}\frac{(x^{2^{n-1}}+x^{-2^{n-1}})^2}{(x^{2^n}+x^{-2^n})}$

$$(x^{1/2}+x^{-1/2})^2(x+x^{-1})(x^2+x^{-2})(x^4+x^{-4})\dots\frac{(x^{2^{n-1}}+x^{-2^{n-1}})}{(x^{2^n}+x^{-2^n})}$$

$$=\frac{(x^{1/2}+x^{-1/2})^2(x+x^{-1})(x+x^{-1})(x^2+x^{-2})(x^4+x^{-4})\dots(x^{2^{n-1}}+x^{-2^{n-1}})}{(x-x^{-1})(x^{2^n}+x^{-2^n})}$$

$$=\frac{(x^{1/2}+x^{-1/2})^2(x^{2^n}-x^{-2^n})}{(x-x^{-1})(x^{2^n}+x^{-2^n})}$$

$$\lim_{n\to\infty} p_n = \frac{(x^{1/2}+x^{-1/2})^2}{(x-x^{-1})} = \frac{(x+1)^2}{x^2-1} = \left(\frac{x+1}{x-1}\right)$$

40. a, G_1, G_2, b are in G.P.

$\Rightarrow \quad G_1 = a^{2/3}b^{1/3},\ G_2 = a^{1/3}b^{2/3}$

a, A, b are in A.P.

$\Rightarrow \quad A = \frac{a+b}{2}$

$$\frac{G_1^3+G_2^3}{G_1G_2} = \frac{a^2b+ab^2}{ab} = a + b = 2A$$

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. $y = \frac{\tan 3x}{\tan x} = \frac{3 - \tan^2 x}{1 - 3\tan^2 x} = \frac{8}{3(1 - 3\tan^2 x)} + \frac{1}{3}$

$$\Rightarrow \quad y \in \left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$$

$$a = \frac{1}{3},\ b = 3,\ s_\infty = \frac{6}{1 - \frac{1}{3}} = 9$$

2.
$$a_{n+1} + 1 = \left(\sqrt{1 + a_n} + 1\right)^2$$

$$\Rightarrow \quad \sqrt{1 + a_{n+1}} = \sqrt{1 + a_n} + 1$$

$\Rightarrow \left\{\sqrt{1 + a_n}\right\}$ is an A.P. with first term $\sqrt{1 + a_1} = 1$ and common difference 1.

$$\Rightarrow \quad \sqrt{1 + a_n} = 1 + (n - 1)1 = n \Rightarrow a_n = n^2 - 1$$

3. $\frac{a_1 + 2}{2} = \sqrt{2a_1} \Rightarrow (a_1 - 2)^2 = 0 \Rightarrow a_1 = 2$

$$(a_n + 2)^2 = 8s_n$$
$$(a_{n-1} + 2)^2 = 8s_{n-1}$$

$$\Rightarrow \quad (a_n + 2)^2 - (a_{n-1} + 2)^2 = 8a_n$$

$$\Rightarrow (a_n - a_{n-1})(a_n + a_{n-1} + 4) = 8a_n$$

$$\Rightarrow \quad a_n^2 - a_{n-1}^2 = 4(a_n + a_{n-1})$$

$$\Rightarrow \quad a_n - a_{n-1} = 4$$

4.
$$\frac{3x + \frac{4xz}{x+z}}{2x - \frac{2xz}{x+z}} + \frac{3z + \frac{4xz}{x+z}}{2z - \frac{2xz}{z+x}} = \frac{3x^2 + 7xz}{2x^2} + \frac{7xz + 3z^2}{2z^2}$$

$$= 3 + \frac{7}{2}\left(\frac{z}{x} + \frac{x}{z}\right) \in (10, \infty)$$

5. $$\frac{5^{a_{n+1}}}{5^{a_n}} = \frac{3n+5}{3n+2}$$

$$\frac{5^{a_2}}{5^{a_1}} \times \frac{5^{a_3}}{5^{a_2}} \times \ldots\ldots\ldots \times \frac{5^{a_n}}{5^{a_{n-1}}} = \frac{8}{5} \times \frac{11}{8} \times \frac{14}{11} \times \ldots\ldots. \frac{3n+2}{3n-1}$$

$$\frac{5^{a_n}}{5^{a_1}} = \frac{3n+2}{5} \qquad \Rightarrow \; 5^{a_n} = 3n+2 \; \Rightarrow \; a_n = \log_5(3n+2)$$

$$[a_n] = 3 \qquad \Rightarrow \; \log_5(3n+2) \in [3, 4)$$

$\Rightarrow \; n \in \{41, 42, \ldots\ldots, \ldots\ldots 207\}$

$$[a_n] = 4 \; \Rightarrow \; \frac{623}{3} \le n < \frac{3123}{3}$$

$\Rightarrow \; n \in \{208, 209, \ldots.., 1041\}$

6. $$\frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b} \ge 2\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$$

$$= 2\left[(a+b+c)\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right) - 3\right] \ge 2\left(\frac{9}{2} - 3\right) = 3$$

$$\frac{3}{\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right)} \le \frac{(b+c)+(c+a)+(a+b)}{3}$$

$$\Rightarrow (a+b+c)\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right) \ge \frac{9}{2}$$

7. $\alpha s = a(1 + r + r^2)$

$s^2 = a^2(1 + r^2 + r^4)$

$$\Rightarrow \quad \alpha^2 = \frac{(1+r+r^2)}{1-r+r^2} = 1 + \frac{2r}{1-r+r^2} = 1 + \frac{2}{\frac{1}{r} + r - 1}$$

$$\Rightarrow \quad \alpha^2 \in \left(1 - \frac{2}{3}, 1 - 0\right) \cup (1+0, 1+2)$$

$$\Rightarrow \quad \alpha^2 \in \left(\frac{1}{3}, 1\right) \cup (1, 3)$$

8. $$p = a + n\left(\frac{b-a}{n+1}\right) = \frac{nb+a}{n+1}$$

$$\frac{1}{q} = \frac{1}{a} + n\left(\frac{\frac{1}{b}-\frac{1}{a}}{n+1}\right) = \frac{na+b}{(n+1)ab}$$

$$q = \frac{(n+1)ab}{na+b}$$

$\therefore$ $$q\left(na + \frac{p(n+1)-a}{n}\right) = (n+1)a\left(\frac{(n+1)p-a}{n}\right)$$

$\Rightarrow$ $$q\left((n^2-1)a + p(n+1)\right) = (n+1)a((n+1)p-a)$$

$\Rightarrow$ $$a^2 + a(q(n-1)-(n+1)p) + pq = 0$$

$D \geq 0 \Rightarrow$ $$(q(n-1)-(n+1)p)^2 - 4pq \geq 0$$

$$(q-p)^2 n^2 + (q+p)^2 - 2n(q^2-p^2) - 4pq \geq 0$$

$\Rightarrow$ $$(q-p)^2 n^2 + (q-p)^2 - 2n(q^2-p^2) \geq 0$$

$\Rightarrow$ $$(q-p)\left((q-p)n^2 + (q-p) - 2n(q+p)\right) \geq 0$$

$\Rightarrow$ $$(q-p)(q(n-1)^2 - p(n+1)^2) \geq 0$$

$$q \in (-\infty, p] \cup \left[\left(\frac{n+1}{n-1}\right)^2 p, \infty\right)$$

9. $a_2^2 = a_1^2 + 2a_1 + 1$

$a_3^2 = a_2^2 + 2a_2 + 1$

$\vdots$

$a_{n+1}^2 = a_n^2 + 2a_n + 1$

Adding, $a_{n+1}^2 = a_1^2 + 2(a_1 + a_2 + \ldots\ldots a_n) + n$

$\Rightarrow$ $$a_1 + a_2 + \ldots\ldots + a_n = \frac{a_{n+1}^2 - n}{2}$$

$\Rightarrow$ $$\frac{\sum_{r=1}^{n} a_r}{n} = -\frac{1}{2} + \frac{a_{n+1}^2}{2n} \geq -\frac{1}{2}$$

10. $$\frac{\underbrace{(x+x+....x)}_{x\text{ times}}+\underbrace{(y+y+....y)}_{y\text{ times}}+\underbrace{(z+z+....z)}_{z\text{ times}}}{(x+y+z)} \geq \left(x^x y^y z^z\right)^{\frac{1}{x+y+z}}$$

$$\geq \frac{x+y+z}{\underbrace{\left(\frac{1}{x}+\frac{1}{x}+....\frac{1}{x}\right)}_{x\text{ times}}+\underbrace{\left(\frac{1}{y}+\frac{1}{y}+....\frac{1}{y}\right)}_{y\text{ times}}+\underbrace{\left(\frac{1}{z}+\frac{1}{z}+....+\frac{1}{z}\right)}_{z\text{ times}}}$$

11. $$\frac{1}{x-1}+\left(\frac{1}{x+1}-\frac{1}{x-1}\right)+\frac{2}{x^2+1}+\frac{4}{x^4+1}+....+\frac{2^n}{x^{2^n}+1}$$

$$=\frac{1}{x-1}+2\left(\frac{1}{x^2+1}-\frac{1}{x^2-1}\right)+\frac{4}{x^4+1}+.....+\frac{2^n}{x^{2^n}+1}$$

$$=\frac{1}{x-1}+4\left(\frac{1}{x^4+1}-\frac{1}{x^4-1}\right)+\frac{8}{x^8+1}+.....+\frac{2^n}{x^{2^n}+1}=\frac{1}{x-1}-\frac{2^{n+1}}{x^{2^{n+1}}-1}$$

$\Rightarrow \quad f(x)=\frac{1}{x-1} \quad \forall\ x>1$

12. $$S(n)=1+\underbrace{\left(\frac{1}{2}+\frac{1}{3}\right)}_{2\text{ terms}}+\underbrace{\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)}_{4\text{ terms}}+\underbrace{\left(\frac{1}{8}+\frac{1}{9}++...+\frac{1}{15}\right)}_{8\text{ terms}}+ ...\underbrace{\left(\frac{1}{2^{n-1}}+\frac{1}{2^{n-1}+1}...\frac{1}{2^n-1}\right)}_{2^{n-1}\text{ terms}}$$

$$<1+\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+...+\frac{1}{8}\right)+...+\left(\frac{1}{2^{n-1}}+\frac{1}{2^{n-1}}+...\frac{1}{2^{n-1}}\right)$$

$= 1 + 1 + 1 + 1 + ... + 1$

$= n$

$$S(n)>\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+\left(\frac{1}{9}+\frac{1}{10}+...+\frac{1}{16}\right)+... +\left(\frac{1}{2^{n-1}+1}+\frac{1}{2^{n-1}+2}+...\frac{1}{2^n}\right)$$

$$>\frac{1}{2}+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+\left(\frac{1}{16}+\frac{1}{16}+...+\frac{1}{16}\right)+...\left(\frac{1}{2^n}+\frac{1}{2^n}+...+\frac{1}{2^n}\right)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots + \frac{1}{2}$$

$$= \frac{n}{2}$$

$\Rightarrow$ $S(2n) > n$

13. $a_n + a_{n-1} = 2n$

$a_{n-1} + a_{n-2} = 2(n-1)$

$\Rightarrow$ $a_n - a_{n-2} = 2$

$a_1, a_2, a_3, \ldots\ldots = 100, -96, 102, -94, 104, -92, \ldots$

$$a_n = \begin{cases} n+99 & \text{if } n \text{ is odd} \\ n-98 & \text{if } n \text{ is even} \end{cases}$$

14. $\{a_n\} = \{2, 2r, 2r^2, 2r^3, 2r^4, \ldots\}$

$2r, 2r^4 \in I \;\&\; 2r^4 \le 200 \quad \Rightarrow r^4 \le 100$

$\Rightarrow$ $r = 2$ or 3

$a_5 = 2(2)^4, 2(3)^4 = 32, 162$

15. **(B)** $$\frac{1}{a^2(1-r^2)}\left[1 + \frac{1}{r^2} + \frac{1}{r^4} + \ldots \frac{1}{r^{2(n-2)}}\right] = \frac{1 - \frac{1}{r^{2(n-1)}}}{\left(1 - \frac{1}{r^2}\right)a^2(1-r^2)}$$

$$= \frac{(1-r^{2n})}{a^2(1-r^2)^2 r^{2n-4}}$$

(C) $$\frac{1}{a^m(1+r^m)}\left[1 + \frac{1}{r^m} + \frac{1}{r^{2m}} + \ldots + \frac{1}{r^{(n-2)m}}\right] = \frac{\left(1 - \frac{1}{r^{(n-1)m}}\right)}{\left(1 - \frac{1}{r^m}\right)a^m(1+r^m)}$$

$$= \frac{(1-r^{mn-m})}{(1-r^m)(r^{mn-m} - r^{mn-2m})}$$

16. $\Rightarrow$ Let x(y – z) = a, y(z – x) = b, z(y – x) = C

$\Rightarrow$ a + b = C

and $b^2 = ac = a(a + b)$

$\Rightarrow \frac{b^2}{a^2} - \frac{b}{a} - 1 = 0 \Rightarrow r = \frac{b}{a} = \frac{1 \pm \sqrt{5}}{2}$

17. $a_n = \frac{n(n+1)(n+2)}{3} - \frac{(n-1)n(n+1)}{3} = n(n+1)$

$b_n = \frac{1}{n(n+1)}$

$$\sum_{r=1}^{n} b_r = \sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

18. $\frac{1}{\alpha} + \frac{1}{\gamma} = \frac{2}{\beta} \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{3}{\beta} = \frac{q}{q} = 1 \Rightarrow \beta = 3$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} \geq \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} \quad \Rightarrow 3\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right) \geq \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)^2 = 1$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} \geq \frac{1}{3}$$

Again $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} \geq \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} \Rightarrow 3\left(\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}\right) \leq \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)^2 = 1$

Put $\beta = 3$ in the equation $27 + 9p + 3q - q = 0 \Rightarrow 9p + 2q + 27 = 0$

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} \leq \frac{1}{3} \Rightarrow \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{-p}{q} \leq \frac{1}{3} \Rightarrow \frac{p}{q} \geq -\frac{1}{3}$$

19. $S = \sum_{n=1}^{\infty}\sum_{k=1}^{n-1} \frac{k}{2^{n+k}} = \sum_{k=1}^{\infty}\left(\frac{k}{2^k}\sum_{n=k+1}^{\infty}\frac{1}{2^n}\right) = \sum_{k=1}^{\infty}\frac{k}{2^k} \times \frac{1}{2^{k+1}} \times 2$

$$\sum_{k=1}^{\infty}\frac{k}{4^k} = \frac{4}{9}$$

20. $n^2 < n^2 + 1 < n^2 + 2 < \dots < n^2 + n$

$$\Rightarrow \frac{1}{n^2+1} + \frac{2}{n^2+2} + \dots + \frac{n}{n^2+n} < \frac{1}{n^2} + \frac{2}{n^2} + \dots \frac{n}{n^2} = \frac{1}{2} + \frac{1}{2n}$$

and $\frac{1}{n^2+1} + \frac{2}{n^2+2} + \dots + \frac{n}{n^2+n} > \frac{1}{n^2+n} + \frac{2}{n^2+n} + \dots \frac{n}{n^2+n} = \frac{1}{2}$

21.(a) $$(1\cdot3\cdot5\ldots(2n-1))^{1/n} < \frac{1+3+5+\ldots(2n-1)}{n} = n \Rightarrow 1\cdot3\cdot5\ldots(2n-1) < n^n$$

(b) $$\frac{1+2+2^2+\ldots+2^{n-1}}{n} > (2^{1+2+\ldots(n-1)})^{1/n} = 2^{\frac{n-1}{2}}$$

$$\Rightarrow \quad 2^n \cdot > 1 + n2^{\frac{n-1}{2}}$$

(c) $$\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+n} > \frac{1}{2n}+\frac{1}{2n}+\ldots+\frac{1}{2n} = \frac{1}{2}$$

22. $$\frac{1}{a_{n+1}-1} = \frac{1}{a_n(a_n-1)} = \frac{1}{a_n-1} - \frac{1}{a_n}$$

$$\frac{1}{a_n-1} - \frac{1}{a_{n+1}-1} = \frac{1}{a_n}$$

$$\Rightarrow \quad \sum_{r=1}^{2018}\frac{1}{a_r} = \frac{1}{a_1-1} - \frac{1}{a_{2019}-1} = 1 - \frac{1}{a_{2019}-1} < 1$$

$$a_{2019} - 1 = a_{2018}(a_{2018}-1) = a_{2018}\, a_{2017}\,(a_{2017}-1)$$

$$\Rightarrow \quad a_{2019} - 1 = a_{2018}a_{2017}a_{2016}\ldots a_2a_1(a_1-1)$$

$$= a_1a_2a_3 \ldots a_{2018}$$

$$2018 < \frac{2018}{\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_{2018}}} < (a_1a_2\ldots a_{2018})^{\frac{1}{2018}}$$

$$\Rightarrow \quad a_{2019} - 1 > (2018)^{2018}$$

$$\Rightarrow \quad 1 - \frac{1}{a_{2019}-1} > 1 - \frac{1}{(2018)^{2018}}$$

23. $$\frac{k}{(k-1)^{4/3}+k^{4/3}+(k+1)^{4/3}} < \frac{k}{(k-1)^{4/3}+((k-1)(k+1))^{2/3}+(k+1)^{4/3}}$$

$$= \frac{\left((k+1)^{2/3}-(k-1)^{2/3}\right)k}{(k+1)^2-(k-1)^2}$$

$$= \frac{1}{4}\left((k+1)^{2/3}-(k-1)^{2/3}\right)$$

$$\Rightarrow \qquad S_n < \frac{1}{4}\sum_{k=1}^{n}\left((k+1)^{2/3}-(k-1)^{2/3}\right)=\frac{1}{4}\left((n+1)^{2/3}+n^{2/3}-1\right)$$

$$\Rightarrow \qquad S_{999} < \frac{1}{4}\left((1000)^{2/3}+(999)^{2/3}-1\right) < \frac{1}{4}(100+100-1) < 50$$

$$S_{26} < \frac{1}{4}\left((27)^{2/3}+(26)^{2/3}-1\right) < \frac{1}{4}(9+9-1)=\frac{17}{4}$$

24. $S = \dfrac{1}{\sqrt{1}+\sqrt{3}}+\dfrac{1}{\sqrt{5}+\sqrt{7}}+\ldots+\dfrac{1}{\sqrt{9997}+\sqrt{9999}}$

$$> \frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{7}+\sqrt{9}}+\ldots\frac{1}{\sqrt{9999}+\sqrt{10001}}$$

$$\Rightarrow \qquad 2S > \frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots+\frac{1}{\sqrt{9999}+\sqrt{10001}}$$

$$=\frac{1}{2}(\sqrt{3}-\sqrt{1}+\sqrt{5}-\sqrt{3}+\sqrt{7}-\sqrt{5}+\ldots+\sqrt{10001}-\sqrt{9999})$$

$$=\frac{1}{2}\left(\sqrt{10001}-\sqrt{1}\right) > \frac{1}{2}(100-1)$$

$$\Rightarrow \qquad S > \frac{99}{4} > 24$$

25. $a_9 = 5a_2 \quad \Rightarrow \quad a_1 + 8d = 5(a_1 + d) \quad \Rightarrow 4a_1 = 3d$

$a_{13} = 2a_6 + 5 \Rightarrow \quad a_1 + 12d = 2(a_1 + 5d) + 5 \quad \Rightarrow a_1 + 5 = 2d$

$\Rightarrow \quad a_1 = 3, d = 4$

26. $a_n, 2 = \sum_{i=0}^{n-1}\sum_{j=i+1}^{n} 2^i 2^j = \dfrac{1}{2}[(1+2+2^n+\ldots+2^n)^2-(1^2+2^2+2^4+\ldots+2^{2n})]$

$$=\frac{(2^{n+1}-1)(2^{n+1}-2)}{3}$$

27. $a_{n+1} - 1 = a_n(a_n - 1)$

$$\frac{1}{a_{n+1}-1}=\frac{1}{a_n-1}-\frac{1}{a_n} \quad \Rightarrow \quad \frac{1}{a_n}=\frac{1}{a_n-1}-\frac{1}{a_{n+1}-1}$$

$$\Rightarrow \quad S=\frac{1}{a_1}+\frac{1}{a_2}+\ldots\frac{1}{a_{2019}}=\frac{1}{a_1-1}-\frac{1}{a_{2020}-1}=1-\frac{1}{a_{2020}-1}$$

$a_i > 1$

$\Rightarrow \quad S < 1$

$$a_{2020} - 1 = a_{2019}(a_{2019} - 1) = a_{2019}\, a_{2018}\, (a_{2018} - 1)$$

$$= a_{2019}a_{2018}a_{2017} \dots a_2\, a_1\, (a_1 - 1)$$

$$= a_1a_2a_3 \dots a_{2019}$$

$$(2019)^{2019} < \left(\frac{2019}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_{2019}}}\right)^{2019} < a_1a_2a_3 \dots a_{2019} = a_{2020} - 1$$

$$\Rightarrow \quad \frac{1}{a_{2020} - 1} < \frac{1}{(2019)^{2019}}$$

$$\Rightarrow \quad S > 1 - \frac{1}{(2019)^{2019}}$$

28. $$\frac{\underbrace{(a+a+\dots a)}_{a\text{ times}} + \underbrace{(b+b+\dots b)}_{b\text{ times}} + \underbrace{(c+c+\dots c)}_{c\text{ times}}}{a+b+c} \geq (a^a b^b c^c)^{\frac{1}{a+b+c}}$$

$$\Rightarrow \quad (a^a b^b c^c)^{1/n} \leq \frac{a^2 + b^2 + c^2}{n}$$

$$\frac{\underbrace{(a+a+\dots a)}_{b\text{ times}} + \underbrace{(b+b+\dots b)}_{c\text{ times}} + \underbrace{(c+c+\dots c)}_{a\text{ times}}}{a+b+c} \geq (a^b b^c c^a)^{\frac{1}{a+b+c}}$$

$$\frac{a^2 + b^2 + c^2}{n} \geq \frac{ab + bc + ca}{n} \geq (a^b b^c c^a)^{\frac{1}{n}}$$

$$(a^a b^b c^c)^{\frac{1}{n}} + (a^b b^c c^a)^{\frac{1}{n}} + (a^c b^a c^b)^{\frac{1}{n}} \leq \frac{a^2 + b^2 + c^2}{n}$$

$$+ \frac{ab + bc + ca}{n} + \frac{ac + ba + cb}{b} = n$$

29. $$\frac{\frac{1}{a}+\frac{1}{d}}{\frac{1}{b}+\frac{1}{c}} = \frac{\frac{1}{a}+\frac{1}{a+3d_1}}{\frac{1}{a+d_1}+\frac{1}{a+2d_1}} = \frac{(a+d_1)(a+2d_1)}{a(a+3d_1)} = 1+\frac{2d_1^2}{a(a+3d_1)} > 1$$

$ad - bc = a(a + 3d_1) - (a + d_1)(a + 2d_1) = -2d_1^2 < 0$

$$\frac{b+c}{2} > \frac{2}{\frac{1}{b}+\frac{1}{c}} \quad \Rightarrow \quad \frac{1}{b}+\frac{1}{c} > \frac{4}{b+c} = \frac{4}{(a+d)}$$

30. **(b)** $(a_1 + a_5) - 2(a_2 + a_4) + 2a_3 = 2a_3 - 4a_3 + 2a_3 = 0$

(d) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 + d$

$= (a_1 + a_5) - 4(a_2 + a_4) + d + 6a_3 = 2a_3 - 8a_3 + d + 6a_3 = 0 + d = d$

31. $S = 3(1 + 2 + 3 + 4 + ... + 2016) = 2^4 \times 3^3 \times 7 \times 2017$

$[\because\ 6^2 + 5^2 + 4^2 - 3^2 - 2^2 - 1^2 = 3\ [(6 + 3) + (5 + 2) + (4 + 1)]$

$= 3(1 + 2 + 3 + 4 + 5 + 6)]$

32. $3 \log_y x = 3 + d$

$3 \log_z y = 3 + 2d$

$7 \log_x z = 3 + 3d$

$\Rightarrow\quad 3^2 \times 7 = 3(1 + d)(3 + 2d)(3 + d)$

$\Rightarrow\quad (1 + d)(3 + d)(3 + 2d) = 21 \quad \Rightarrow \quad d = \frac{1}{2}$

$\therefore\quad 6\log_y x = 7 \Rightarrow\ x^6 = y^7$

$3\log_z y = 4 \Rightarrow\ y^3 = z^4$

$x^{18} = y^{21} = z^{28}$

33. $a(1 + r + r^2 + r^3) = 15$

$a^2(1 + r^2 + r^4 + r^6) = 85 \ \Rightarrow \quad \dfrac{(1+r+r^2+r^3)^2}{(1+r^2+r^4+r^6)} = \dfrac{225}{85}$

$$\Rightarrow \quad \frac{(1-r^4)^2(1-r^2)}{(1-r)^2(1-r^8)} = \frac{45}{17} = \frac{(1-r^4)(1+r)}{(1+r^4)(1-r)}$$

$\Rightarrow \quad 14r^4 - 17r^3 - 17r^2 - 17r + 14 = 0$

$\Rightarrow \quad 14\left(r^2 + \frac{1}{r^2}\right) - 17\left(r + \frac{1}{r}\right) - 17 = 0$

$\Rightarrow \quad 14(t^2 - 2) - 17t - 17 = 0 \qquad t = r + \frac{1}{r}$

$\Rightarrow \quad 14t^2 - 17t - 45 = 0 = (7t + 9)(2t - 5)$

$\Rightarrow \quad r + \frac{1}{r} = \frac{5}{2} \qquad \Rightarrow \qquad r = \frac{1}{2}, 2$

$r = \frac{1}{2}, a = 8 \quad \text{or} \quad r = 2, a = 1$

34. $S_n = \sum_{r=1}^{n} \frac{r^2 + r - 1}{(r+2)!} = \sum_{r=1}^{n} \frac{r^2 + 2r - (r+1)}{(r+2)!} = \sum_{r=1}^{n}\left(\frac{r}{(r+1)!} - \frac{r+1}{(r+2)!}\right)$

$= \frac{1}{2!} - \frac{n+1}{(n+2)!}$

$\Rightarrow \quad S = \frac{1}{2}$

35. $f_n(x) = \frac{1}{2}\left[\frac{1}{1-x} - \frac{1}{1+x} + \frac{1}{1-x^2} - \frac{1}{1+x^2} + \frac{1}{1-x^4} - \frac{1}{1+x^4} + \ldots\right]$

$= \frac{1}{2}\left[-\frac{1}{1-x} - \frac{1}{1+x} - \frac{1}{1+x^2} - \frac{1}{1+x^4} - \ldots\right.$

$\left. - \frac{1}{1+x^{2n-1}} + \frac{2}{1-x} + \frac{1}{1-x^2} + \frac{1}{1-x^4} + \ldots + \frac{1}{1-x^{2^{n-1}}}\right]$

$= \frac{1}{2}\left[-\frac{1}{1-x^2} - \frac{1}{1+x^2} - \frac{1}{1+x^4} - \ldots\right.$

$\left. - \frac{1}{1+x^{2^{n-1}}} + \frac{2}{1-x} + \frac{1}{1-x^4} + \frac{1}{1-x^8} + \ldots + \frac{1}{1-x^{2^{n-1}}}\right]$

$= \frac{1}{2}\left[\frac{2}{1-x} - \frac{2}{1-x^{2^n}}\right] = \frac{1}{1-x} - \frac{1}{1-x^{2^n}}$

$\therefore \quad f(x) = \begin{cases} \dfrac{x}{1-x} & x \in (0,1) \\ \dfrac{1}{1-x} & x \in (1, \infty) \end{cases}$

36. $$f(x) = \sum_{r=1}^{n} \frac{rx^{r-1}}{(x+1)(x+2)\ldots(x+r)}$$

$$f(x) = \left(1 - \frac{x}{1+x}\right) + \sum_{r=2}^{n} \frac{(x+r)x^{r-1} - x^r}{(x+1)(x+2)\ldots(x+r)}$$

$$= \left(1 - \frac{x}{1+x}\right) + \sum_{r=2}^{n} \left(\frac{x^{r-1}}{(x+1)(x+2)\ldots(x+r-1)} - \frac{x^r}{(x+1)(x+2)\ldots(x+r)} \right)$$

$$= 1 - \frac{x^n}{(x+1)(x+2)\ldots(x+n)}$$

$$1 - f(x) = \prod_{r=1}^{n} \frac{x}{x+r}$$

$$\Rightarrow \quad \frac{-f'(x)}{1-f(x)} = \frac{1}{x} \sum_{r=1}^{n} \frac{r}{x+r} \quad \Rightarrow \quad f'(x) = -\frac{x^{n-1}}{(x+1)(x+2)\ldots(x+n)} \sum_{r=1}^{n} \frac{r}{x+r}$$

37. If n is even

$$S = 1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 + \ldots + 2\cdot n^2$$

$$= \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n/2} (2r)^2 = \frac{n(n+1)(2n+1)}{6} + 4\,\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)(n+1)}{6}$$

$$= \frac{n(n+1)^2}{2}$$

If n is odd

$$S = \frac{(n-1)n^2}{2} + n^2 = \frac{n^2(n+1)}{2}$$

38. $$a_k a_{n-(k-1)} = (a_1 + (k-1)d)\,(a_n - (k-1)d)$$

$$= a_1 a_n + d(k-1)\,(a_n - a_1) - (k-1)^2 d^2$$

$$= a_1 a_n + d^2(k-1)\,(n-k)$$

$$\le \begin{cases} a_1 a_n + d^2\left(\frac{n+1}{2} - 1\right)\left(n - \frac{n+1}{2}\right) = a_1 a_n + \frac{(n-1)^2}{4} d^2 & \text{If } n \text{ is odd} \\ a_1 a_n + \frac{d^2}{4} n(n-2) & \text{If } n \text{ is even} \end{cases}$$

39. Let a, ar, ar^2 be the sides r > 0, a > 0

$a(1 + r) > ar^2 \Rightarrow r^2 - r - 1 < 0 \Rightarrow r \in \left(0, \frac{1+\sqrt{5}}{2}\right)$

$a(1 + r^2) > ar \Rightarrow r > 0$

$a(r^2 + r) > a \Rightarrow r^2 + r - 1 > 0 \Rightarrow r \in \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$

$\Rightarrow r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$

40. $b^2 = \frac{2a^2c^2}{a^2+c^2} \Rightarrow ((a + c)^2 - 2ac)\, b^2 = 2a^2c^2$

$\Rightarrow 2b^4 - ac\, b^2 - a^2c^2 = 0 = (2b^2 + ac)(b^2 - ac)$

$\Rightarrow b^2 = \left(-\frac{a}{2}\right)c$ or $b^2 = ac$

SECTION-3

COMPRHENSION BASED QUESTIONS

Comprehension (Q.1 To Q.3)

Put $\left(t + 2(n - 1), \frac{n}{2}(2t + (n - 1)2)\right)$ to the curve

$\Rightarrow n(t + (n - 1)) = \alpha(t + 2(n - 1))^2 + \beta(t + 2(n - 1)) + \gamma$

$= (n - 1)(t + (n - 1)) + t + (n - 1)$

$\Rightarrow (n - 1)^2 (4\alpha - 1) + (n - 1)(2\beta + 4\alpha t - t - 1) + (\alpha t^2 + \beta t + \gamma - t) = 0 \ \forall\, n \in N$

$\Rightarrow 4\alpha - 1 = 0,\ 2\beta + (4\alpha - 1)t - 1 = 0\ \&\ \alpha t^2 + (\beta - 1)t + \gamma = 0 \begin{cases} a_1 \\ b_1 \end{cases}$

$\Rightarrow \alpha = \frac{1}{4}, \beta = \frac{1}{2}$

If $\gamma = 0, \frac{1}{4}t^2 - \frac{1}{2}t = 0$

$t = 0, 2$

$\Rightarrow a_1 = 0, b_1 = 2$

Comprehension (Q.4 to Q.6)

$$2(S_n - S_{n-1}) = (S_n - S_{n-1})S_n - S_n^2 = -S_{n-1}S_n$$

$$\frac{1}{S_n} - \frac{1}{S_{n-1}} = \frac{1}{2}$$

$\Rightarrow \left\{\frac{1}{S_n}\right\}$ is an A.P. with comon difference $\frac{1}{2}$

$$\frac{1}{S_1} = \frac{1}{b_1} = 1$$

$$\Rightarrow \quad \sum_{r=1}^{n}\frac{1}{S_r} = \frac{n}{2}\left(2(1)+(n-1)\frac{1}{2}\right) = (5n-1)\frac{n}{4}$$

$$\frac{1}{S_n} = 1 + (n-1)\frac{1}{2} = \frac{n+1}{2} \Rightarrow S_n = \frac{2}{n+1}$$

$$b_n = S_n - S_{n-1} = \frac{2}{n+1} - \frac{2}{n} = \frac{-2}{n(n+1)}$$

also a_{81} lies in 12th row, 3rd column

$$\Rightarrow \quad a_{81} = b_{13}r^2 = -\frac{1}{91}r^2 = -\frac{4}{91}$$

$$\Rightarrow \quad r = 2$$

Sum of numbers in m^{th} row $= b_m + b_{m+1} + b_{m+2} + ... + b_{2m-1}$

$$\frac{b_m(2^m-1)}{2-1} = \frac{2(1-2^m)}{m(m+1)}$$

Comprehension (Q.7 to Q.9)

$$\left(5 - a + \frac{3}{b}\right)(5 + a + 3b) = (5 + 3)^2 = 64$$

$$\Rightarrow \quad 5 - a + \frac{3}{b} = 1 \text{ and } 5 + a + 3b = 64$$

$$\Rightarrow \quad b = \frac{55 \pm 7\sqrt{61}}{6}, a = \frac{63 \mp 7\sqrt{61}}{2}$$

$$\text{or} \quad 5 - a + \frac{3}{b} = 2 \text{ and } 5 + a + 3b = 32$$

$$\Rightarrow \quad b = 4 \pm \sqrt{15}, a = 15 \mp 3\sqrt{15}$$

or $5 - a + \frac{3}{b} = 4$ & $5 + a + 3b = 16$

$\Rightarrow$ $b = 3, \frac{1}{3}, a = 2, 10$

or $5 - a + \frac{3}{b} = 8$ & $5 + a + 3b = 8$

$\Rightarrow$ $b = 1, a = 0$

Hence $b_{max} = \frac{55 + 7\sqrt{61}}{6}, a_{max} = \frac{63 + 7\sqrt{61}}{2}$

Comprehension (Q.10 to Q.12)

$2S_n = S_n - S_{n-1} + \frac{1}{S_n - S_{n-1}}$

$\Rightarrow$ $2S_n(S_n - S_{n-1}) = (S_n - S_{n-1})^2 + 1 = S_n^2 + S_{n-1}^2 - 2S_nS_{n-1} + 1$

$\Rightarrow$ $S_n^2 = S_{n-1}^2 + 1 = (S_{n-2}^2 + 2) = \ldots\ldots\ldots$

$S_n^2 = S_1^2 + n - 1 = n \Rightarrow S_n = \sqrt{n}$

$\Rightarrow$ $a_n^2 + 1 - 2\sqrt{n}\, a_n = 0$

$\Rightarrow$ $a_n = \sqrt{n} \pm \sqrt{n-1}$

$\Rightarrow$ $a_n = \sqrt{n} - \sqrt{n-1}$ $[\because S_n = \sqrt{n}]$

$$\lim_{n\to\infty} \sqrt{n}\left(\sqrt{n} - \sqrt{n-1}\right) = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n} + \sqrt{n-1}} = \frac{1}{2}$$

$$\frac{1}{\sqrt{k} + \sqrt{k+1}} < \frac{1}{2\sqrt{k}} < \frac{1}{\sqrt{k} + \sqrt{k-1}}$$

$$\sqrt{k+1} - \sqrt{k} < \frac{1}{2\sqrt{k}} < \sqrt{k} - \sqrt{k-1}$$

$$\sum_{k=1}^{100} \sqrt{k+1} - \sqrt{k} < \sum_{k-1}^{100} \frac{1}{2\sqrt{k}} < \frac{1}{2} + \sum_{k=2}^{100} (\sqrt{k} - \sqrt{k-1})$$

$$9 < \sqrt{101} - 1 < \frac{1}{2} \sum_{k=1}^{100} \frac{1}{\sqrt{k}} < \frac{1}{2} + (10 - 1) = \frac{19}{2}$$

$$18 < \sum_{k=1}^{100} \frac{1}{\sqrt{k}} < 19$$

Comprehension (Q.13 to Q.15)

$$\sum_{r=1}^{n}\left((r+1)^2-r^2\right)\phi(r)=\sum_{r=1}^{n}(r+1)^2\left(\phi(r)-\phi(r+1)\right)+\sum_{r=1}^{n}\left((r+1)^2\phi(r+1)-r^2\phi(r)\right)$$

$$=-\sum_{r=1}^{n}(r+1)+(n+1)^2\phi(n+1)-1^2\phi(1)$$

$$=-(1+2+3+\ldots+(n+1))+(n+1)^2\phi(n+1)$$

$$=-\frac{(n+1)(n+2)}{2}+(n+1)^2\phi(n+1)$$

$$\sum_{r=0}^{9}P(r)=1^2+2^2+\ldots 10^2=\frac{10\times11\times21}{6}=385$$

$$\sum_{r=0}^{\infty}\frac{1}{Q(r)}=\sum_{r=0}^{\infty}\frac{2}{(r+1)(r+2)}=2\sum_{r=0}^{\infty}\left(\frac{1}{r+1}-\frac{1}{r+2}\right)=2$$

$$P(n)-Q(n)=(n+1)^2-\frac{(n+1)(n+2)}{2}=\frac{n(n+1)}{2}$$

Comprehension (Q.16 to Q.17)

Let number of terms = 2n

$$k=\frac{\frac{n}{2}(2a+(n-1)d)}{\frac{n}{2}(2(a+nd)+(n-1)d)}=\frac{2+(n-1)d}{2+(3n-1)d}$$

$$=1-\frac{2d}{\left(\frac{2-d}{n}\right)+3d}$$

$\Rightarrow$ For k to be constant, d = 0 or d = 2,

$$k=1,\ \frac{1}{3}$$

Comprehension (Q.18 to Q.19)

18. $$\sum_{n=2}^{\infty}\frac{a_n}{a_{n-1}a_{n+1}}=\lim_{n\to\infty}\sum_{n=2}^{n}\frac{a_{n+1}-a_{n-1}}{a_{n-1}a_{n+1}}=\lim_{n\to\infty}\sum_{n=2}^{n}\left(\frac{1}{a_{n-1}}-\frac{1}{a_{n+1}}\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{a_1}+\frac{1}{a_2}-\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)=2$$

19. $$\sum_{n=2}^{\infty}\frac{1}{a_{n-1}a_{n+1}}=\lim_{n\to\infty}\sum_{n=2}^{n}\frac{a_n}{a_{n-1}a_na_{n+1}}=\lim_{n\to\infty}\sum_{n=2}^{n}\frac{a_{n+1}-a_{n-1}}{a_{n-1}a_na_{n+1}}$$

$$=\lim_{n\to\infty}\sum_{n=2}^{n}\left(\frac{1}{a_{n-1}a_n}-\frac{1}{a_na_{n+1}}\right)=\lim_{n\to\infty}\left(\frac{1}{a_1a_2}-\frac{1}{a_na_{n+1}}\right)=1$$

Comprehension (Q.20 to Q.22)

I[st] A.P. $a_1, a_2, \ldots\ldots\ldots, a_n$

2[nd] A.P. $b_1, b_2, \ldots\ldots, b_n$

$$\frac{a_n}{b_1}=\frac{b_n}{a_1}=4 \Rightarrow a_n=4b_1,\ b_n=4a_1$$

$$\frac{a_1+a_n}{b_1+b_n}=2 \Rightarrow \frac{a_1+a_n}{4a_1+\frac{a_n}{4}}=2 \Rightarrow a_n=14a_1$$

$$b_1=\frac{a_n}{4}=\frac{7}{2}a_1$$

Comprehension (Q.29 to Q.30)

$$xy^{\log_{10}y}=x^{2\log_{10}x} \qquad \ldots(1)$$

$$x^{\log_{10}x}(xy)^{\log_{10}(xy)}=y^{2\log_{10}y}$$

$$\Rightarrow \quad x^{2\log_{10}x+\log_{10}y}=y^{\log_{10}y-\log_{10}x} \qquad \ldots(2)$$

$$\log_{10}^2 y+\log_{10}x=2\log_{10}^2 x \qquad \ldots(3)$$

$$\Rightarrow \quad 2\log_{10}^2 x+2\log_{10}x\log_{10}y-\log_{10}^2 y=0 \qquad \ldots(4)$$

(3) + (4)

$\Rightarrow$ $\log_{10}x+2\log_{10}x\log_{10}y=0$

$\Rightarrow$ $x=1,\ y=1$

or $\log_{10}y=-\frac{1}{2},\ y=\frac{1}{\sqrt{10}}$

$\Rightarrow$ $2\log_{10}^2 x-\log_{10}x-\frac{1}{4}=0$

$\log_{10}(x_1x_2)=\frac{1}{2} \Rightarrow x_1x_2=\sqrt{10}$

$$\Rightarrow \frac{x_1x_2}{y_1y_2}=10\sqrt{10}=\sqrt{1000}$$

SECTION-4

MATCH THE COLUMN

1. (A) $\alpha+\gamma=2\beta \Rightarrow \alpha+\beta+\gamma=9=3\beta \Rightarrow \beta\gamma\alpha=3$

Put $x=3 \Rightarrow k=3^3-9(3)^2+26(3)=24$

(B) $\alpha\gamma=\beta^2 \Rightarrow \alpha\beta\gamma=64=\beta^3 \Rightarrow \beta=4$

Put $x=4 \Rightarrow 64-14(16)+4k-64=0 \Rightarrow k=56$

(C) $\frac{2}{\beta}=\frac{1}{\alpha}+\frac{1}{\gamma} \Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{1}{1/6}=\frac{3}{\beta} \Rightarrow \beta=\frac{1}{2}$

Put $x=\frac{1}{2} \Rightarrow \frac{6}{8}-\frac{k}{4}+\frac{6}{2}-1=0 \Rightarrow k=11$

(D) $\alpha+\gamma=3 \Rightarrow \alpha+\beta+\gamma=0=3+\beta \Rightarrow \beta=-3$

Put $x=-3 \Rightarrow -27+3k+6=0 \Rightarrow k=7$

2.(A) $A=\frac{n(n+1)(2n+1)}{6}$

$$B=\sum_{m=1}^{n}\frac{m(m+1)}{2}-\frac{1}{2}\frac{n(n+1)}{2}$$

$$\frac{1}{6}\sum_{m=1}^{n}(m+2)m(m+1)-m(m+1)(m-1)-\frac{n(n+1)}{4}$$

$$=\frac{n(n+1)(n+2)}{6}-\frac{n(n+1)}{4}=\frac{n(n+1)(2n+1)}{12}$$

(B) $$\frac{\Sigma a(b^2+c^2)}{abc}\geq\frac{a(2bc)+b(2ca)+c(2ab)}{abc}=6$$

(C) $$2\sum\frac{x^2-1+1}{1-x}=2\Sigma-(x+1)+\frac{1}{1-x}=-8+2\Sigma\frac{1}{1-x}\geq-8+2\left(\frac{9}{2}\right)=1$$

$$\left[\because \frac{3}{\Sigma\frac{1}{1-x}}\leq\frac{\Sigma(1-x)}{3}=\frac{2}{3}\Rightarrow\Sigma\frac{1}{1-x}\geq\frac{9}{2}\right]$$

(D) $\because$ x, y, z $\in$ N $\Rightarrow$ $\frac{1}{xy} \le 1$

$\Rightarrow$ $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{2x} \le 3$

$\therefore$ x = y = z = 1

3. (A) $xy^2z^2 = 16 \Rightarrow \frac{x + \frac{y}{2} + \frac{y}{2} + \frac{z}{2} + \frac{z}{2}}{5} \ge \left(\frac{xy^2z^2}{16}\right)^{\frac{1}{5}} = 1$

$\Rightarrow x + y + z \ge 5$

(B) $a + b = 3, \frac{3}{\frac{2}{a} + \frac{2}{a} + \frac{1}{b}} \le \frac{\frac{a}{2} + \frac{a}{2} + b}{3} = 1 \Rightarrow \frac{4}{a} + \frac{1}{b} \ge 3$

(C) $a + c = 2b \quad a + b > c \Rightarrow \frac{b}{c} > \frac{2}{3}$ and $b + c > a \Rightarrow \frac{b}{c} < 2 \Rightarrow \frac{2b}{c} \in \left(\frac{4}{3}, 4\right)$

(D) $3^{|\sin x|} \in [1, 3], 2^{-|\sec y|} \in \left(0, \frac{1}{2}\right], 5\cos z \in [-5, 5]$

$a = 3^{|\sin x|}\, 2^{-|\sec y|} + 5\cos z$

$\Rightarrow a \in \left(-5, 5 + \frac{3}{2}\right], a \in \left(-5, \frac{13}{2}\right]$

4. (A) $$\frac{\left(\frac{49a}{3}\right)3 + \left(\frac{3b}{2}\right)2 + c}{6} \ge \left\{\left(\frac{49a}{3}\right)^3 \left(\frac{3b}{2}\right)^2 c\right\}^{1/6} = \left(\frac{7^6 a^3 b^2 c}{12}\right)^{1/6}$$

$\Rightarrow \quad 49a + 3b + c \ge 6 \times 7 = 42$

(B) $x = -t, t > 0$

$2x^3 - \frac{3}{x^2} = -\left(2t^3 + \frac{3}{t^2}\right) \le -5$

$$\because \frac{t^3 + t^3 + \frac{1}{t^2} + \frac{1}{t^2} + \frac{1}{t^2}}{5} \ge \left\{(t^3)^2 \frac{1}{(t^2)^3}\right\}^{1/5} = 1$$

$\Rightarrow \quad 2t^3 + \frac{3}{t^2} \ge 5$

(C) $$\left(\frac{\frac{x^3}{5}+\frac{x^3}{5}+\frac{x^3}{5}+\frac{x^3}{5}+\frac{x^3}{5}+\frac{8-x^3}{3}+\frac{8-x^3}{3}+\frac{8-x^3}{3}}{8}\right)^8 \geq \left(\frac{8-x^3}{3}\right)\left(\frac{x^3}{5}\right)$$

$\Rightarrow \quad (8 - x^3)\, x^5 \leq (3^3\, 5^5)^{1/3} = 3 \times 5^{5/3}$

(D) $$\frac{(x+x+x+x+x+x+x)+(y+y+y+y+y)}{12} \geq (x^7y^5)^{1/12}$$

$\Rightarrow \quad (7x + 5y) \geq 12a^{1/12}$

$\therefore \quad 12a^{1/12} \geq 12$

$\Rightarrow \quad a \geq 1$

5. $b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}$

$= 1, r, r^2, (2r^2 - r), (2r - 1)^2, (2r - 1)(3r - 2), (3r - 2)^2, (3r - 2)$

$(4r - 3), (4r - 3)^2, (4r - 3)(5r - 4)$

$b_5 + b_6 = 198$

$\Rightarrow \quad (2r - 1)(5r - 3) = 198$

$\Rightarrow \quad r = 5$

$r = -\frac{39}{10}$ (rejected)

$b_7 = (3 \times 5 - 2)^2 = 169$

$b_8 = (3 \times 5 - 2)(4 \times 5 - 3) = 221$

$b_9 = (4 \times 5 - 3)^2 = 289$

$b_{10} = (4 \times 5 - 3)(5 \times 5 - 4) = 357$

SECTION-5

Subjective Type Questions

1. $S_n = \sum_{r=1}^{n} a_r = \sum_{k=1}^{n}\sum_{j=1}^{k} 2j = \sum_{k=1}^{n} k(k+1) = \frac{1}{3}\sum_{k=1}^{n}\left((k(k+1)(k+2)-(k-1)k(k+1)\right)$

$$S_n = \frac{1}{3}n(n+1)(n+2)$$

$$a_n = S_n - S_{n-1} = \frac{1}{3}(n+1)(n)\,((n+2)-(n-1)) = n(n+1)$$

$$\sum_{r=1}^{n}\frac{1}{a_r} = \sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right) = 1-\frac{1}{n+1} = \frac{n}{n+1}$$

$$\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = e^{\lim_{n\to\infty} -\frac{n}{n+1}} = e^{-1}$$

$$\frac{1}{\lambda} = e \quad \Rightarrow \quad \left[\frac{1}{\lambda}\right] = 2$$

2. $a-b = \sum_{r=1}^{1001} r^2\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right) = \sum_{r=-1}^{1001}\frac{2r^2}{4r^2-1}$

$$= \sum_{r=1}^{1001}\frac{1}{2}+\frac{1}{4}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$$

$$= \frac{1001}{2}+\frac{1}{4}\left(1-\frac{1}{2003}\right) \in (500, 501)$$

3. Consider sequence of terms

$1, 2, 1, 2, 2, 1, \ldots\ldots\ldots\ldots, 1, \underbrace{2, 2, \ldots\ldots, 2}_{n \text{ times}}$

number of terms $= \underbrace{n}_{\text{number of 1's}} + \underbrace{(1+2+3+\ldots..+n)}_{\text{number of 2's}} = n+\frac{n(n+1)}{2} = \frac{n(n+3)}{2}$

$$\frac{n(n+3)}{2} \le 2018 \quad \Rightarrow \quad n \le 62$$

First 2018 terms of sequence will be

$1, 2, 2, 1, 2, 2, 2, 1, 2, ..., 1, \underbrace{2, 2, ...2}_{62}, 1, 2, 2$

Sum = $1 \times 62 + (1 + 2 + 3 + ... 62)2 + 1 + 2 + 2$

$= 3973$

4. $\left(\frac{a+b+c+d}{4}\right)^2 \le \frac{a^2+b^2+c^2+d^2}{4} \Rightarrow (8 - e)^2 \le 4(16 - e^2)$

$\Rightarrow \quad 5e^2 - 16e \le 0$

$\Rightarrow \quad e \in \left[0, \frac{16}{5}\right]$

5. $a + b = \frac{1}{6}\sum_{i=1}^{6}(a_i + b_i) = 1$

$\sum_{i=1}^{6}(a_i - a)^2 + \sum_{i=1}^{6}a_ib_i = 6a^2 - 2a\,\Sigma a_i + \Sigma a_i^2 + \Sigma a_ib_i$

$= 6a^2 - 2a(6a) + \sum_{i=1}^{6}a_i(a_i + b_i)$

$= -6a^2 + 6a = 6a(1 - a) = 6ab$

6. $a_1 = S_1 = 1 + 3 + 4 = 8$

$a_n = S_n - S_{n-1} = n^2 + 3n - (n - 1)^2 - 3(n - 1) = 2(n + 1), n \ge 2$

$a_1 + a_3 + ... + a_{21} = 8 + (8 + 12 + 16 + ... + 44) = 8 + \frac{10}{2}(44 + 8) = 268$

7. $\frac{a_1^2 + a_2^2 + a_3^2}{b_1 + b_2 + b_3} = \frac{14d^2}{d^2(1 + r + r^2)} = \frac{14}{1 + r + r^2} = m$

$mr^2 + mr + (m - 14) = 0$

$D = m^2 - 4m(m - 14) = 56m - 3m^2 = k^2 \qquad k \in I$

Also $0 < r < 1 \qquad 1 + r + r^2 = \frac{14}{m} \in (1, 3) \Rightarrow m \in [5, 13]$

Check $m = \{5, 6, ..., 13\}$ for which $56m - 3m^2$ is perfect square

$\Rightarrow \quad m = 8$

8. $S = \sum_{k=1}^{99} k((k+1)^2 - k^2) + 100 = \frac{100 \times 99 \times 401}{6} + 100$

$= 661750$

9. $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{3^i 3^j 3^k} = \left(\frac{1}{1-\frac{1}{3}}\right)^3 - 3\left(\sum_{i=0}^{\infty} \frac{1}{3^{2i}} \frac{1}{\left(1-\frac{1}{3}\right)} - \sum_{i=0}^{\infty} \frac{1}{3^{3i}}\right) - \sum_{i=0}^{\infty} \frac{1}{3^{3i}}$

$(i \neq j \neq k)$

$= \frac{27}{8} - \frac{3}{2} \times 3\left(\frac{1}{1-\frac{1}{9}}\right) + 2\frac{1}{1-\frac{1}{27}}$

$= \frac{27}{8} - \frac{81}{16} + \frac{27}{13} = \frac{81}{208}$

10. $S = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n3^m + m3^n)}$ ....(1)

Inter change $m \to n$

$S = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{n^2 m}{3^n (m3^n + n3^m)}$

$S = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{n^2 m}{3^n (n3^m + m3^n)}$ ....(2)

$(1) + (2) \Rightarrow$

$2S = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{mn(m3^n + n3^m)}{3^m 3^n (n3^m + m3^n)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{mn}{3^m 3^n}$

$= \left(\sum_{m=1}^{\infty} \frac{m}{3^m}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

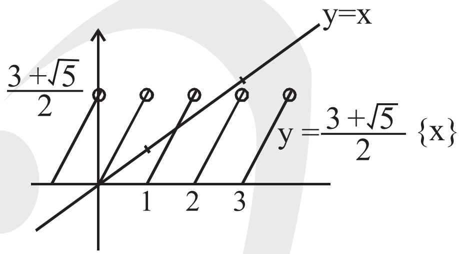
$\Rightarrow \quad S = \frac{9}{32}$

11. $[x]^2 = x\{x\} = x^2 + \{x\}^2 - 2x\{x\}$

$$\frac{x}{\{x\}} = \frac{3 \pm \sqrt{5}}{2}$$

Solution lies in interval (1, 2)

$\Rightarrow \quad x = \frac{(3+\sqrt{5})}{2}(x-1)$



$$\left(\frac{\sqrt{5}+1}{2}\right)x = \frac{3+\sqrt{5}}{2} = \frac{(\sqrt{5}+1)^2}{4}$$

$$x = \frac{\sqrt{5}+1}{2}$$

$\frac{1}{2^n}(1+\sqrt{5})^n > 100 \Rightarrow n \geq 10$

12. $\frac{n(n+1)}{2} - (m+k) = 17(n-2) \quad 3 \leq \frac{n^2 - 33n + 68}{2} = m + k \leq 2n - 1$

$\Rightarrow n \in [31, 35] \quad n = \{31, 32, 33, 34, 35\}$

$\therefore$ Maximum value of m + k = 35 + 34 = 69

13. The number are 16, 24, 36, 54, 81

14. $x^4 - 16x^3 + px^2 - 256x + q = 0$ (roots x_1, x_2, x_3, x_4) $\quad x_1, x_2, x_3, x_4$ are in G.P. $\Rightarrow x_1x_4 = x_2x_3$

$$(x_1 + x_4) + (x_2 + x_3) = 16$$

$x_1x_4(x_2 + x_3) + x_2x_3(x_1 + x_4) = 256$

$\Rightarrow \quad x_1x_4(16) = 256 \quad \Rightarrow \quad x_1x_4 = x_2x_3 = 16$

$$P = x_1x_4 + x_2x_3 + (x_1 + x_4)(x_2 + x_3)$$

$$x_1x_2x_3x_4 = (16)^2 \quad \Rightarrow \quad (x_1 x_2 x_3 x_4)^{1/4} = 2^2 = 4$$

$$\frac{x_1 + x_2 + x_3 + x_4}{4} = 4$$

$\Rightarrow \quad x_1 = x_2 = x_3 = x_4 = 4$

$\Rightarrow \quad P = 6 \times 16 = 96, q = 256$

15. $\sum_{r=1}^{2018} \frac{x_r^2}{1-x_r} = \sum_{r=1}^{2018} \left(-(x_r+1)+\frac{1}{1-x_r}\right)$

$$= \sum_{r=1}^{2018}\left[-(x_r+1)+\frac{1-x_r+x_r}{1-x_r}\right] = \sum_{r=1}^{2018}\left[-(x_r+1)+1+\frac{x_r}{1-x_r}\right]$$

$= -1 + 1 = 0$

16. $T_1 = 2014,\ T_2 = 2^3 + 1^3 + 4^3 = 73,\ T_3 = 7^3 + 3^3 = 370$

$T_4 = 370, \ldots\ldots, T_{2018} = 370$

17. $y - x = d(a_1 + a_2 + \ldots. + a_{20018}) = 1009d\ (2a_1 + 2017d)$

$z = (a_1 + 1008d) + (a_1 + 1009d) = 2a_1 + 2017d$

$\Rightarrow \frac{y-x}{z} = 1009d$

18. $\sum_{i=1}^{2018} \frac{x_i^2}{1-x_i} = \sum_{i=1}^{2018} -(x_i+1)+\frac{1}{1-x_i} = -2019 + \sum_{i=1}^{2018}\frac{1}{1-x_i}$

$$\frac{(1-x_1)+(1-x_2)+\ldots(1-x_{2018})}{2018} \ge \frac{2018}{\frac{1}{1-x_1}+\frac{1}{1-x_2}+\ldots+\frac{1}{1-x_{2018}}}$$

$$\Rightarrow \quad \sum_{i=1}^{2018}\frac{1}{1-x_i} \ge \frac{(2018)^2}{2017}$$

$$\therefore \quad \sum_{i=1}^{2018}\frac{x_i^2}{1-x_i} \ge \frac{(2018)^2}{2017} - 2019 = \frac{(2018)^2-((2018)^2-1)}{2017} = \frac{1}{2017}$$

$\Rightarrow$ The least value of k satisfying $k\sum_{i=1}^{2018}\frac{x_i^2}{1-x_i} \ge 1$.

$= 2017$

19. $[x] = a$ if $x \in \left[\sqrt{a^2}, \sqrt{(a+1)^2}\right)$

Number of values of $x = (a + 1)^2 - a^2 = 2a + 1$ for which $[x] = a$

$$S = \sum_{a=1}^{30}\frac{2a+1}{2a+1} + \frac{1}{2(31)+1} \times 40 = 30 + \frac{40}{63}$$

$[S] = 30$

20. Adding, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{3}{2018}$

$$2xz-2yz+1=\frac{z}{2018}$$

$$2yz-2zx+1=\frac{x}{2018}$$

$$2yz-2xy+1=\frac{y}{2018}$$

Adding, $x+y+z=2018\times 3$

$\because \quad \dfrac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\le\dfrac{x+y+z}{3} \quad \Rightarrow \quad x=y=z=2018$

21. $\Sigma a(1+c)(1+a)=(1+a)(1+b)(1+c)$

$\Rightarrow \quad a^2c+b^2a+c^2b+a^2+b^2+c^2=1+abc$

$\Rightarrow \quad 1+abc\ge 3abc+3(abc)^{2/3}$

Put $\quad (abc)^{1/3}=t \Rightarrow 2t^3+3t^2-1\le 0 \Rightarrow \quad (2t-1)(t^2+2t+1)\le 0$

$\Rightarrow \quad t\le\frac{1}{2} \quad \Rightarrow \quad abc\le\frac{1}{8}$

22. $-\dfrac{a^3}{(a-b)(c-a)}-\dfrac{b^3}{(b-c)(a-b)}-\dfrac{c^3}{(c-a)(b-c)}$

$$=-\frac{a^3(b-c)+b^3(c-a)+c^3(a-b)}{(a-b)(b-c)(c-a)}$$

$$=-\frac{(b-c)(c-a)(a-b)(-a-b-c)}{(a-b)(b-c)(c-a)}$$

$=a+b+c\ge 4$

23. $3<\dfrac{3}{\frac{1}{k+a}+\frac{1}{k+b}+\frac{1}{k+c}}\le\dfrac{(k+a)+(k+b)+(k+c)}{3}=k+\dfrac{a+b+c}{3}$

$\Rightarrow \quad 3-k<\frac{a+b+c}{3} \quad \forall\ a, b, c>0$

$\Rightarrow \quad 3-k<\text{minimum of } \frac{a+b+c}{3}$

Now, $\frac{a+b+c}{3} \geq (abc)^{1/3} = 1$

$\Rightarrow \quad 3 - k < 1 \quad \Rightarrow \quad k > 2$

24. $\frac{1}{1+x_2} + \frac{1}{1+x_3} + \ldots + \frac{1}{1+x_{2018}} = \frac{x_1}{1+x_1}$

$$\Rightarrow \quad \frac{x_1}{1+x_1} \geq 2017\left(\frac{1}{(1+x_2)(1+x_3)\ldots.(1+x_{2018})}\right)^{\frac{1}{2017}}$$

Similarly,
$$\frac{x_2}{1+x_2} \geq 2017\left(\frac{1}{(1+x_1)(1+x_3)\ldots.(1+x_{2018})}\right)^{\frac{1}{2017}}$$

$$\vdots$$

$$\frac{x_{2018}}{1+x_{2018}} \geq 2017\left(\frac{1}{(1+x_1)(1+x_2)\ldots.(1+x_{2017})}\right)^{\frac{1}{2017}}$$

Multiplying

$$\frac{x_1 x_2 \ldots. x_{2018}}{(1+x_1)(1+x_2)\ldots..(1+x_{2018})} \geq \frac{(2017)^{2018}}{(1+x_1)(1+x_2)\ldots.(1+x_{2018})}$$

$$\Rightarrow \quad x_1 x_2 \ldots.. x_{2018} \geq (2017)^{2018}$$

25. $\frac{1}{(n+1)\sqrt{n}} = \frac{\sqrt{n}}{(n+1)n} = \sqrt{n}\left(\frac{1}{n} - \frac{1}{n+1}\right) = \frac{1}{\sqrt{n}} - \frac{\sqrt{n}}{n+1}$

$$\Rightarrow \quad \frac{1}{(n+1)\sqrt{n}} > \frac{1}{\sqrt{n}} - \frac{\sqrt{n+1}}{n+1} = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$$

$$\Rightarrow \quad \sum_{n=1}^{\infty} \frac{1}{(n+1)\sqrt{n}} > \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = 1$$

Also,
$$\sum_{n=1}^{\infty} \frac{1}{(n+1)\sqrt{n}} = \sum_{n=1}^{\infty} \frac{2}{\sqrt{n+1}\sqrt{n}(2\sqrt{n+1})} < 2\sum \frac{1}{\sqrt{n}\sqrt{n+1}\left(\sqrt{n+1}+\sqrt{n}\right)}$$

$$= 2\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right) = 2$$

26. $\lim_{n\to\infty} 2\left(1-\frac{1}{2}\right)\left(1+\frac{1}{2^1}\right)\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)....\left(1+\frac{1}{2^{2^n}}\right)$

$$= \lim_{n\to\infty} 2\left(1-\frac{1}{2^{2^{n+1}}}\right) = 2$$

27. $\lim_{n\to\infty} \sum_{k=1}^{n}\left(\frac{2^k}{3^k-2^k}-\frac{2^{k+1}}{3^{k+1}-2^{k+1}}\right)$

$$\lim_{n\to\infty}\left(\frac{2}{3-2}-\frac{2^{n+1}}{3^{n+1}-2^{n+1}}\right) = \lim_{n\to\infty}\left(2-\frac{\left(\frac{2}{3}\right)^{n+1}}{1-\left(\frac{2}{3}\right)^{n+1}}\right) = 2$$

28. $$\frac{1}{x_{k+1}} = \frac{1}{x_k(x_k+1)} = \frac{1}{x_k}-\frac{1}{x_k+1}$$

$\Rightarrow$ $$\sum_{k=1}^{100}\frac{1}{x_k+1} = \sum_{k=1}^{100}\left(\frac{1}{x_k}-\frac{1}{x_{k+1}}\right) = \frac{1}{x_1}-\frac{1}{x_{101}} = 2-\frac{1}{x_{101}}$$

$$x_2 = \frac{3}{4},\ x_3 = \frac{3}{4}+\frac{9}{16} > 1$$

$\Rightarrow$ $$x_{101} > 1$$

$\therefore$ $$0 < \frac{1}{x_{101}} < 1$$

$\Rightarrow$ $$1 < 2-\frac{1}{x_{101}} < 2$$

29. $\sum_{i=1}^{n}\frac{a_i}{p-a_i} = p\sum_{i=1}^{n}\frac{1}{p-a_i} - n$

$$\sum_{i=1}^{n}\frac{(p-a_i)}{n} \geq \frac{n}{\sum_{i=1}^{n}\frac{1}{(p-a_i)}}$$

$\Rightarrow$ $$p\sum_{i=1}^{n}\frac{1}{p-a_i} \geq \frac{n^2}{n-1}$$

$\Rightarrow$ $$\sum_{i=1}^{n}\frac{a_i}{p-a_i} \geq \frac{n^2}{n-1} - n = \frac{n}{n-1} > 1$$

If $0 < a < b \Rightarrow \frac{a}{b} < \frac{a+c}{b+c}$ $\quad c > 0$

$$\therefore \qquad \frac{a_i}{p-a_i} < \frac{a_i + a_i}{p - a_i + a_i} \Rightarrow \frac{a_i}{p-a_i} < \frac{2a_i}{p}$$

$$\Rightarrow \qquad \sum_{i=1}^{n} \frac{a_i}{p-a_i} < 2\sum \frac{a_i}{p} = 2$$

30. $S_i = \frac{20}{2}(2 + 19di)$

$$\frac{2S_3S_1 - (S_1+S_3)S_2}{S_1+S_3-2S_2} = \frac{200(2+19d_1)(2+19d_3) - 100(2+19d_2)(4+19(d_1+d_3))}{10(4+19(d_1+d_3)-4-38d_2)}$$

$$= 10\left(\frac{8+76(d_1+d_3)+722d_1d_3-8-38(d_1+d_3+2d_2)-361d_2(d_1+d_3)}{19(d_1+d_3-2d_2)}\right)$$

$$= 10\left(\frac{38(d_1+d_3-2d_2)+361(2d_1d_3-d_2(d_1+d_3))}{19(d_1+d_3-2d_2)}\right)$$

$= 20$

31. Coefficient of $x^2 = 1(-3) + 1(5) + 1(-7) + ... + (-23)25$

$$= \frac{1}{2}\left[(1-3+5-7+...-23+25)^2 - (1^2+3^2+5^2+7^2+...25^2)\right]$$

$$= \frac{1}{2}\left[13^2 - \sum_{k=1}^{13}(2k-1)^2\right] = \frac{1}{2}\left[169 - 4\frac{13\times14\times27}{6} + 4\times\frac{13\times14}{2} - 13\right]$$

$= -1378$

32. Case-I

$a + b - c < a < c + a - b < b < c < b + c - a < a + b + c$

$\Rightarrow \qquad a + b - c + a + b + c = 2b$

$\Rightarrow \qquad a = 0$ which is not possible

Case-II

$a + b - c < a < b < c + a - b < c < b + c - a < a + b + c$

$(a + b - c) + (a + b + c) = b + c = 2(c + a - b)$

$\Rightarrow \quad b + 2a = c$

and $\quad 4b = 2c \Rightarrow c = 2b$

$\Rightarrow \quad b = 2a \Rightarrow c = 4a$

Sequence is

–a, a, 2a, 3a, 4a, 5a, 7a which is not an A.P.

33. $a_1(1 + r + r^2 + r^3) = 20$

$a_1r^4(1 + r + r^2 + r^3) = 320$

$\Rightarrow \quad r^4 = 16 \Rightarrow r = 2$

$N = a_1r^{12}(1 + r + r^2 + r^3) = 2^{12} \times 20 = 2^{14} \times 5^1$

34. $S = \sum_{r=1}^{\infty} (2r-1)^2 \frac{1}{2^{r-1}} = 2\sum_{r=2}^{\infty} r(r-1)\frac{1}{2^{r-2}} + \sum_{r=1}^{\infty} \frac{1}{2^{r-1}}$

$\sum_{r=0}^{\infty} x^r = \frac{1}{1-x} \Rightarrow \sum_{r=1}^{\infty} rx^{r-1} = \frac{1}{(1-x)^2} \Rightarrow \sum_{r=2}^{\infty} (r-1)x^{r-2} = \frac{2}{(1-x)^3}$

$\Rightarrow \quad S = 4 \times 8 + 2 = 34$

35. $S_n = \sum_{k=1}^{n} \frac{(k+3)(k+2)(k+1)-1}{(k+3)!} = \sum_{k=1}^{n}\left(\frac{1}{k!} - \frac{1}{(k+3)!}\right)$

$$= 1 + \frac{1}{2!} + \frac{1}{3!} - \frac{1}{(n+1)!} - \frac{1}{(n+2)!} - \frac{1}{(n+3)!}$$

$\Rightarrow \lim_{n\to\infty} S_n = 1 + \frac{1}{2} + \frac{1}{6} = \frac{5}{3}$

36. $a^n + b^n = \sqrt{ab}(a^{n-1} + b^{n-1})$

$\Rightarrow \quad \left(a^{\frac{2n-1}{2}} - b^{\frac{2n-1}{2}}\right)(a^{1/2} - b^{1/2}) = 0$

$\Rightarrow \quad \boxed{n = \frac{1}{2}}$

Solutions

CHAPTER 3

PERMUTATION AND COMBINATION

SECTION-1

SINGLE CHOICE QUESTIONS

1. Number of ways = ${}^9C_4 + {}^9C_4 - {}^8C_3 = 196$

2. Every element has 3 possibilities

(i) It belongs to A only

(ii) It belongs to B only

(iii) It belongs to none

$\therefore$ Number of ways = $\frac{3^n - 1}{2} + 1 = \frac{3^n + 1}{2}$ [$\because$ Null (A & B) are not repeated]

3. $({}^{32}C_1)({}^{32}C_1) - [8({}^4C_1 \times {}^4C_1)$ + $8({}^4C_1 \times {}^4C_1)]$

both from same row; both from same column

= 768

4. $15! = 2^{11}\, 3^6\, 5^3\, 7^2\, 11^1\, 13^1$

Number of ways = Number of ways 15! can be expressed as product of 2 relatively prime divisors = $2^{6-1} = 32$

5. Let A = block 12 appear

B = block 34 appear

C = block 567 appear

$n(A \cup B \cup C) = (8! + 8! + 7!) - (7! + 6! + 6!) + 5!$

$= 661 \times 5!$

number of ways = $9! - 661 \times 5!$

6. Number of ways = 8! × 2!

SRG GGGGBBBB

7. Number of ways = $\left(\frac{100!}{(20!)^5 \times 5!} \times 5!\right) \times (4!)^5$

8. All different or (2 Alike, 1 different) or (3 Alike)

$$= {}^{100}C_3 + {}^{100}C_2 \times 2 + {}^{100}C_1$$
$$= {}^{101}C_3 + {}^{101}C_2 = {}^{102}C_3$$

9. 'k' are to be selected from (n – k + 1) gaps between 'n – k' not selected objects.

$\Rightarrow$ Number of ways = ${}^{n-k+1}C_k$

10. n–k empty boxes must be selected from k + 1 gaps between 'k' non empty boxes.

11. Distribute 'k' alike objects (representing $a_1, a_2, ..., a_k$ here) over n persons (which represents numbers 1, 2, 3, ...n).

12. $x > 0, y > 0, x + y \le 99$

number of ways = ${}^{99-2+2}C_2 = {}^{99}C_2$

$x = 0, |y| \le 99$

number of ways = 198

total number of ways = 2(198) + 4 ${}^{99}C_2$ + 1 = 19801

13. Number of subsets = ${}^{100}C_3 \times 3 + ({}^{100}C_1)^3$

= 1004851

14. The number of integers will be same as number of permutation of $(x_1, x_2, x_3, x_4, x_5)$ of five non negative integers so that $x_1 + x_2 + x_3 + x_4 + x_5 = 8$

Number of integers = ${}^{8+4}C_4 = {}^{12}C_4 = 495$

15. $x + y + z = 2n + 1$

$x + y > z \Rightarrow 2n + 1 - z > z \Rightarrow z < n + \frac{1}{2}$

$\therefore$ $1 \le x, y, z \le n$

$\therefore$ Number of ways $= {}^{2n+1-3+2}C_2 - {}^{3}C_1\,{}^{2n+1-(n+3)+2}C_2$

$$= {}^{2n}C_2 - 3 \cdot {}^{n}C_2$$

$$= \frac{2n(2n-1) - 3n(n-1)}{2} = \frac{n(n+1)}{2}$$

17. Number of ways = ${}^5C_2 \times 3!\left(\frac{1}{2!}-\frac{1}{3!}\right) = 20$

18. Number of ways = ${}^{31}C_{10} + {}^{32}C_{10} + {}^{33}C_{10} + \ldots.. + {}^{40}C_{10}$

$$= -{}^{31}C_{11} + ({}^{31}C_{11} + {}^{31}C_{10}) + {}^{32}C_{10} + {}^{33}C_{10} + \ldots.. + {}^{40}C_{10})$$

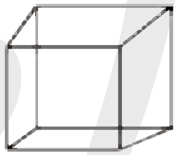
$$= -{}^{31}C_{11} + {}^{41}C_{11}$$

19. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$x + y + z \le 12,\ x, y, z \in N$

number of triplets (x, y, z) $= {}^{12-3+3}C_3 = {}^{12}C_3$

20. Number of line segments = ${}^8C_2 = 28$



21. Let n = number of women

$n \times 3 = 9 \times 4 \quad \Rightarrow \quad n = 12$

22. Number of 6-digit numbers = ${}^6C_3\, 5^6 - {}^5C_2\, 5^5$

$= 90 \times 5^5 = 281250$

23. Guests can be accomodated as shown $\begin{bmatrix} G & X \\ X & G \\ G & X \\ X & G \end{bmatrix}$ or $\begin{bmatrix} X & G \\ G & X \\ X & G \\ G & X \end{bmatrix}$

$\therefore$ number of ways = $2 \times 4! = 48$

24. A – – A = 4 × 3

A – – – A = 4 × 3 × 2

A – – – – A = 4 × 3 × 2 × 1

Total number of ways = 12 + 24 + 24 = 60

25. Number of ways = $3^8 - ({}^3C_1\, 2^8 - {}^3C_2\, 1^8) = 5796$

26. Exponent of 2 in 100! $= \left[\frac{100}{2}\right]+\left[\frac{100}{2^2}\right]+\left[\frac{100}{2^3}\right]+\left[\frac{100}{2^4}\right]+\left[\frac{100}{2^5}\right]+\left[\frac{100}{2^6}\right]$

$= 97$

Exponent of 3 in 100! $= \left[\frac{100}{3}\right]+\left[\frac{100}{3^2}\right]+\left[\frac{100}{3^3}\right]+\left[\frac{100}{3^4}\right]$

$= 48$

Exponent 12 in 100! $= 48$

27. 3, 3, 5, 5 ; 2, 2, 8, 8, 8

Number of ways $= {}^5C_4 \frac{4!}{2!2!} \frac{5!}{2!3!} = 300$

28. Number of functions $= {}^7C_3\,(2^4 - {}^2C_1) = 14 \times {}^7C_3$

29. Number of interior points $= {}^nC_4$

30. Number of ways $= \sum_{r=0}^{n} {}^{2n}C_r = \frac{2^{2n} + {}^{2n}C_n}{2} = 2^{2n-1} + \frac{(2n)!}{(n!)^2 2}$

31. $10800 = 2^4\, 3^3\, 5^2$

$9000 = 2^3\, 3^2\, 5^3$

$\therefore$ Number of common divisors $= 4 \times 3 \times 3 = 36$

32. Number of ways $= (n-2)!\ {}^{n-1}C_2\ 2!$

$= (n-2)\,(n-1)!$

$3600 = 5 \times 6!$

$\Rightarrow \quad n = 7$

33. $x < z < y$

$x < z = y$

$x = z < y$

$z < x < y$

Number of ways $= {}^9C_3 + {}^9C_2 + {}^9C_2 + {}^{10}C_3$

$= 2({}^{10}C_3) + {}^9C_2 = 276$

34. $2A + 2D = 9 \times 8 \times {}^3C_2 - 8$

$2A + 2A = 2 \times {}^9C_1 - 1$

Total number of ways $= 225$

35. Number of ways $= \dfrac{7!}{2!\,2!}\left[{}^{10}C_7 - \left\{\left({}^9C_7 + {}^8C_7 + {}^7C_7\right) - \left({}^7C_7\right)\right\}\right]$

$= 1260\,[120 - 44]$

$= 1260 \times 76$

$= 95760$

36. Different sequence like 122, 222, 345, etc are possible. No. of sets = no. of ways to distribute 3 alike dice over 6 numbers 1, 2, 3, 4, 5, 6 with repetitions allowed

$$= {}^{3+6-1}C_{6-1} = {}^8C_5 = 56$$

SECTION-2

ONE OR MORE THAN ONE CORRECT

2. $z = 2$, number of $(x, y) = 1 \times 1$

$z = 3$, number of $(x, y) = 2 \times 2$

$z = 4$, number of $(x, y) = 3 \times 3$

$z = n + 1$, number of $(x, y) = n \times n$

Alternate: $x < y < z$ or $y < x < z$

$\Rightarrow$ number of $(x, y, z) = 2 \times {}^{n+1}C_3$

or $x = y < z$

$\Rightarrow$ number of $(x, y, z) = {}^{n+1}C_2$

Total number of $(x, y, z) = {}^{n+1}C_2 + 2\,{}^{n+1}C_3$

3. There are two possibilities for a_n, either append a nonzero digit to a legal $(n - 1)$ string or append a zero to an illegal $(n - 1)$ string.

$\Rightarrow$ $a_n = a_{n-1} \times 9 + (10^{n-1} - a_{n-1}) \times 1$

$\Rightarrow \qquad a_n = 8a_{n-1} + 10^{n-1}$

$a_n = 8(8\,a_{n-2} + 10^{n-2}) + 10^{n-1} = 8^2\,a_{n-2} + 8 \times 10^{n-2} + 10^{n-1}$

$= 8^2(8a_{n-3} + 10^{n-3}) + 8 \times 10^{n-2} + 10^{n-1}$

$= 8^3\,a_{n-3} + 8^2 \times 10^{n-3} + 8 \times 10^{n-2} + 10^{n-1}$

$= 8^{n-1}\,a_{n-(n-1)} + (10^{n-1} + 8 \times 10^{n-2} + 8^2 \times 10^{n-3} + \ldots\ldots (n-1)$ terms)

$$= 8^{n-1} \times 9 + \frac{10^{n-1}\left(\left(\frac{8}{10}\right)^{n-1} - 1\right)}{\frac{8}{10} - 1}$$

$$= \frac{10(10^{n-1} - 8^{n-1})}{2} + 9.8^{n-1}$$

$$a_n = \frac{10^n + 8^n}{2}$$

4. $a_1 = 1$, $a_2 = 2$

A gets to n^{th} step either coming from (n – 1)th step, which he can reach in a_{n-1} ways or he can get to n^{th} step by coming from (n – 2)nd step, which he reached in a_{n-2} ways.

$a_n = a_{n-1} + a_{n-2}$

$a_3 = 1 + 2 = 3$

$a_4 = 2 + 3 = 5$

$a_5 = 3 + 5 = 8$

$a_6 = 5 + 8 = 13$

$a_7 = 8 + 13 = 21$

$a_8 = 13 + 21 = 34$

$a_9 = 21 + 34 = 55$

$a_{10} = 34 + 55 = 89$

$a_{11} = 55 + 89 = 144$

5. Consider 1, 2, 3,....., 50 in a row

x_1 a x_2 b x_3 — x_1 is number of numbers in the left of a

x_2 is number of numbers between a & b

x_3 is number of numbers in the right of b

$x_1 + x_2 + x_3 = 48$

$x_1 \geq 0, 0 \leq x_2 \leq 4, x_3 \geq 0$

number of subsets $= {}^{48+2}C_2 - {}^{48-5+2}C_2$

$= {}^{50}C_2 - {}^{45}C_2 = 235$

number of subsets so that $|a - b| \geq 5 = {}^{48-4+2}C_2 = {}^{46}C_2$

$\left[\begin{array}{l} \because x_1 + x_2 + x_3 = 48 \\ x_1 \geq 0, x_2 \geq 4, x_3 \geq 0 \end{array}\right]$

6. 1 digit plaindromes → 9

2 digit palindromes → 9 × 1

3 digit palindromes → 9 × 10

4 digit palindromes → 9 × 10

5 digit palindromes → 9×10^2

6 digit palindromes → 9×10^2

1 $\underline{0}$ $\underline{0}$ -------------- → 10

1 0 1 -------------- → 10

∴ Number at 2(9 + 90 + 900 + 10) = 2018, rank is 1019101

1 ------------------------- → 10^3

2 0 ---------------------- → 10^2

2 1 0 --------------------- → 10

2 1 1 → 10

2 1 2 → 10

2 1 3 → 10

2 1 4 0 4 1 2 → 1

Rank of 2140412 $= 2(9 + 90 + 900) + 1000 + 100 + 40 + 1$

$= 1998 + 1000 + 100 + 40 + 1$

$= 3139$

7. Number of ways $= \dfrac{{}^{2n}C_1 \; {}^{2n-3-(k-1)+1}C_{k-1}}{k}$

$= \dfrac{2n}{k} \; {}^{2n-1-k}C_{k-1}$

or ${}^{2n-k+1}C_k - {}^{2n-k-1}C_{k-2}$

8. If x is odd

$x = 2k + 1,$ $y = 2k + 1, z = 2k + 1, 2k + 2, ... 98 - 4k \rightarrow (98 - 6k)$ ways

$y = 2k + 2, z = 2k + 2, 2k + 3, ..., 97 - 4k \rightarrow (96 - 6k)$ways

$y = 49 - k, z = 49 - k, 50 - k \rightarrow 2$ ways

$\therefore$ number of triplets $= \sum_{k=0}^{16}\left((98-6k)+(96-6k)+.....+2\right)$

$= \sum_{k=0}^{16}(49-3k)\ (50-3k)$

$= 14722$

If x is even

$x = 2k, y = 2k, z = 2k, 2k + 1, ..., 100 - 4k \rightarrow (101 - 6k)$ ways

$y = 2k + 1, z = 2k + 1, ..., 99 - 4k \rightarrow (99 - 6k)$ ways

$\vdots$

$y = 50 - k, z = 50 - k \rightarrow 1$ way

$\therefore$ Number of triplets $= \sum_{k=0}^{16}\left((101-6k)+(99-6k)+.....+1\right)$

$= \sum_{k=0}^{16}(51-3k)^2$

$= 16065$

9. $x + y = 9 \times 10 \times 10 \times 10 \times 10 \times 10 = 9 \times 10^5$

$x = y = \dfrac{1}{2} \times 9 \times 10^5 = 450000$

10. $a_n = a_{n-1} + n$

$a_1 = 2$

$\therefore$ $$a_6 = 2 + 2 + 3 + 4 + 5 + 6 = 22$$

$$a_{10} = 2 + 2 + 3 + 4 + ... + 10 = 56$$

11. Number of handshakes $= {}^{2n}C_2 - n = n(2n - 1) - n$

$$= 2n(n - 1)$$

12. (a) (2A, 3D) + (2A, 2A, 1D) + (5D)

$$= {}^3C_1\, {}^4C_3 \frac{5!}{2!} + {}^3C_2\, {}^3C_1 \frac{5!}{2!\,2!} + 5!$$

$$= 1110$$

(b) 2A, 3D $= {}^3C_1\, {}^4C_3 \dfrac{5!}{2!} = 720$

(c) 2A, 2A, 1D $= {}^3C_2\, {}^3C_1 \dfrac{5!}{2!\,2!} = 270$

13. Total number of paths $= {}^6C_2\, {}^5C_2 + {}^7C_2\, {}^4C_2 - {}^6C_2\, {}^4C_2$

$$= 186$$

Number of paths without crossing N $= {}^7C_2\, {}^4C_2 - {}^6C_2\, {}^4C_2$

$$= 36$$

Number of paths without crossing M $= {}^6C_2\, {}^5C_2 - {}^6C_2\, {}^4C_2$

$$= 60$$

14. Number of words having COUNT in them (COUNT ; N, T, I, I, Y) $= \dfrac{6!}{2!}$

Number of words with all vowels separated $= \dfrac{6!}{2!\,2!} \times {}^7C_4 \times \dfrac{4!}{2!} = 15 \times 7!$

15. a_n = number of sets containing 'n' + number of sets not containing 'n'

Number of sets containing 'n' = {n} + subsets to be chosen from {1, 2, ..., n – 2} and contain 'n'.

$$= 1 + a_{n-2}$$

Number of sets not containing 'n' = subset to be chosen from {1, 2, 3, ..., n – 1}

$$= a_{n-1}$$

$\therefore$ $$a_n = (a_{n-2} + 1) + a_{n-1}$$

16. (A) $^8C_4\, 4! + {}^8C_3\, 3! \times 1 + {}^8C_2 \times 2! = 2072$

(D) $\dfrac{^8C_4 \times 4!}{4} + \dfrac{^8C_3 \times 3!}{2} + \dfrac{^8C_2 \times 2!}{2} = 616$

A	D	D	C	C	B	B	A	A	C	A	B	A	B	B	A
B	C	A	B	D	A	C	D	B	A	C	A	B	A	A	B

A		A	C	B		B	A	A		A	B	A	B		
C	B		B	A	C		C	B	C		C			B	A

17. In 1st round, number are of form 15k + 1, so last number is 991

In 2nd round, first number is 991 + 15 – 1000 = 6

number are of the form 15k + 6, so last number is 996

In 3rd round, first number is 996 + 15 – 1000 = 11, so number are of the form 15k + 11, so last number is 986.

In 4th round, first number is 986 + 15 – 1000 = 1

18. C, O, LL, EE, G

number of ways $= 2A + 20A\ ;\ 2A + 2D\ ;\ 4D$

$= 1 + {}^2C_1\, {}^4C_2 + {}^5C_4 = 18$

19. 1 ------------------- $^8C_4 = 70$

2 3 ----------------- $^6C_3 = 20$

2 4 ----------------- $^5C_3 = 10$

2 5 ----------------- $^4C_3 = 4$

2 6 7 8 9

SECTION-3

COMPRHENSION BASED QUESTIONS

Comprehension (Q.1 To Q.3)

All possibilities over selection of 6 books are

5 alike, 1 different $\rightarrow 1 \times {}^2C_1 = 2$ ways

4 alike, 2 others alike $\rightarrow {}^2C_1 \times {}^2C_1 = 4$ ways

4 alike, 2 different $\rightarrow {}^2C_1\ {}^2C_2 = 2$ ways

3 alike, 3 others alike $\rightarrow {}^3C_2 = 3$ ways

3 alike, 2 other alike, 1 different $= {}^3C_1\ {}^2C_1 = 6$ ways

2 alike, 2 other alike, 2 other alike = 1 way

1. Total ways = 18 ways
2. 4 alike, 2 different + 3 alike, 2 other alike, 1 different + 2 alike, 2 other alike, 2 other alike

$$= 2 + 6 + 1 = 9 \text{ ways.}$$

3. 2 alike, 2 other alike, 2 other alike

$$= 1 \text{ way}$$

Comprehension (Q.4 to Q.5)

The number of ways nobody gets back her coats $= 5! - ({}^5C_1 4! - {}^5C_2 3! + {}^5C_3 2! - {}^5C_4 1! + {}^5C_5)$

$$= 44$$

4. Number of ways = 44 × 44 =
5. Number of ways $= (5!)^2 - ({}^5C_1(4!)^2 - {}^5C_2(3!)^2 + {}^5C_3(2!)^2 - {}^5C_4(1!)^2 + {}^5C_5)$

$$= 11844$$

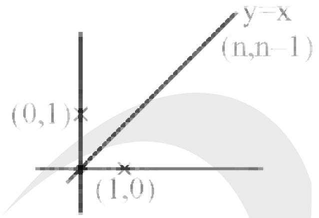
Comprehension (Q.6 to Q.8)

The number of paths from (1, 0) to (n, n – 1) such that y < x at every interior points

$= {}^{n-1+n-1}C_{n-1} - {}^{n+(n-2)}C_{n-2}$

$= {}^{2n-2}C_{n-1} - {}^{2n-2}C_{n-2}$

$= {}^{2n-2}C_{n-1} - \left(\frac{n-1}{n}\right) {}^{2n-2}C_{n-1}$



$= \frac{1}{n}\left({}^{2n-2}C_{n-1}\right)$ $\left[{}^{2n-2}C_{n-2} = \frac{(2n-2)!}{n!(n-2)!} = \frac{n-1}{n}\frac{(2n-2)!}{((n-1)!)^2}\right]$

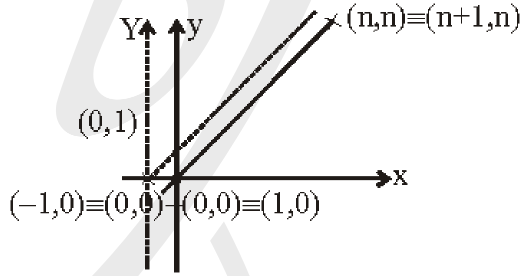
6. Number of paths $= \frac{2}{n}\left({}^{2n-2}C_{n-1}\right)$

7. Shift origin to (–1, 0) w.r.t. new system

Number of paths = Number of paths from (1, 0) to (n + 1, n)

$= \frac{1}{n+1}\,{}^{2(n+1)-2}C_{(n+1)-1}$

$= \frac{{}^{2n}C_n}{n+1}$



8. Number of paths

$= 2\frac{{}^{2n}C_n}{n+1}$

Comprehension (Q.9 to Q.11)

9. There are 2 possibilities either string ends with '0' or 1

_ _ _ _ _ _ _ 0 + _ _ _ _ _ _ _ _ 0 1

$\Rightarrow \quad a_n = a_{n-1} + a_{n-2}$

10. There are 3 possibilities ends with '0' or '01' or '011'

_ _ _ _ _ _ _ 0 + _ _ _ _ _ _ _ _ 0 1 + _ _ _ _ _ _ _ _ 0 1 1

$b_n = b_{n-1} + b_{n-2} + b_{n-3}$

11. $b_1 = 2, b_2 = 2^2, b_3 = 2^2 + 2 + 1 = 7$

$\therefore \quad b_4 = 7 + 4 + 2 = 13$

$b_5 = 13 + 7 + 4 = 24$

$b_6 = 24 + 13 + 7 = 44$

Comprehension (Q.12 to Q.13)

12. ${}^7C_5 \frac{8!}{5!3!} + {}^6C_5 \frac{8!}{5!3!} = 1512$

13. $x_1Rx_2Rx_3Rx_4Bx_5Bx_6Bx_7Bx_8Bx_9$

$x_1 + x_2 + \ldots.. + x_9 = 5$

number of ways $= {}^{5+8}C_8 = 1287$

Comprehension (Q.14 to Q.16)

Sol. AAA, SSSS, I I, NN, T, O

14. First arrange letters other than 2N, 1O.

Number of ways $= \frac{10!}{3!4!2!}$

Now select places for NNO between other letters

$x_1L_1x_2L_2x_3 \ldots.. x_{10}L_{10}x_{11}$

$x_1 + x_2 + x_3 + \ldots.. + x_{11} = 3$

x_i denote number of places for 2N, 1O

number of ways $= {}^{3+10}C_{10} \times 2$ arrangement among 2N & 1O

$\therefore$ Total number of ways $= \frac{10!}{3!4!2!} \times {}^{13}C_{10} \times 2 = \frac{13!}{3!3!4!}$

15. ${}^{3-1+4}C_4 \times {}^{6+7}C_7 \times \frac{6!}{2!2!} = 2700 \times {}^{13}C_7$

16. For first A to precede the first S, we must have an arrangement of the form

$x_1 \text{ S } x_2 \text{ S } x_3 \text{ S } x_4 \text{ S } x_5$

where in the place x_i, we put from 0 to 3A's with condition that $x_1 \geq 1, x_2, x_3, x_4, x_5 \geq 0$ the number of solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 3, x_1 \geq 1, x_k \geq 0, 2 \leq k \leq 5$ is ${}^{3-1+4}C_4 = {}^6C_4$

Once the first condition has been fulfilled, we have arranged the 3A's and 4S's in allowable configurations, arranging 3 + 4 = 7 letters, creating 8 spaces in between, say, as in

$y_1L_1y_2L_2y_3L_3$ .... $y_7L_7y_8$

Choosing the spaces for N and O is equivalent to solving the equation

$y_1 + y_2 + ... y_8 = 3, y_k \geq 0, 1 \leq k \leq 8$

and so there are ${}^{3+7}C_7$ ways of doing this. Once the spaces have been chosen for N and O, we may arrange them in just two ways so that first N precedes O. Once the second condition has been fulfilled we have to arrange 3 + 4 + 2 + 1 = 10 letters in allowable configurations creating 11 spaces between then. We now arrange remaining letters.

$x_1L_1x_2L_2x_3L_3$ ..... $L_{10}x_{11}$, $\quad x_1 + x_2 + ... + x_{11} = 3$

in ${}^{3+10}C_3 \times \frac{3!}{2!}$ ways

$\Rightarrow$ Total number of ways = ${}^{6}C_4 \times {}^{10}C_7 \times 2 \times {}^{13}C_3 \times 3$

Comprehension (Q.17 to Q.18)

17. Minimum number of locks = ${}^{11}C_6 = \frac{11\times10\times9\times8\times7}{5\times4\times3\times2} = 462.$

18. Minimum number of locks each criminal must carry = ${}^{10}C_5 = \frac{10\times9\times8\times7\times6}{5\times4\times3\times2} =$ 252.

Comprehension (Q.19 to Q.20)

R + W + G + B = 20

R, W, G, B are whole numbers

R = number of red balls

W = number of white balls

G = number of green balls

B = number of blue balls

19. $G \leq 3, R \geq 0, W \geq 0, B \geq 0$

number of ways = ${}^{20+3}C_3 - {}^{20-4+3}C_3 = 802$

20. Let W = 2k

R + 2k + G + B = 20 2k = number of white balls

R + G + B = 20 – 2k

$$\text{number of ways} = \sum_{k=0}^{10} {}^{20-2k+2}C_2 = \sum_{k=0}^{10} (11-k)\,(21-2k)$$

$$= 2\left(\frac{10\times11\times21}{6}\right) - 43\left(\frac{10\times11}{2}\right) + 231 \times 11$$

$$= 948$$

Comprehension (Q.21 to Q.22)

21. Number of ways $= ({}^9C_4\,{}^8C_6 - {}^8C_3\,{}^7C_5) + ({}^9C_5\,{}^8C_5 - {}^8C_4\,{}^7C_4) + ({}^9C_6\,{}^8C_4 - {}^8C_5\,{}^7C_3)$

$= 2352 + 4606 + 3920$

$= 10878$

22. Number of ways $= ({}^8C_3\,{}^7C_5 + {}^8C_4\,{}^7C_6) + ({}^8C_4\,{}^7C_4 + {}^8C_5\,{}^7C_5) + ({}^8C_5\,{}^7C_3 + {}^8C_6 + {}^7C_4)$

$= 1666 + 3626 + 2940$

$= 8232$

Comprehension (Q.23 to Q.25)

23. ${}^{m-n+1}C_n\, n!$

24. ${}^{m-n+1}C_{\frac{n}{2}}\, n!$

25. $\underbrace{{}^{\frac{m-1}{2}}C_n\, 2^n\, n!}_{\text{Middle seat empty}} + \underbrace{{}^{\frac{m-1}{2}}C_{n-1}\, 2^{n-1}\, n!}_{\text{Middle seat occupied}}$

Comprehension (Q.26 to Q.28)

M,M ; A, A ; T, T ; H ; E ; I ; C ; S

26. At odd places → 2A, 2OA, 1D,

2A, 3D

$$\text{number of ways} = {}^3C_2\,{}^6C_1\,\frac{5!}{2!2!}\times\frac{6!}{2!} + {}^3C_1\,{}^7C_3\,\frac{5!}{2!}\times\frac{6!}{2!\,2!} = (113)\frac{5!\,6!}{8}$$

27. MM TT HEICS

number of ways = $7! \times {}^8C_2 = 28 \times 7!$

28. Number of ways = $\frac{7!}{2!\,2!} \times {}^8C_4 \times \frac{4!}{2!} = 210 \times 7!$

SECTION-4

MATCH THE COLUMN

1. (A) Number of ways = $3^5 - ({}^3C_1 2^5 - {}^3C_2 1^5) = 150$

(B) Number of ways = ${}^{5-3+2}C_2 = 6$

(C) Number of ways = $\frac{150}{3!} = 25$

(D) Number of ways = 2 (1, 1, 3 and 1, 2, 2)

2. (A) x b b b x c x c x c x

number of ways = $\frac{4!}{3!} \times {}^5C_4 = 20$

(B) 2b, 1b ; 2c, 1c or 2b, 1b ; 1c, 1c, 1c or 2b, 1b ; 3c

x bb x c x c x b x c x, bbcbcc ; bcbbc, bccbbc (same way starting with c)

cbbcbc, cbcbbc

bbcccb, bcccbb

number of ways = $12 \times {}^6C_4 = 180$

(C) bcbcbc = ${}^7C_4$

bccbcb or bcbccb = $2 \times {}^6C_3$

bbccbc or bccbbc or bbcbcc or bcbbcc = $4 \times {}^5C_2$

bcccbb or bbcccb = $2 \times {}^4C_1$

bbbccc = ${}^4C_4$

Total ways = 2(35 + 40 + 40 + 8 + 1) = 248

(D) bcbcbc, cbcbcb

bccbcb, cbbcbc

bcbccb, cbcbbc

number of ways = ${}^7C_1 \times 2 + 4 = 18$

3. H → Horizontal step, V → Vertical step

(A) x H x V x H x V x H x V x

5 diagonal steps may be distributed in 7 places in any fashion.

The number of ways is equal to number of ways of distributing 5 identical objects over 7 persons

$$= {}^{5+6}C_6 = {}^{11}C_6 = 462$$

(B) Number of ways HVHV

$$= {}^{6+4}C_4 = {}^{10}C_4 = 210$$

(C) HVHVHVHV

number of ways $= {}^{4+8}C_8 = {}^{12}C_8 = 495$

(D) Number of ways $= {}^{11}C_6$

4. (A) ${}^nC_r\,(2^m - 1)^r$

$\longrightarrow$ every element belong to atleast one subset

(B) ${}^nC_r(2^m - 1)^{n-r}$

$\longrightarrow$ remaining elements don't belong to all subsets simultaneously.

(C) Every element has choices only A_1, only A_2, ..., only A_m, none

$\Rightarrow$ Number of ways $= (m + 1)^n$

(D) $(2^m - 1)^n$

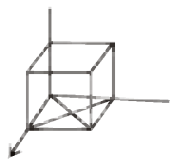
5. (A) no. of ways $= ({}^4C_2)\,3 + 6 = 24$

(B) no. of ways $= 10 + 8 + 6 + 4 + 2 = 30$

(C) each edge has '6 face diagonals skew with it.

$\therefore$ no. of ways $= 12 \times 6 = 72$

(D) no. of ways $= 5 \times 6 \;+\; 4 \times 6$



$\downarrow$ in same face $= 54$

$\downarrow$ face diagonal & edge in different faces

6. (A) $a > b,\ a > c,\ a > d$

$\Rightarrow$ no. of ways $= {}^{10}C_4 \times 3! = 1260$

(B) $a < b,\ a < c,\ a < d$

number of ways $= {}^9C_4 \times 3! = 756$

(C) $a \le b \le c \le d$

$\Rightarrow$ no. of ways $= {}^{4+9-1}C_{9-1} = {}^{12}C_8 = 495$

(D) $a \ge b \ge c \ge d$

$\Rightarrow$ no. of ways $= {}^{4+10-1}C_{10-1} - 1 = {}^{13}C_9 - 1 = 714$

$\downarrow$

(omitting oooo)

7. (A) $N = {}^{8+3}C_3 = 165$

(B) $N = {}^{8-4+3}C_3 = 35$

(C) $N = {}^8C_4 4! - ({}^4C_1\, {}^7C_3 3! - {}^4C_2\, {}^6C_2 2! + {}^4C_3\, {}^5C_1 - 1)$

$= 1006$

(D) $N = {}^4C_2 \times 4 \times 3 = 72$

SECTION-5

SUBJECTIVE

1. $*(a) = 7 \quad \Rightarrow \quad a = P_1^6$

$*(g) = 13 \quad \Rightarrow \quad g = P_1^{12}$, P_1, P_2 are prime

$g = ar^6 \quad \Rightarrow \quad P_1^6 r^6 = P_2^{12} \quad \Rightarrow \quad r = \frac{P_2^2}{P_1}$

$432 = d - c = P_1^6(r^3 - r^2) = P_1^6\left(\frac{P_2^6}{P_1^3} - \frac{P_2^4}{P_1^2}\right) = P_1^6 P_2^6 - P_1^4 P_2^4$

$\Rightarrow 2^4 3^3 = P_1^3 P_2^4 (P_2^2 - P_1)$

$\Rightarrow \quad P_2 = 2, P_1 = 3$

$\Rightarrow \quad b = 972$

2. $\_\ \_\ \_\ \_\ \underline{5} = 10^4$ ways

$\_\ \_\ \_\ \underline{5}\ \_ = 10^4$ ways

$\_\ \_\ \underline{5}\ \_\ \_ = 10^4$ ways

$\_\ \underline{5}\ \_\ \_\ \_ = 10^4$ ways

$\underline{5}\ \_\ \_\ \_\ \_ = 10^4$ ways

Total $= 5 \times 10^4$ ways

e.g 50555 should be counted 4 times 50555 is counted in numbers having 5 at unit, tens, hundreds & ten thousands place.

3. (4 integers have same remainder) + (2 leaving remainder 0, 1 leaving remainder 1 and other 3) + (2 leaving remainder 1, and two 3) + (2 leaving remainder 0 and two 2) + (2 leaving remainder 2 and one leaving 1 and other 3)

$N = (3\ {}^{26}C_4 + {}^{27}C_4) + ({}^{26}C_2\ {}^{27}C_1\ {}^{26}C_1) + ({}^{26}C_2\ {}^{26}C_2) + ({}^{27}C_2\ {}^{26}C_2) + ({}^{26}C_2\ {}^{27}C_1\ {}^{26}C_1)$

$= 738400$

4. $x_1 + x_2 + x_3 = x, \quad x_4 + x_5 = y, \quad z = x_6$

$x + 3y = 21 \quad - 5x_6 \quad x_6 = 1, 2$

$x + 3y = 16, 11$

$(x, y, x_6) = (4, 4, 1), (7, 3, 1), (10, 2, 1), (5, 2, 2)$

Number of 6-tuples $= {}^{4-3+2}C_2\ {}^{4-2+1}C_1 + {}^{7-3+2}C_2\ {}^{3-2+1}C_1$

$+ {}^{10-3+2}C_2\ {}^{2-2+1}C_1 + {}^{5-3+2}C_2\ {}^{2-2+1}C_1$

$= 9 + 30 + 36 + 6$

$= 81$

5. Number of ways $= {}^6C_4 + {}^6C_2\ {}^7C_1\ {}^7C_1 + {}^6C_1({}^7C_3 + {}^7C_3) + {}^7C_2\ {}^7C_2$

${}^6C_4$	${}^6C_2\ {}^7C_1\ {}^7C_1$	${}^6C_1({}^7C_3 + {}^7C_3)$	${}^7C_2\ {}^7C_2$
all of form 3k	two of form 3k, one 3k + 1, one 3k + 2	one of form 3k, 3 of 3k + 1 or 3k + 2	two of form 3k + 1 & two of 3k + 2

$= 15 + 735 + 420 + 441 = 1611$

6. Total number of elements in set S

$= \frac{4 \times 6}{4} = 6$

Also total number of elements in set S

$= \frac{2 \times n}{3} = \frac{2n}{3}$

$\Rightarrow \quad \frac{2n}{3} = 6$

$\Rightarrow \quad n = 9$

7. With smallest length formed by vertices $A_1A_3, A_2A_4, \ldots A_{20}A_2$

Number of Δs = 12×20

With smallest length formed by $A_1A_4, A_2A_5, \ldots, A_{20}A_3$

Number of Δs = 10×20

With smallest length formed by $A_1A_5, A_2A_6, \ldots, A_{20}A_4$, number of Δs = 6×20

With smallest length formed by $A_1A_6, A_2A_7, \ldots, A_{20}A_5$, number of Δs = 4×20

$N = (12 + 10 + 6 + 4) \times 20 = 640$

8. The number of points of intersection on the $A_iA_{i+9} = 2 \times 8 = 16$

Since, the lengths of these 200 diagonals are equal, they all are tangent to a same circle. Since from any point outside the circle exactly two tangents lines to the circle can be introduced, so any three of the 200 diagonals can't intersect at one common point

$\Rightarrow$ Total number of interior point of intersection of 200 diagonals = $\dfrac{200 \times 16}{2}$

$= 1600$

9. $N = {}^6C_3 + 4\,{}^6C_4 + 5\,{}^6C_5 + {}^6C_6 = 111$

${}^6C_3$	$4\,{}^6C_4$	$5\,{}^6C_5$	${}^6C_6$
Using 3 points on the circle	Using 2 points on the circle and 1 interior point	using 1 point on the circle and 2 interior points	using all 3 interior points

10. 1 _ _ _ _ _ _ $= 5 \times 5! = 600$

2 _ _ _ _ _ _ $= 5 \times 5! = 600$

3 _ _ _ _ _ _ $= 5 \times 5! = 600$

41 _ _ _ _ _ $= 4 \times 4! = 96$

42 _ _ _ _ _ $= 4 \times 4! = 96$

431 _ _ _ _ $= 3 \times 3! = 18$

4321 _ _ _ $= 2 \times 2! = 4$

43251 _ _ $= 2!$

43256 _ _ $= 2!$

2018^{th} number is 4325671

12. Various forms of the numbers can be

(i) abc15 (ii) ab150 (iii) ab155 (iv) a15b0

(v) a15b5 (vi) 15ab0 (vii) 15ab5

Number of numbers of form abc15 = 300 {102, 105, ----999}

Number of numbers of form ab150 = 30 {12, 15, ----, 99}

Number of number of the form ab155 = 30 {10, 13,-----97}

Number of number of the form a15b0 = 30 {12, 15,-----99}

Number of number of the form a15b5 = 30 {10, 13,-----97}

Number of number of the form 15ab0 = 34 {00, 03,-----99}

Number of number of the form 15ab5 = 33 {01, 04,-----97}

Number of common numbers in number of form abc 15 & a15b5 = 3

Number of common numbers in number of form ab150 & 15ab0 = 1

Number of common numbers in number of form ab155 & 15ab5 = 1

Number of common numbers in number of form abc 15 & 15ab5 = 3

$\Rightarrow$ Total number of numbers = 300 + 30 × 4 + 34 + 33 – 8 = 487 – 8 = 479

13. 6 A like $\rightarrow$ 5

4 A like + 2 others alike $\rightarrow 5 \times 4 \times \dfrac{6!}{4!2!} = 300$

3 Alike + 3 others alike $\rightarrow {}^5C_2 \times \dfrac{6!}{3!3!} = 200$

2 Alike + 2 other alike + 2 other alike $\rightarrow {}^5C_3 \dfrac{6!}{2!2!2!} = 900$

N = 900 + 200 + 300 + 5 = 1405

14. 0 1 2 3 0 4 $\rightarrow$ $\dfrac{6!}{2!} - 5!$

0 1 2 3 1 3 $\rightarrow$ $\dfrac{6!}{2!2!} - \dfrac{5!}{2!2!}$

0 1 2 3 2 2 $\rightarrow$ $\dfrac{6!}{3!} - \dfrac{5!}{3!}$

N = 240 + 150 + 100 = 490

15. Out of all groups of 4 consecutive even numbers, there is exactly one of the form $8k + 4$ which is divisible by 4 but not by 8.

Number of four digit even numbers $= 9 \times 9 \times 9 \times 4$

$\therefore \quad N = 9 \times 9 \times 9 = 729$

16. Elements 2, 3, 5, 7, 8 have unique choice they all belong to both A & B.

Remaining elements each have choices

{1, 4, 6, 9, 10, 11, 12} → only A / only B / none, exacluding the case all elements belonging to none

$N = 3^7 - 1 = 2186$

17. Let $x_1x_2x_3x_4x_5x_6x_7x_8$ be the eight digit number

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 4$

$x_1 \geq 1$ & $x_i \geq 0 \quad i > 1 \quad x_i \in I$

$\Rightarrow \quad N = {}^{4-1+7}C_7 = {}^{10}C_7 = 120$

18. If $a > 1 \Rightarrow a! + b! + c!$ is even, but 3^d is odd

$\Rightarrow$ $a = 1$, if $b > 2$, then $a! + b! + c!$ is of the form $3k + 1$, but 3^d is of form $3k$

$\Rightarrow$ $a = 1$, $b = 1$ or 2

If $a = 1, b = 1 \Rightarrow c! = 3^d - 2 \Rightarrow c \leq 2$ $\qquad$ ($\because$ $3^d - 2$ is not divisible by 3)

$c = 1, d = 1$

If $a = 1, b = 2 \Rightarrow c! = 3^d - 3$ is not divisible by 9 ($\because$ $d > 1$) but divisible by 3

$\Rightarrow c < 6$

$\therefore$ check for $c = 3, 4, 5$

$\Rightarrow$ $c = 3, d = 2$ & $c = 4, d = 3$

$(a, b, c, d) = (1, 1, 1, 1); (1, 2, 3, 2), (1, 2, 4, 3)$

19. {0, 3, 6, 9}, {1, 4, 7}, {2, 5, 8}

Number of $(3k + 1)$ form numbers = 0, number of $(3k + 2)$ form numbers = 3

or Number of $(3k + 1)$ form numbers = 3, number of $(3k + 2)$ form numbers = 0

or Number of $(3k + 1)$ form numbers = number of $(3k + 2)$ form numbers

$N = (1 + 1)\, 2^4 + 2^4\left(({}^3C_0)^2 + ({}^3C_1)^2 + ({}^3C_2)^2 + ({}^3C_3)^2\right) - 1$

$= 32 + 16(20) - 1 = 351$

20. 1, 2, 3, 4, 5 → only A / only B / none / both A,B   Total = 4^5

for $A \subseteq B$ 1, 2, 3, 4, 5 → only B / both A,B / none   no. of ways = 3^5

for $B \subseteq A$ no. of ways = 3^5

for $A = B$ no. of ways = $2^5 \times 1$

$\therefore \quad N = 4^5 - (2.3^5 - 2^5) = 4^5 - 2.3^5 + 2^5 = 570$

21. $N = {}^{32}C_3 - (32 + 32 \times 28 + 16 \times 26)$

$= 3616$

22. $$N = \left(\underbrace{8\times 9\times 9}_{\text{3 digit no.s having no.3}} - \underbrace{7\times 8\times 8}_{\text{3 digit no.s having no.3 \& 5}}\right) + \left(\underbrace{9\times 9}_{\text{1 '3' at first place}} - \underbrace{8\times 8}_{\text{1 '3' of first place and no.5}}\right) +$$

$$2 \times \left(\underbrace{8\times 9}_{\text{exactly one '3' at 2nd or 3rd place}} - \underbrace{7\times 8}_{\text{exactly one '3' at 2nd or 3rd place and no.5}}\right) = 249$$

23. Let, a_n = no. of ways in which A can get back the ball after 'n' passes.

b_n = no. of ways in which the ball goes to a fixed person other than A after n trials.

$\therefore \quad 3^n = a_n + 3b_n \ \& \ a_n = 3b_{n-1}$

$\Rightarrow \quad 3^{n-1} = a_{n-1} + a_n$

$a_1 = 0$

$\Rightarrow \quad a_2 = 3^1 - a_1 = 3$

$a_3 = 3^2 - 3$

$a_4 = 3^3 - 3^2 + 3$

$\vdots$

$$a_7 = 3^6 - 3^5 + 3^4 - 3^3 + 3^2 - 3 = \frac{-3((-3)^6 - 1)}{-3-1} = 546$$

$$\boxed{N = 546 = 13 \times 7 \times 6}$$

24. Number can be formed using the digits (9, 9, 3), (9, 8, 4), (9, 7, 5), (9, 6, 6), (8, 8, 5), (8, 7, 6), (7,7,7)

$\therefore N = \frac{3!}{2!} \times 3 + 3! \times 3 + 1 = 28.$

25. If a = 1, $2^a + (4-1)^b + (1+4)^c = 3 + (-1)^b + 4\lambda$

$\Rightarrow$ b is even

$\therefore$ No. of triplets = 1 × 2 × 5 = 10

If a ≠ 1, $2^a + 3^b + 5^c = 4\lambda + (-1)^b + 1$

$\Rightarrow$ b is odd.

$\therefore$ number of triplets = 4 × 3 × 5 = 60

$\Rightarrow$ N = 10 + 60 = 70

26. Number of numbers of the form 7K = 14

Number of numbers of the form 7K + 1 = 15

Number of numbers of the form 7K + 2 = 15

Number of numbers of the form 7K + 3 = 14

Number of numbers of the form 7K + 4 = 14

Number of numbers of the form 7K + 5 = 14

Number of numbers of the form 7K + 6 = 14

N = 1 + 15 + 15 + 14 = 45

27. CC, LL, UU, A, S

A _ _ _ _ (2 alike + 2OA) + (2 alike + 2 different) + 4 diff.

$$= {}^3C_2 \frac{4!}{2!2!} + {}^3C_1 {}^3C_2 \frac{4!}{2!} + 4! = 150$$

CAC _ _ (2 alike) + (2D) = ${}^2C_1 + {}^3C_2\, 2! = 8$

CALC _ L or S or U

$$N = 150 + 8 + 3 = 161$$

28. 0 constants = 1

1 consonant = $^5C_1(2)$

2 consonants = $^5C_2(2)^2$

3 consonants = $(^5C_3 - 3)2^3$

4 consonants = 2^4

N = 1 + 10 + 40 + 56 + 16 = 123

29. $N = P_1^3$ or P_1P_2

$30 = P_1 + P_1^2$ or $30 = P_1 + P_2$

$N = 5^3, 7 \times 23, 11 \times 19, 13 \times 17$

30.

a	b	c
2^3	$2^0, 2^1, 2^2, 2^3$	2^4
$2^0, 2^1, 2^2$	2^3	2^4
5^3	$5^0, 5^1, 5^2$	5^3
5^3	5^3	$5^0, 5^1, 5^2, 5^3$
$5^0, 5^1, 5^2$	5^3	5^3

number of ways = 7 × 10 = 70

31. Distribute 1 to 8 places.

Now distribute 4 to 8 places

Number of ways = $^{4+7}C_7 - {}^8C_1 = 322$

$\Rightarrow$ N = 322

32. $0, 0, 1, 2, 4 \rightarrow 3 \times \frac{4!}{2!} = 36$

$0, 1, 1, 2, 3 \rightarrow \frac{4 \times 4!}{2!} = 48$

$0, 1, 2, 2, 2 \rightarrow 4 \times \frac{4!}{3!} = 16$

Total number of numbers = 36 + 48 + 16 = 100

33. Number of dolls each girl has got = $^6C_4 = 15$

$\therefore$ Total number of dolls = 15 × 3 = 45

34. 3^m is of form $4k + 1$ or $4k + 3$ according as m is even or m is odd

5^n is of form $4k + 1$

7^p is of form $4k + 1$, if p is even, $4k + 3$ if p is odd.

$\Rightarrow$ Number of divisors of form $4k + 1 = 3 \times 5 \times 8 + 3 \times 5 \times 8 = 240$

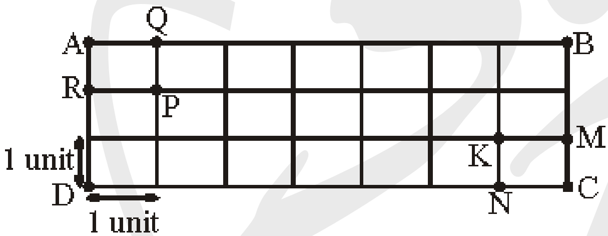
[3^m, 7^p, m & p both odd or both even]

35. Sum $= [(2 + 4 + 6 + 8) \times {}^5C_2 \times 2! + (1 + 3 + 5 + 7 + 9) \times {}^4C_1\ {}^4C_1 \times 2!]$
$(10^0 + 10^1 + 10^2)$

$= (400 + 800) \times 111$

$= 133200$

36.



6 westward, 1 eastward and 1 northward move

number of ways $= \dfrac{8!}{6!} = 56$

37. $a + b = d_i$, $c + d = \dfrac{2016}{d_i}$ $2016 = 2^5 \cdot 3^2 \cdot 7^1$

$$\text{number of (a, b, c, d)} = \sum_{i=1}^{36}(d_i - 1)\left(\frac{2016}{d_i} - 1\right)$$

$$= \sum_{i=1}^{36}\left(2017 - \left(d_i + \frac{2016}{d_i}\right)\right)$$

$= 2017 \times 36 - 2(2^0 + 2^1 + \ldots 2^5)(3^0 + 3^1 + 3^2)(7^0 + 7^1)$

$= 59508$

38. $2^{100} + 2^{100} + 2^{|C|} = 2^{|A \cup B \cup C|} = 2^{101} + 2^{|C|}$

$\Rightarrow$ $|C| = 101$, $|A \cup B \cup C| = 102$

Minimum value $|A \cap B \cap C| = 98$, when $n(A \cap B) = 99$, $n(A \cap C)$

$= 99$, $n(B \cap C) = 99$

39. 1 →
- only A
- only B
- only C
- both A and B not C
- A and C, not B
- B and C, not A

$$N = 6^{2003}$$

number of divisors = $(2004)^2 = 4016016$

40.

0,1,2,3,4

$\underline{1}\ \underline{\ }\ \underline{\ }\ \underline{\ }\quad = 5^3$

0,1,2,3,4

$\underline{1}\ \underline{\ }\ \underline{\ }\ \underline{9}\quad = 5^2$

0,1,2,3,4

$\underline{1}\ \underline{\ }\ \underline{9}\ \underline{9}\quad = 5^1$

$\underline{1}\ \underline{9}\ \underline{9}\ \underline{9}\quad = 1$

Number of ways = $5^3 + 5^2 + 5^1 + 1 = 156$

41. Consider the sum 1 + 1 + 1 + ... + 1, being 10 terms in all. We can break then into 1 or more parts (10 parts at the most) by either putting or not putting parenthesis after the 9 + signs.

This can be done in 2^{10-1} ways.

For example 1 + 1 + 1, 1 + 1, 1 + 1 + 1 + 1, 1 = 3, 2, 4, 1 is one case and so on.

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CHAPTER 4 BINOMIAL THEOREM

SECTION-1

SINGLE CHOICE QUESTIONS

1. General term = $\dfrac{10!}{r!s!(10-r-s)!}\, 2^{\frac{r}{2}}\, 3^{\frac{s}{3}}\, 5^{\frac{10-r-s}{6}}$

(r, s) = (4, 0), (10, 0), (4, 6) for rational term.

2. $\sum\limits_{r=0}^{n-1}(a_r + a_{r+1})^2 = \sum\limits_{r=0}^{n-1}({}^nC_r + {}^nC_{r+1})^2 = \sum\limits_{r=0}^{n-1}{}^{n+1}C_{r+1}^2 = {}^{2n+2}C_{n+1} - 2$

3. Coefficient of x^r in $(1 + x)^n (1 - 2x + 3x^2 - 4x^3 + ... + (-1)^r(r + 1) x^r)$

$= \text{coefficient of } x^r \text{ in } (1 + x)^{n-2} (1-(-x)^{r+1}) + \dfrac{(-1)^{r+1}(r+1)x^{r+1}}{(1+x)}$

$= {}^{n-2}C_r$

4.

		$(1 + x + x^2 + x^3)^5$	$= a_0 + a_1x + a_2x^2 + ... + a_{15}x^{15}$
Put	$x = 1$	4^5	$= a_0 + a_1 + a_2 + a_3 + ... + a_{15}$
	$x = -1$	0	$= a_0 - a_1 + a_2 - a_3 + ... - a_{15}$
		$4^5 - 0$	$= 2(a_1 + a_3 + a_5 + ... + a_{15})$
	$\Rightarrow$	$a_1 + a_3 + a_5 + ... + a_{15} = 512$	

5. $2(1 + x^3)^{100} = \sum\limits_{r=0}^{100}\left(a_r x^r - \cos\dfrac{\pi}{2}(x+r)\right)$

Put x = 1, $2^{101} = a_0 + a_1 + a_2 + ... + a_{100} + \sum\limits_{r=0}^{100}\sin\left(\dfrac{r\pi}{2}\right)$

$x = -1, \qquad 0 = a_0 - a_1 + a_2 - a_3 + \ldots.. + a_{100} - \sum_{r=0}^{100} \sin\left(\frac{r\pi}{2}\right)$

Adding, $\qquad 2^{101} = 2(a_0 + a_2 + a_4 + \ldots.. + a_{100})$

$\Rightarrow \qquad a_0 + a_2 + a_4 + \ldots.. + a_{100} = 2^{100}$

6. $T_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}}$

$\Rightarrow \qquad 2T_n = n\sum \frac{1}{{}^nC_r} = nS_n$

7. $(1 + n)^{m+1} - n^{m+1} = 1 + {}^{m+1}C_1 n + {}^{m+1}C_2 n^2 + {}^{m+1}C_3 n^3 + \ldots.. + {}^{m+1}C_m n^m$

Put n = 1, 2, 3, ... n and add

$(1 + n)^{m+1} - 1^{m+1} = n + {}^{m+1}C_1 S_1 + {}^{m+1}C_2 S_2 + {}^{m+1}C_3 S_3 + \ldots.. + {}^{m+1}C_m S_m$

$\Rightarrow \qquad {}^{m+1}C_1 S_1 + {}^{m+1}C_2 S_2 + {}^{m+1}C_3 S_3 + \ldots.. + {}^{m+1}C_m S_m = (n + 1)^{m+1} - (n + 1)$

8. $\sum_{r=1}^{n} \frac{r(r+1)}{2r} {}^nC_r^2 = \frac{1}{2}\sum_{r=1}^{n} (r\,{}^nC_r^2 + {}^nC_r^2)$

$= \frac{1}{2}\sum_{r=1}^{n} (n\,{}^{n-1}C_{r-1}\,{}^nC_{n-r} + {}^nC_r^2)$

$= \frac{1}{2}(n\,{}^{2n-1}C_{n-1} + {}^{2n}C_n - 1)$

9. $\sum_{r=0}^{n} \frac{{}^nC_r^2}{r+1} = \frac{1}{(n+1)} \sum_{r=0}^{n} {}^nC_r\;{}^{n+1}C_{r+1}$

$= \frac{1}{(n+1)} \sum_{r=0}^{n} {}^nC_{n-r}\;{}^{n+1}C_{r+1} = \frac{{}^{2n+1}C_{n+1}}{(n+1)}$

10. $2({}^3C_3 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2) + {}^{n+1}C_2$

$= 2({}^{n+1}C_3) + {}^{n+1}C_2$

$= \frac{(n+1)n(n-1)}{3} + \frac{(n+1)n}{2}$

$= \frac{n(n+1)\,(2n+1)}{6}$

11. $S = \sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{(r+1)^2} = \frac{1}{n+1} \sum_{r=0}^{n} (-1)^r \frac{{}^{n+1}C_{r+1}}{r+1}$

$$= \frac{1}{(n+1)}\left[\sum_{r=0}^{n} \frac{(-1)^r\, {}^nC_r}{r+1} + \sum_{r=0}^{n-1} \frac{(-1)^r\, {}^nC_{r+1}}{r+1}\right]$$

$$= \frac{1}{(n+1)}\left[\frac{1}{n+1} + \sum_{r=0}^{n-1} \frac{(-1)^r\, {}^{n-1}C_r}{r+1} + \sum_{r=0}^{n-2} \frac{(-1)^r\, {}^{n-1}C_{r+1}}{r+1}\right]$$

$$= \frac{1}{(n+1)}\left[\frac{1}{n+1} + \frac{1}{n} + \sum_{r=0}^{n-2} \frac{(-1)^r\, {}^{n-2}C_r}{r+1} + \sum_{r=0}^{n-3} (-1)^r \frac{{}^{n-2}C_{r+1}}{r+1}\right]$$

$$= \frac{1}{(n+1)}\left[\frac{1}{(n+1)} + \frac{1}{n} + \frac{1}{(n-1)} + \ldots.. \frac{1}{3} + \frac{1}{2} + 1\right]$$

Alternate : $(1 - x)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^r$

$$\frac{1-(1-x)^{n+1}}{x(n+1)} = \sum_{r=0}^{n} \frac{(-1)^r\, {}^nC_r\, x^r}{r+1}$$

$$\frac{1}{(n+1)}\int_0^1 \left(\frac{1-(1-x)^{n+1}}{1-(1-x)}\right) dx = \sum_{r=0}^{n} \frac{(-1)^r\, {}^nC_r}{(r+1)^2}$$

$$= \frac{1}{n+1}\int_0^1 \frac{1-x^{n+1}}{1-x} dx$$

$$= \frac{1}{(n+1)}\int_0^1 (1+x+x^2+\ldots+x^n)dx$$

$$= \frac{1}{n+1}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots..\frac{1}{n}+\frac{1}{n+1}\right)$$

12. $S = \sum_{r=0}^{n} (r-2)\, {}^nC_r + 2 + n$

$= n.2^{n-1} - 2^{n+1} + 2 + n$

$= (n - 4)\, 2^{n-1} + n + 2$

13. $S = \sum_{r=0}^{n} \frac{(-1)^r \, {}^nC_r}{(4r+2)} = \int_0^1 x(1-x^4)^n dx = \frac{1}{2}\int_0^1 (1-x^2)^n dx$

$$I_n = \int_0^1 (1-x^2)^n dx = x(1-x^2)^n \Big|_0^1 + 2n\int_0^1 x^2(1-x^2)^{n-1} dx = 2nI_{n-1} - 2nI_n$$

$$I_n = \frac{2n}{(2n+1)} I_{n-1} = \frac{2^2 n(n-1)}{(2n+1)(2n-1)} I_{n-2} = \ldots\ldots$$

$$= \frac{2^n n(n-1)(n-2)\ldots\ldots 1}{(2n+1)(2n-1)(2n-3)\ldots\ldots 3} I_0$$

$$= \frac{2^n \, n!}{3\cdot 5\cdot 7\ldots\ldots(2n-1)(2n+1)}$$

14. Coefficient of x^n in

$$\left(1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots+\frac{(-1)^{n-1}x^{n-1}}{(n-1)!}+\frac{(-1)^n x^n}{n!}\right)\left(\frac{(-1)^n x^n}{n!}+\frac{(-1)^{n-1}x^{n-1}}{(n-1)!}+\ldots+-\frac{x}{1!}+1\right)$$

$$= \frac{(-1)^n}{n!}+\frac{(-1)^n}{1!(n-1)!}+\frac{(-1)^n}{2!(n-2)!}+\ldots++\frac{(-1)^n}{n!}$$

$$= \frac{(-1)^n}{n!}\left({}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n\right) = \frac{(-2)^n}{n!}$$

15. Coefficient of x^{64} in $(x^3 + 1)^{130}(x + 1) = {}^{130}C_{21}$

16. ${}^nC_1 + {}^nC_2 = 36 = {}^{n+1}C_2 \quad \Rightarrow \quad n = 9$

$$\frac{9C_2(2^x)^7(2^{-2x})^2}{9C_1(2^x)^8(2^{-2x})^1} = 7 \cdot 2^{-3x-1} = 7$$

$\Rightarrow \qquad x = -\frac{1}{3}$

17. $S = \sum_{r=1}^{n} (-1)^{r-1}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{r}\right) {}^nC_r$

$$= \sum_{r=1}^{n}\left({}^nC_r(-1)^{r-1}\int_0^1 (1+x+x^2+\ldots.+x^{r-1})dx\right)$$

$$= \sum_{r=0}^{n}\int_0^1\left(\frac{1-x^r}{1-x}\right) {}^nC_r(-1)^{r-1}\, dx$$

$$= -\int_0^1 \left(\sum_{r=0}^{n} \frac{{}^nC_r (1-x^r)(-1)^r}{1-x} \right) dx$$

$$= -\int_0^1 \frac{0-(1-x)^n}{(1-x)} dx = \int_0^1 (1-x)^{n-1} dx = \int_0^1 x^{n-1} dx = \frac{1}{n}$$

18. $\sum_{r=0}^{n} {}^nC_r (1-x)^r (-1)^r = x^n$

$\Rightarrow \quad \sum_{r=1}^{n} {}^nC_r (-1)^{r-1} (1-x)^{r-1} = \frac{1-x^n}{1-x}$

$\Rightarrow \quad \int_1^x \left(\sum_{r=1}^{n} {}^nC_r (-1)^{r-1} (1-x)^{r-1} \right) dx = \int_1^x (1+x+x^2+x^3+\ldots+x^{n-1}) dx$

$\Rightarrow \quad -\sum_{r=1}^{n} \frac{{}^nC_r (-1)^{r-1} (1-x)^r}{r} = \frac{x-1}{1} + \frac{x^2-1}{2} + \frac{x^3-1}{3} + \ldots + \frac{x^n-1}{n}$

$\Rightarrow \quad \sum_{r=1}^{n} \frac{{}^nC_r (1-x)^r (-1)^{r-1}}{r} = \frac{1-x}{1} + \frac{1-x^2}{2} + \frac{1-x^3}{3} + \ldots + \frac{(1-x^n)}{n}$

19. $S = {}^nC_0 - ({}^{n-1}C_0 + {}^{n-1}C_1) + ({}^{n-1}C_1 + {}^{n-1}C_2) - ({}^{n-1}C_2 + {}^{n-1}C_3) + \ldots + \ldots + (-1)^{m-1} ({}^{n-1}C_{m-2} + {}^{n-1}C_{m-1})$

$= (-1)^{m-1}\, {}^{n-1}C_{m-1}$

20. $\sum_{r=0}^{n} r^3 \left(\frac{n+1-r}{r} \right)^2 = \sum_{r=0}^{n} r(n+1-r)^2 = \sum_{r=0}^{n} (n-r)(r+1)^2$

$$= (n+1) \sum_{r=0}^{n} (r+1)^2 - \sum_{r=0}^{n} (r+1)^3$$

$$= \frac{(n+1)^2 (n+2)(2n+3)}{6} - \frac{(n+1)^2 (n+2)^2}{4}$$

$$= \frac{n(n+1)^2 (n+2)}{12}$$

21. Coefficient of $x^n = {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n$

$$= \frac{1}{2}\, 2^{2n+1} = 2^{2n}$$

22. $S=\sum_{i=0}^{n-1}\left(\sum_{j=i+1}^{n}[(n-j)^nC_{n-i}+(n-i)^nC_{n-j}]\right)$

$$= n\sum_{i=0}^{n-1}\left(\sum_{j=i+1}^{n}({}^nC_i+{}^nC_j)\right)-S$$

$\Rightarrow$ $$S=\frac{n^2}{2}\sum_{i=0}^{n}{}^nC_i = n^2 2^{n-1}$$

23. $\left(\frac{1}{{}^nC_0}-\frac{1}{{}^nC_n}\right)+\left(-\frac{1}{{}^nC_1}+\frac{1}{{}^nC_{n-1}}\right)+\ldots..=0$

If r is odd, n – r is even

24. $\sum_{r=0}^{n}\frac{{}^nC_r}{r+1}a^{n-r}b^r=\frac{1}{n+1}\sum_{r=0}^{n}{}^{n+1}C_{r+1}a^{n-r}b^r$

$$=\frac{1}{(n+1)b}[(a+b)^{n+1}-a^{n+1}]$$

Put $a = x^2$, $b = 2 - x^2$

$\Rightarrow$ $$\sum_{r=0}^{n}\frac{{}^nC_r}{r+1}x^{2n-2r}(2-x^2)^r=\frac{2^{n+1}-x^{2n+2}}{(n+1)(2-x^2)}$$

25. $n\sum_{r=1}^{n}r^2\,{}^{n-1}C_{r-1}p^rq^{n-r}=n\sum_{r=2}^{n}(r+1)(r-1)\,{}^{n-1}C_{r-1}p^rq^{n-r}+n\sum_{r=1}^{n}{}^{n-1}C_{r-1}p^rq^{n-r}$

$$= n(n-1)\sum_{r=3}^{n}(r-2)\,{}^{n-2}C_{r-2}p^rq^{n-r}+3n(n-1)\sum_{r=2}^{n}{}^{n-2}C_{r-2}p^rq^{n-r}+n\sum_{r=1}^{n}{}^{n-1}C_{r-1}p^rq^{n-r}$$

$$= n(n-1)(n-2)\sum_{r=3}^{n}{}^{n-3}C_{r-3}p^rq^{n-r}+3n(n-1)p^2(q+p)^{n-2}+np(q+p)^{n-1}$$

$$= n(n-1)(n-2)p^3+3n(n-1)p^2+np$$

$$= np((n-1)(n-2)p^2+3(n-1)p+1$$

26. $T_{r+1}\ge T_r \quad\Rightarrow\quad {}^{18}C_r\left(\frac{1}{2}\right)^{18-r}\left(\frac{1}{3}\right)^{r}\ge {}^{18}C_{r-1}\left(\frac{1}{2}\right)^{19-r}\left(\frac{1}{3}\right)^{r-1}$

$\Rightarrow$ $\quad 2(19-r)\ge 3r \quad\Rightarrow\quad r\le\frac{38}{5}$

$$T_{r+1}\ge T_{r+2}\quad\Rightarrow\quad r+1\ge\frac{38}{5}$$

$\Rightarrow \quad r \geq \frac{33}{5} \quad \Rightarrow \quad r = 7$

$$\text{Greatest coefficient} = {}^{18}C_7\left(\frac{1}{2}\right)^{11}\left(\frac{1}{3}\right)^{7}$$

27. $\frac{{}^nC_r 2^r}{{}^nC_{r+1} 2^{r+1}} = \frac{1}{4} \quad \Rightarrow \quad n = 2 + 3r$

$\frac{{}^nC_{r+1} 2^{r+1}}{{}^nC_{r+2} 2^{r+2}} = \frac{4}{10} \quad \Rightarrow \quad 4n = 9r + 14$

$\Rightarrow \quad n = 8$

28. $a = {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n$

$b = {}^{2n}C_n$

29. Coefficient of x^r in $\left(\frac{(x+a)^{n-1}\left(\left(\frac{x+b}{x+a}\right)^n - 1\right)}{\frac{x+b}{x+a} - 1}\right)$

$= \text{coefficient of } x^r \text{ in } [(x+a)^n - (x+b)^n]$

$= {}^nC_{n-r}(a^{n-r} - b^{n-r})$

30. $T_{r+1} = {}^{2n}C_r(x\sin\alpha)^{2n-r}\left(\frac{\cos\alpha}{x}\right)^r$

For term independent of x, r = n

$\Rightarrow \quad T_{n+1} = {}^{2n}C_n\left(\frac{\sin 2\alpha}{2}\right)^n \leq \frac{{}^{2n}C_n}{2^n}$

SECTION-2

ONE OR MORE THAN ONE CORRECT

1. $\sum_{r=1}^{n}(r\,{}^nC_r)[(n-r)\,{}^nC_r] = n\sum_{r=1}^{n}\left[{}^{n-1}C_{r-1}\left((n-r)\,{}^nC_{n-r}\right)\right]$

$$= n^2\sum_{r=1}^{n} {}^{n-1}C_{r-1}\,{}^{n-1}C_{n-r-1}$$

$$= n^2\, {}^{2n-2}C_{n-2} = n(n-1)\, {}^{2n-2}C_{n-1}$$

$$\left[\because n^2\, {}^{2n-2}C_{n-2} = n^2 \frac{(2n-2)!}{n!(n-2)!} = \frac{n(n-1)(2n-2)!}{(n-1)!(n-1)!} = n(n-1)\, {}^{2n-2}C_{n-1}\right]$$

2. $-\sum_{r=1}^{n} (-1)^r r\, {}^nC_r^2 = -n\sum_{r=1}^{n} (-1)^r\, {}^{n-1}C_{r-1}\, {}^nC_r$

$$= -n \text{ coefficient of } x^n \text{ in } (1-x^2)^{n-1}(1-x)$$

3. $S = \sum_{r=0}^{n-1} \frac{{}^nC_r(-1)^{n-(r+1)}}{r+1} = \frac{(-1)^{n-1}}{(n+1)} \sum_{r=0}^{n-1} {}^{n+1}C_{r+1}(-1)^r$

$$= \frac{(-1)^n}{n+1}\left[(1-1)^{n+1} - 1 - (-1)^{n+1}\right]$$

$$= \frac{[(-1)^n - 1](-1)^n}{n+1} = \begin{cases} 0 & \text{if } n \text{ is even} \\ \dfrac{2}{n+1} & \text{if } n \text{ is odd} \end{cases}$$

4. $(x^3-1)^n = (x-1)^n(1+x+x^2)^n$

Coefficient of x^n in $(x^3-1)^n = {}^nC_0a_0 - {}^nC_1a_1 + {}^nC_2a_2 - {}^nC_3a_3 + \ldots + (-1)^n\,{}^nC_na_n$

$\Rightarrow$ $\sum_{r=0}^{n} {}^nC_r(-1)^r a_r$ = coefficient of x^n in $(x^3-1)^n$

$$= \begin{cases} {}^nC_{\frac{2n}{3}}(-1)^{\frac{2n}{3}} = {}^nC_{\frac{n}{3}} & \text{if } n \text{ is multiple of } 3 \\ 0 & \text{if } n \text{ is not multiple of } 3 \end{cases}$$

5. $S = \sum_{r=0}^{9} \frac{{}^9C_r(-1)^r}{r+8}$

$$x^7(1-x)^9 = \sum_{r=0}^{9} {}^9C_r(-1)^r x^{r+7}$$

$$\int_0^1 x^7(1-x)^9\,dx = \sum_{r=0}^{9} \frac{{}^9C_r(-1)^r}{r+8} = \frac{7!\,9!}{17!}$$

$$= \frac{7\times6\times5\times4\times3\times2}{17\times16\times15\times14\times13\times12\times11\times10} = \frac{1}{17\times16\times13\times11\times5}$$

6. $\alpha = {}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + ...$

$$\beta = {}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5 + ...$$

$$\alpha + \beta = (x + a)^n \text{ and } \alpha - \beta = (x - a)^n$$

$$(x + a)^{2n} - (x - a)^{2n} = (\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$$

$$(x + a)^{2n} + (x - a)^{2n} = (\alpha + \beta)^2 + (\alpha - \beta)^2 = 2(\alpha^2 + \beta^2)$$

$$(x + a)^n (x - a)^n = (x^2 - a^2)^n = (\alpha + \beta)(\alpha - \beta) = \alpha^2 - \beta^2$$

7. $T_{r+1} > T_r \quad \Rightarrow \quad {}^{24}C_rx^r > {}^{24}C_{r-1}x^{r-1}$

$$\Rightarrow \quad \frac{(25-r)x}{r} > 1$$

$$\Rightarrow \quad x > \frac{r}{25-r}$$

$T_{r+1} > T_{r+2} \quad \Rightarrow \quad x < \dfrac{r+1}{24-r}$

Put r = 12

$\therefore \quad \Rightarrow \quad \dfrac{12}{13} < x < \dfrac{13}{12}$

8. $({}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m + ... + {}^mC_m) + 2({}^{n-1}C_m + {}^{n-2}C_m + ... + {}^mC_m)$

$$+ 2({}^{n-2}C_m + {}^{n-3}C_m + ... + {}^mC_m) + ... + 2({}^{m+1}C_m + {}^mC_m) + 2({}^mC_m)$$

$$= {}^{n+1}C_{m+1} + 2({}^nC_{m+1} + {}^{n-1}C_{m+1} + {}^{n-2}C_{m+1} + ... + {}^{m+2}C_{m+1} + {}^{m+1}C_{m+1})$$

$$= {}^{n+1}C_{m+1} + 2\,{}^{n+1}C_{m+2} = {}^{n+1}C_{m+2} + {}^{n+2}C_{m+2}$$

9. $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$

$x \to -\dfrac{1}{x} \qquad (1 - x + x^2)^n = a_0x^{2n} - a_1x^{2n-1} + a_2x^{2n-2} - a_3x^{2n-3} + ... + a_{2n}$

$$a_0a_1 - a_1a_2 + a_2a_3 - a_3a_4 + ... - a_{2n-1}a_{2n}$$

$$= \text{coefficient of } x^{2n+1} \text{ in } (1 + x^2 + x^4)^n = 0$$

10. $\dfrac{1}{\sqrt{4x+1}}\left[\left(\dfrac{1+\sqrt{4x+1}}{2}\right)^n - \left(\dfrac{1-\sqrt{4x+1}}{2}\right)^n\right]$

$$= \frac{1}{2^{n-1}}[{}^nC_1 + {}^nC_3(4x + 1) + {}^nC_5(4x + 1)^2 + ... + {}^nC_{11}(4x + 1)^5]$$

$\Rightarrow \qquad n = 11 \text{ or } 12$

11. $$2!\sum_{r=0}^{n}\frac{{}^{n}C_r(-2)^r}{(r+2)(r+1)} = \frac{2}{(n+1)(n+2)}\frac{1}{4}\sum_{r=0}^{n}{}^{n+2}C_{r+2}(-2)^{r+2}$$

$$= \frac{1}{2(n+1)(n+2)}\left[(1-2)^{n+2} - 1 + (n+2)2\right]$$

$$= \frac{(-1)^n + 2n + 3}{2(n+1)(n+2)} = \begin{cases} \dfrac{1}{n+1} & \text{if } n \text{ is even} \\ \dfrac{1}{n+2} & \text{if } n \text{ is odd} \end{cases}$$

12. $n(1 + cx + x^2)^{n-1}(c + 2x) = \sum_{r=0}^{2n} r a_r x^{r-1}$

Put $x = -1$, $\quad -n(2 - c)^n = \sum_{r=0}^{2n} r a_r (-1)^{r-1}$

Put $x = 1$, $\quad n(2 + c)^n = \sum_{r=0}^{2n} r . a_r$

$\Rightarrow \quad \sum_{r=0}^{2n}(2r+1)a_r = 2n(2 + c)^n + (2 + c)^n = (2n + 1)(2 + c)^n$

13. $a_r = a_{2n-r}\ \forall\ r \in \{0, 1, 2, ..., n\}$

$a_0 + a_1 + a_2 + ... + a_{n-1} + a_n + a_{n+1} + ... + a_{2n} = 3^n$

$\Rightarrow \quad a_0 + a_1 + a_2 + ... + a_{n-1} = \frac{(3^n - a_n)}{2}$

$$(1 - x + x^2)^n = a_0x^{2n} - a_1x^{2n-1} + a_2x^{2n-2} - a_3x^{2n-3} + ... + a_{2n}$$

$\Rightarrow \quad a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + a_{2n}^2 = \text{coefficient of } x^{2n} \text{ in } (1 + x^2 + x^4)^n$

$$= a_n$$

$\Rightarrow \quad a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + (-1)^{n-1}a_{n-1}^2 = \frac{a_n - (-1)^n a_n^2}{2}$

$$n(1 + 2x)(1 + x + x^2)^{n-1} = \sum_{r=1}^{2n} r a_r x^{r-1}$$

$$n(1 + 2x)(1 + x + x^2)^n = (1 + x + x^2)\sum_{r=1}^{2n} r a_r x^{r-1}$$

Equating coefficient of x^r from both the sides

$$n(a_r + 2a_{r-1}) = (r+1)a_{r+1} + ra_r + (r-1)a_{r-1}$$

$$\Rightarrow \quad (r+1)a_{r+1} = (n-r)a_r + (2n-r+1)a_{r-1}$$

14. $\left({}^nC_1\,{}^nC_2 \ldots {}^nC_{n-1}\right)^{\frac{1}{n-1}} < \dfrac{{}^nC_1 + {}^nC_2 + \ldots + {}^nC_{n-1}}{n-1}$

$$\Rightarrow \quad P < \left(\frac{2^n - 2}{n-1}\right)^{n-1}$$

Also, $\left({}^nC_0\,{}^nC_1\,{}^nC_2 \ldots {}^nC_{n-1}\,{}^nC_n\right)^{\frac{1}{n+1}} < \dfrac{{}^nC_0 + {}^nC_1 + C_2 + \ldots + {}^nC_n}{n+1}$

$$\Rightarrow \quad P < \left(\frac{2^n}{n+1}\right)^{n+1}$$

15. ${}^{2n+1}C_0^2 + {}^{2n+1}C_1^2 + {}^{2n+1}C_2^2 + \ldots + {}^{2n+1}C_{2n+1}^2 = {}^{4n+2}C_{2n+1}$

${}^{2n+1}C_0^2 - {}^{2n+1}C_1^2 + {}^{2n+1}C_2^2 - {}^{2n+1}C_3^2 + \ldots - {}^{2n+1}C_{2n+1}^2 = 0$

$$\Rightarrow \quad \sum_{r=0}^{n} {}^{2n+1}C_{2r}^2 = \sum_{r=0}^{n} {}^{2n+1}C_{2r+1} = \frac{1}{2}\left({}^{4n+2}C_{2n+1}\right)$$

16. $(25^n - 8^n) - (20^n - 3^n)$

$= 17[25^{n-1} + 25^{n-2}8 + 25^{n-3}8^2 + \ldots + 8^{n-1} - 20^{n-1} - 20^{n-2}3 - \ldots - 3^{n-1}]$

$= 17 \times 5k = 85\ k$

17. If $m < k$

Coefficient of $x^m = {}^kC_m + {}^{k+1}C_m + {}^{k+2}C_m + \ldots + {}^nC_m$

$= -{}^kC_{m+1} + ({}^kC_{m+1} + {}^kC_m) + {}^{k+1}C_m + \ldots + {}^nC_m$

$= {}^{n+1}C_{m+1} - {}^kC_{m+1}$

If $k < m < n$

Coefficient of $x^m = {}^mC_m + {}^{m+1}C_m + {}^{m+2}C_m + \ldots + {}^nC_m$

$= {}^{n+1}C_{m+1}$

18. Coefficient of x^{10} in $(1-x^3)^{50}(1-3x+3x^2-x^3)$

$$= -3(-{}^{50}C_3) = 3({}^{50}C_3)$$

Coefficient of x^{10} in $\dfrac{(1+x^4)^{30}(3+2x^2)}{x^{60}} = 2\ {}^{30}C_{17}$

19. Coefficient of $x^{50} = {}^{1000}C_{50} + 2\ {}^{999}C_{49} + 3\ {}^{998}C_{48} + ... + 49\ {}^{952}C_2 + 50\ {}^{951}C_1 + 51\ {}^{950}C_0$

$$= \left({}^{950}C_0 + {}^{951}C_1 + {}^{952}C_2 + ... + {}^{1000}C_{50}\right) + \left({}^{950}C_0 + {}^{951}C_1 + {}^{952}C_2 + ... + {}^{999}C_{49}\right)$$

$$+ ...... + \left({}^{950}C_0 + {}^{951}C_1\right) + {}^{950}C_0$$

$$= {}^{1001}C_{50} + {}^{1000}C_{49} + {}^{999}C_{48} + ... + {}^{953}C_2 + {}^{952}C_1 + {}^{952}C_0$$

$$= {}^{1002}C_{50}$$

20. ${}^5C_2 x^3 x^{2\log_{10}x} = 106$

$\Rightarrow$ $$x^{3+2\log_{10}x} = 10^5$$

$\Rightarrow$ $$(3+2\log_{10}x)\log_{10}x = 5$$

$\Rightarrow$ $$\log_{10}x = -\frac{5}{2}, 1$$

$\Rightarrow$ $$x = 10^{-5/2}, 10$$

21. $P_n = {}^nC_0 - {}^nC_2 + {}^nC_4 - {}^nC_6 + ...$

$$= \text{Re}\left[(1+i)^n\right] = \text{Re}\left(2^{n/2}\left(e^{i\frac{\pi}{4}}\right)^n\right)$$

$$= 2^{n/2}\cos\left(\frac{n\pi}{4}\right)$$

$$Q_n = {}^nC_1 - {}^nC_3 + {}^nC_5 - {}^nC_7 + ...$$

$$= \text{Im}\left((1+i)^n\right) = 2^{n/2}\sin\left(\frac{n\pi}{4}\right)$$

22. $f(x) = (1+x^2-x^3)^{1000} = a_0 + a_1x + ... + a_{20}x^{20} + ...$

$f(-x) = (1+x^2+x^3)^{1000} = a_0 - a_1x + ... + a_{20}x^{20} + ...$

$a = d$

$g(x) = (1-x^2+x^3)^{1000} = b_0 + b_1x + ... + b_{20}x^{20} + ...$

$g(-x) = (1 - x^2 - x^3)^{1000} = b_0 - b_1x + ... + b_{20}x^{20} + ...$

$\Rightarrow\ b = c$

Clearly d is the largest.

23. $\beta_{100} = {}^{100}C_{100} + {}^{101}C_{100} + {}^{102}C_{100} + ... {}^{200}C_{100}$

$= {}^{201}C_{101}$

$\beta_{99} = {}^{99}C_{99} + {}^{100}C_{99} + {}^{101}C_{99} + ... + {}^{200}C_{99}$

$= {}^{201}C_{100}$

24. $\left(5\sqrt{3}+8\right)^{2n+1} - \left(5\sqrt{3}-8\right)^{2n+1} = 2\left({}^{2n+1}C_1(5\sqrt{3})^{2n}8 + {}^{2n+1}C_3(5\sqrt{3})^{2n-2}8^3 + ...\right)$

$\Rightarrow\quad [x] + \{x\} - N = 2k,\ k \in I$

$0 < \{x\} < 1 \quad \text{and } 0 < N < 1$

$\Rightarrow\quad -1 < \{x\} - N < 1 \ \text{ and } \{x\} - N \in I$

$\Rightarrow\quad \{x\} - N = 0$

$\Rightarrow\quad xN = x\{x\} = (11)^{2n+1}$

25. $a_0 + a_2 + a_4 + ... + a_{38} + a_{40} = \dfrac{4^{20} + 2^{20}}{2} = 2^{19}(2^{20} + 1)$

$a_{40} = 2^{20}$

$\Rightarrow\quad N = 2^{19}(2^{20} - 1)$

26. Put $x = i$ $\quad\Rightarrow\quad i^n = (a_0 - a_2 + a_4 - a_6 + ...) + i\,(a_1 - a_3 + a_5 - a_7 + ...)$

$$\Rightarrow\quad a_1 - a_3 + a_5 - a_7 + ... = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n = 4k+1,\ k \in I \\ -1 & \text{if } n = 4k+3 \end{cases}$$

27. $(13^n - 3^n) + 7^n = 10k + 7^n$

$\Rightarrow$ last digit is 3 if $n = 4k + 3,\ k \in I$

28. $2^{4n} - 2^n(7n + 1) = 2^n[(1 + 7)^n - 7n - 1]$

$$= 2^n({}^nC_2 7^2 + {}^nC_3 7^3 + ...)$$

29. $S_n = \sum_{k=0}^{n} \frac{{}^{n+k}C_n}{2^k}$

$$= \text{coefficient of } x^n \text{ in } \left((1+x)^n + \frac{(1+x)^{n+1}}{2} + \frac{(1+x)^{n+2}}{2^2} + ... + \frac{(1+x)^{n+n}}{2^n}\right)$$

$$= \text{coefficient of } x^n \text{ in } \frac{(1+x)^n\left(\left(\frac{1+x}{2}\right)^{n+1} - 1\right)}{\left(\frac{1+x}{2} - 1\right)}$$

$$= \text{coefficient of } x^n \text{ in } \frac{1}{2^n}\frac{\left((1+x)^{2n+1} - 2^{n+1}(1+x)^n\right)}{(x-1)}$$

$$= \text{coefficient of } x^n \text{ in } \frac{1}{2^n}(2^{n+1}(1+x)^n - (1+x)^{2n+1})(1 + x + x^2 + ... \infty)$$

$$= \frac{1}{2^n}[2^{n+1}({}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n) - ({}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n)]$$

$$= \frac{1}{2^n}\left[2^{2n+1} - \frac{1}{2}2^{2n+1}\right] = 2^n$$

30. Coefficient of $x^t = {}^tC_t + {}^{t+1}C_t + {}^{t+2}C_t + ... + {}^nC_t = {}^{n+1}C_{t+1}$

Coefficient of $x^{t+1} = {}^{t+1}C_{t+1} + {}^{t+2}C_{t+1} + {}^{t+3}C_{t+1} + ... + {}^nC_{t+1} = {}^{n+1}C_{t+2}$

${}^{n+1}C_{t+1} = {}^{n+1}C_{t+2} \Rightarrow \quad n - t = t + 2$

$\Rightarrow \quad t = \frac{n-2}{2}$ their sum $= {}^{n+2}C_{t+2}$

SECTION-3

COMPREHENSION TYPE QUESTIONS

COMPREHENSION (Q.1 TO Q.2):

$$\sum_{r=0}^{n} {}^nC_r a^{n-r} b^r e^{i(rA-(n-r)B)} = \sum_{r=0}^{n} {}^nC_r (ae^{-iB})^{n-r} (be^{iA})^r$$

$$= (ae^{-iB} + be^{iA})^n$$

$$= [(a\cos B + b\cos A) + i(b\sin A - a\sin B)]^n$$

$$= c^n$$

COMPREHENSION-2 (Q.3 TO Q.5) :

$$\sum_{r=0}^{n} \frac{r^2}{{}^nC_r} = a_n + \sum_{r=0}^{n} \frac{(r-1)(r+1)}{{}^nC_r}$$

$$= a_n + (n+1) \sum_{r=0}^{n} \frac{r+2-3}{{}^{n+1}C_{r+1}}$$

$$= a_n + (n+1)(n+2) \sum_{r=0}^{n} \frac{1}{{}^{n+2}C_{r+2}} - 3(n+1) \sum_{r=0}^{n} \frac{1}{{}^{n+1}C_{r+1}}$$

$$= a_n + (n+1)(n+2)\left(a_{n+2} - 1 - \frac{1}{n+2}\right) - 3(n+1)(a_{n+1} - 1)$$

$$= a_n + (n+1)(n+2)a_{n+2} - 3(n+1)a_{n+1} - n(n+1)$$

COMPREHENSION-3 (Q.6 TO Q.8) :

Clearly $A_k = 4$

$\therefore \quad 4 + B_k x + C_k x^2 + D_k x^3 + \ldots = (4 + B_{k-1}x + C_{k-1}x^2 + \ldots -2)^2$

$$= 4 + 4B_{k-1}x + (4C_{k-1} + B_{k-1}^2)x^2 + \ldots$$

$\Rightarrow \quad B_k = 4B_{k-1}$ and $C_k = 4C_{k-1} + B_{k-1}^2$

$\therefore \quad B_k = 4B_{k-1} = 4^2 B_{k-2} = \ldots = 4^{k-1}B_1 = -4^k$

$$C_k = B_{k-1}^2 + 4C_{k-1} = 4^{2k-2} + 4C_{k-1}$$
$$= 4^{2k-2} + 4(4^{2k-4}) + 4^2\, C_{k-2}$$
$$= 4^{2k-2} + 4^{2k-3} + 4^2\,(4^{2k-6} + 4C_{k-3})$$
$$= 4^{2k-2} + 4^{2k-3} + 4^{2k-4} + 4^3 C_{k-3}$$
$$= 4^{2k-2} + 4^{2k-3} + 4^{2k-4} + \ldots + 4^k + 4^{k-1}C_1$$
$$= 4^{2k-2} + 4^{2k-3} + 4^{2k-4} + \ldots + 4^k + 4^{k-1}$$
$$= \frac{4^{k-1}(4^k-1)}{(4-1)} = \frac{4^{2k-1}-4^{k-1}}{3}$$

COMPREHENSION-4 (Q.9 TO Q.10) :

9. $\text{Sum} = 1\,{}^{n-1}C_{r-1} + 2\,{}^{n-2}C_{r-1} + 3\,{}^{n-3}C_{r-1} + \ldots (n-r+1)^{r-1}C_{r-1}$

$$= ({}^{r-1}C_{r-1} + {}^{r}C_{r-1} + {}^{r+1}C_{r-1} + \ldots + {}^{n-1}C_{r-1}) + ({}^{r-1}C_{r-1} + {}^{r}C_{r-1} + \ldots + {}^{n-2}C_{r-1})$$
$$+ \ldots({}^{r-1}C_{r-1} + {}^{r}C_{r-1}) + {}^{r-1}C_{r-1}$$
$$= {}^{n}C_r + {}^{n-1}C_r + {}^{n-2}C_r + \ldots + {}^{r+2}C_r + {}^{r+1}C_r + {}^{r}C_r$$
$$= {}^{n+1}C_{r+1}$$

10. $\text{A.M.} = \dfrac{{}^{n+1}C_{r+1}}{{}^{n}C_r} = \dfrac{n+1}{r+1}\,\dfrac{(r+1)\,{}^{n+1}C_{r+1}}{(n+1)\,{}^{n}C_r} = \dfrac{n+1}{r+1}$

SECTION-4

MATCH THE COLUMN

1. (A) $S = \dfrac{1}{n} \displaystyle\sum_{0\le i<j\le n}\sum (n-i+n-j)\,{}^{n}C_{n-i}\,{}^{n}C_{n-j} = 2\sum_{0\le i<j\le n}\sum {}^{n}C_i\,{}^{n}C_j - S$

$\Rightarrow$ $S = \displaystyle\sum_{0\le i<j\le n}\sum {}^{n}C_i\,{}^{n}C_j = \frac{1}{2}\left[\left(\sum_{i=0}^{n}{}^{n}C_i\right)^2 - \sum_{i=0}^{n}{}^{n}C_i^2\right] = \frac{1}{2}\,(2^{2n} - {}^{2n}C_n)$

(B) $S = n[(1\,{}^{n}C_1) + (2.\,{}^{n}C_2) + (3\,{}^{n}C_3) + \ldots((n-1)\,{}^{n}C_{n-1}) + (n\,{}^{n}C_n)]$

$$= n\sum_{r=1}^{n} r\,{}^{n}C_r = n^2\sum_{r=1}^{n}{}^{n-1}C_{r-1} = n^2\,2^{n-1}$$

Alternate : $S = \sum_{0 \le i < j \le n}\sum (i\,{}^nC_i + j\,{}^nC_j)$

$$S = \sum_{0 \le i < j \le n}\sum ((n-i)\,{}^nC_{n-i} + (n-j)\,{}^nC_{n-j})$$

$$\Rightarrow \quad 2S = n\sum_{0 \le i < j \le n}\sum ({}^nC_i + {}^nC_j) = n^2\sum_{i=0}^{n}{}^nC_i = n^2 2^n$$

$$\Rightarrow \quad S = n^2\, 2^{n-1}$$

(C)
$$S = \sum_{0 \le i < j \le n}\sum \left((n-i+n-j)({}^nC_{n-i} - {}^nC_{n-j})^2\right)$$

$$2S = 2n\sum_{0 \le i < i \le n}\sum ({}^nC_i - {}^nC_j)^2 = 2n\left[n\sum_{i=0}^{n}{}^nC_i^2 - 2\sum_{0 \le i < j \le n}\sum {}^nC_i\,{}^nC_j\right]$$

$$S = n\left[(n+1)\sum_{i=0}^{n}{}^nC_i^2 - ({}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n)^2\right]$$

$$= [(n+1)\,{}^{2n}C_n - 2^{2n}]n$$

2. (A) $32^{32} = 2^{160} = (3-1)^{160} = 3k + 1$

$2^{(32)^{32}} = 2\cdot(1+7)^k = 7\mu + 2$

(D) $32^{(32)^{32}} = 2^{5(3k+1)} = 4(1+7)^{\lambda} = 7\mu + 4$

(B) $5^{99} = 5(26-1)^{49} = 13k - 5 = 13\mu + 8$

(C) $(18+2)^{13} + (9+4)^{20} = 9k + 2^{13} + 4^{20} = 9k + 2\,(9-1)^4 + 2(9-1)^{13}$

$= 9\mu + 2 - 2 = 9\mu$

3. (A) ${}^nC_3\left(\frac{x}{a}\right)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2} \quad \Rightarrow \quad n - 6 = 0$

$\Rightarrow \quad a^3 = 8 \quad \Rightarrow \quad a = 2$

(B) $\sum_{p=1}^{4}\sum_{r=p}^{4}\frac{4!}{r!(4-r)!}\frac{r!}{(r-p)!p!} = \sum_{p=1}^{4}{}^4C_p\sum_{r=p}\frac{(4-p)!}{(r-p)!(4-r)!}$

$= \sum_{p=1}^{4}{}^4C_p 2^{4-p} = 3^4 - 2^4 = 65$

(C) Coefficient of x^{13} in $(1-x)(1-x^4)^4 = -(-{}^4C_3) = 4$

(D) $\sum_{r=0}^{4}{}^4C_r(r-2)^2 = 16$

4. (A) Coefficient of $x^4 = \frac{6!}{2!\,4!} 2^2 + \frac{6!}{3!\,2!} 2^3\, 3 + \frac{6!}{2!\,4!} 2^4\, 3^2 = 3660$

(B) ${}^{11}C_0\, {}^{22}C_{11} - {}^{11}C_1\, {}^{20}C_{11} + {}^{11}C_2\, {}^{18}C_{11} - {}^{11}C_3\, {}^{16}C_{11} + \ldots$

$= \text{coefficient of } x^{11} \text{ in } {}^{11}C_0(1+x)^{22} - {}^{11}C_1(1+x)^{20} + \ldots$

$= \text{coefficient of } x^{11} \text{ in } ((1+x)^2 - 1)^{11}$

$= \text{coefficient of } x^{11} \text{ in } x^{11}(2+x)^{11} = 2^{11} = 2048$

(C) ${}^5C_1\, {}^5C_5 - {}^5C_2\, {}^{10}C_5 + \ldots + {}^5C_5\, {}^{25}C_5$

$= \text{coefficient of } x^5 \text{ in } [1 - (1 - (1+x)^5)^5]$

$= \text{coefficient of } x^5 \text{ in } [1 - (-x\,(1 + (1+x) + (1+x)^2 + (1+x)^3 + (1+x)^4))^5]$

$= \text{coefficient of } x^5 \text{ in } [1 + x^5\,(1 + (1+x) + (1+x)^2 + (1+x)^3 + (1+x)^4)^5]$

$= 5^5 = 3125$

(D) Coefficient of $x^4 = \frac{11!}{4!\,7!} + \frac{11!}{2!\,8!} + \frac{11!}{9!} + \frac{11!}{2!\,9!}$

$= 990$

5. (A) $(\sqrt{2}-1)^{10} + (\sqrt{2}+1)^{10} = 2({}^{10}C_0 2^5 + {}^{10}C_2 2^4 + {}^{10}C_4 2^3 + {}^{10}C_6\, 2^2 + {}^{10}C_8 2 + {}^{10}C_{10})$

$= 6726$

$\Rightarrow \quad x + \frac{1}{x} = 6726$

$x = (\sqrt{2}-1)^{10}$

$\Rightarrow \quad x^2 - 6726x + 1 = 0$

$\Rightarrow \quad x = 3363 - \sqrt{(3363)^2 - 1}$

(B) $(\sqrt{5}+2)^7 - (\sqrt{5}-2)^7 = 2({}^7C_1 5^3\, 2 + {}^7C_3\, 5^2\, 2^3 + {}^7C_5\, 5.2^5 + {}^7C_7\, 2^7) = 24476$

Let $x = (\sqrt{5}+2)^7$

$x - \frac{1}{x} = 24476$

$\Rightarrow \quad x^2 - 24476x - 1 = 0$

$\Rightarrow \quad x = 12238 + \sqrt{(12238)^2 + 1}$

(C) $(17)^{256} = (290-1)^{128} = 1000\,k + {}^{128}C_{126}\, 290 - {}^{128}C_{127}(290) + 1$

$\Rightarrow \quad N = 681$

(D) Let $x = (2+\sqrt{3})^6,\ (2+\sqrt{3})^6 + (2-\sqrt{3})^6$

$$= 2\,({}^6C_0 2^6 + {}^6C_2\, 2^4\, 3 + {}^6C_4\, 2^2\, 3^2 + {}^6C_6\, 3^3) = 2702$$

$$x + \frac{1}{x} = 2702$$

$\Rightarrow$ $x^2 - 2702x + 1 = 0$

$\Rightarrow$ $x = 1351 + \sqrt{(1351)^2 - 1}$

SECTION-5

SUBJECTIVE TYPE QUESTOINS

1. $S = \sum_{k=1}^{n} (-1)^{k-1}\, {}^nC_k \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}\right)$

$$= \sum_{k=1}^{n}\left((-1)^{k-1}\, {}^nC_k \sum_{r=1}^{k}\int_0^1 x^{r-1}dx\right)$$

$$= \sum_{k=1}^{n}(-1)^{k-1}\, {}^nC_k \int_0^1 \frac{1-x^k}{1-x}dx$$

$$= -\int_0^1 \sum_{k=0}^{n}(-1)^k\, {}^nC_k\, \frac{(1-x^k)}{(1-x)}dx$$

$$= -\int_0^1 -\frac{(1-x)^n}{1-x}dx$$

$$= \int_0^1 (1-x)^{n-1}dx = \int_0^1 x^{n-1}dx = \frac{1}{n}$$

2. $\left(15+\sqrt{220}\right)^{19} + (15-\sqrt{220})^{19} = 2\,({}^{19}C_0\, 15^{19} + {}^{19}C_2\, 15^{17}\,(220) + \dots) = 10k_1$

$$\left(15+\sqrt{220}\right)^{82} + \left(15-\sqrt{220}\right)^{82} = 10\,k_2$$

and $\left(15-\sqrt{220}\right) = \frac{5}{15+\sqrt{220}} < \frac{1}{3}$

$\Rightarrow$ $\left(15-\sqrt{220}\right)^{19} + (15-\sqrt{220})^{82} < 1$

$\therefore$ $[x] = 10k - 1$

3. $x = (\sqrt{3}+1)^{2018}$ and $y = (\sqrt{3}-1)^{2018}$

$$[x] + \{x\} + y = (\sqrt{3}+1)^{2018} + (\sqrt{3}-1)^{2018}$$

$$[x] + 1 = 2^{1009}\left[(2+\sqrt{3})^{1009} + (2-\sqrt{3})^{1009}\right]$$

$$= 2^{1009} \times 2[{}^{1009}C_0\, 2^{1009} + {}^{1009}C_2\, 2^{1007}\, 3^1 + \ldots + {}^{1009}C_{1006} 2^3\, 3^{503} + {}^{1009}C_{1008}\, 2 \times 3^{504}]$$

$$[x] + 1 = 2^{1011}\underbrace{\left[{}^{1009}C_0 2^{1008} + \ldots + {}^{1009}C_{1006} 2^2\, 3^{503} + {}^{1009}C_{1008} 3^{504}\right]}_{\text{odd}}$$

$[\because\ \{x\} + y \in (0, 2) \Rightarrow \{x\} + y = 1]$

$\Rightarrow$ N is divisible by 2^{1011}, hence divisible by $(16)^{252}$

4. $S = {}^{50}C_6 - {}^5C_1\, {}^{40}C_6 + \ldots + {}^5C_4\, {}^{10}C_6$

$= \text{Coefficient of } x^6 \text{ in } {}^5C_0(1 + x)^{50} - {}^5C_1(1 + x)^{40} + \ldots + {}^5C_4(1 + x)^{10} - {}^5C_5(1 + x)^0$

$= \text{Coefficient of } x^6 \text{ in } ((1 + x)^{10} - 1)^5$

$= \text{Coefficient of } x^6 \text{ in } x^5\, ({}^{10}C_1 + {}^{10}C_2x + \ldots + {}^{10}C_{10}x^9)^5$

$= {}^5C_4\, ({}^{10}C_1)^4\, ({}^{10}C_2)$

$= 5 \times 10^4 \times 5 \times 9$

$= 2250000$

5. $(7 - 1)^{2007} + (7 + 1)^{2007} = 49k + 2007 \times 7 \times 2 = 49\lambda + 21$

6. ${}^{10}C_0\, {}^{20}C_{10} - {}^{10}C_1\, {}^{18}C_{10} + \ldots\ {}^{10}C_5\, {}^{10}C_{10}$

$= \text{Coefficient of } x^{10} \text{ in } {}^{10}C_0(1 + x)^{20} - {}^{10}C_1(1 + x)^{18} + {}^{10}C_2(1 + x)^{16} - {}^{10}C_3(1 + x)^{14} + \ldots - {}^{10}C_9(1 + x)^2 + {}^{10}C_{10}$

$= \text{Coefficient of } x^{10} \text{ in } ((1 + x)^2 - 1)^{10}$

$= \text{Coefficient of } x^{10} \text{ in } x^{10}\, (2 + x)^{10}$

$= 2^{10}$

7. $\dfrac{b}{a} = \dfrac{T_4}{T_3} = \dfrac{{}^nC_3x^3}{{}^nC_2x^2} = \dfrac{(n-2)x}{3}$

$\dfrac{c}{b} = \dfrac{T_5}{T_4} = \dfrac{{}^nC_4x^4}{{}^nC_3x^3} = \dfrac{(n-3)x}{4}$

$$\frac{d}{c}=\frac{T_6}{T_5}=\frac{{}^nC_5x^5}{{}^nC_4x^4}=\frac{(n-4)x}{5}$$

$$\frac{b^2-ac}{c^2-bd}=\frac{\left(\frac{b^2-ac}{ab}\right)}{\left(\frac{c^2-bd}{bc}\right)}\frac{ab}{bc}=\left(\frac{\frac{b}{a}-\frac{c}{b}}{\frac{c}{b}-\frac{d}{c}}\right)\frac{a}{c}$$

$$=\left(\frac{\frac{n-2}{3}-\frac{n-3}{4}}{\frac{n-3}{4}-\frac{n-4}{5}}\right)\frac{a}{c}=\frac{5}{3}\frac{a}{c}$$

8. Coefficient of x^5y^5 in

$$(1+x)^5(1+y)^5(x+y)^5=\sum_{r=0}^{5}{}^5C_r\,{}^5C_{5-r}\,{}^5C_{5-r}=\sum_{r=0}^{5}{}^5C_r^3$$

9. $T_{r+1}={}^{6561}C_r\,7^{\frac{6561-r}{3}}11^{\frac{r}{9}}$

$r=9k,\ k=0,1,2,3,\ldots,729$

$\Rightarrow$ Number of rational terms = 730

10. $S=\sum_{i=1}^{6}\left(\sum_{j=i}^{6}{}^6C_j\,{}^jC_i\right)=\sum_{j=1}^{6}\left({}^6C_j\sum_{i=1}^{j}{}^jC_i\right)$

$$=\sum_{j=0}^{6}{}^6C_j(2^j-1)=3^6-2^6=665$$

11. $S={}^{50}C_0({}^{51}C_1+{}^{51}C_2+\ldots{}^{51}C_{51})+{}^{50}C_1({}^{51}C_2+{}^{51}C_3+\ldots{}^{51}C_{51})+\ldots$

$+{}^{50}C_{49}({}^{51}C_{50}+{}^{51}C_{51})+{}^{50}C_{50}{}^{51}C_{51}$

$=({}^{50}C_0{}^{51}C_1+{}^{50}C_1{}^{51}C_2+\ldots+{}^{50}C_{50}{}^{51}C_{51})+({}^{50}C_0\,{}^{51}C_2+{}^{50}C_1\,{}^{51}C_3+\ldots+{}^{50}C_{49}{}^{51}C_{51})$

$+({}^{50}C_0{}^{51}C_3+{}^{50}C_1{}^{51}C_4+\ldots+{}^{50}C_{48}{}^{51}C_{51})+\ldots+({}^{50}C_0{}^{51}C_{50}+{}^{50}C_1{}^{51}C_{51})+{}^{50}C_0\,{}^{51}C_{51}$

$={}^{101}C_{50}+{}^{101}C_{49}+{}^{101}C_{48}+\ldots+{}^{101}C_2+{}^{101}C_1+{}^{101}C_0$

$=\frac{2^{101}}{2}=2^{100}$

12. $|T_{r+1}| > |T_r| \Rightarrow {}^{10}C_r\, 2^{10-r}\left(\frac{3}{8}|x|\right)^r > {}^{10}C_{r-1}\, 2^{11-r}\left(\frac{3}{8}|x|\right)^{r-1}$

$$\Rightarrow \frac{11-r}{r}\frac{3}{8}|x| > 2$$

$$\Rightarrow |x| > \frac{16r}{3(11-r)}$$

$|T_{r+1}| > |T_{r+2}| \Rightarrow |x| < \frac{16(r+1)}{3(10-r)}$

Put r = 3 $\Rightarrow 2 < |x| < \frac{64}{21}$

$$\Rightarrow x \in \left(-\frac{64}{21}, -2\right) \cup \left(2, \frac{64}{21}\right)$$

13. Coefficient of $x^{\frac{n(n+1)}{2}-7} = -7 + (1\cdot 6 + 2\cdot 5 + 3\cdot 4) - (1\cdot 2\cdot 4) = 13$

14. $\frac{3(27)^{673}}{28} = \frac{3(28-1)^{673}}{28} = \frac{28k+25}{28}$

$\Rightarrow \left\{\frac{3^{2020}}{28}\right\} = \frac{25}{28}$

15. $S = \sum_{r=1}^{2n-1} \frac{r(-1)^r}{{}^{2n}C_r} = -\frac{1}{{}^{2n}C_1} + \frac{2}{{}^{2n}C_2} - \frac{3}{{}^{2n}C_3} + \ldots\ldots\ldots\ldots - \frac{(2n-1)}{{}^{2n}C_{2n-1}}$ ...(1)

$S = -\frac{(2n-1)}{{}^{2n}C_{2n-1}} + \frac{2n-2}{{}^{2n}C_{2n-2}} - \frac{2n-3}{{}^{2n}C_3} + \ldots\ldots - \frac{1}{{}^{2n}C_1}$ ...(2)

Add (1) and (2)

$$S = n\left(-\frac{1}{{}^{2n}C_1} + \frac{1}{{}^{2n}C_2} - \frac{1}{{}^{2n}C_3} + \ldots\ldots\ldots - \frac{1}{{}^{2n}C_{2n-1}}\right)$$

$$= \frac{n(2n+1)}{(2n+2)}\left(-\left(\frac{1}{{}^{2n+1}C_1} + \frac{1}{{}^{2n+1}C_2}\right) + \left(\frac{1}{{}^{2n+1}C_2} + \frac{1}{{}^{2n+1}C_3}\right) - \left(\frac{1}{{}^{2n+1}C_3} + \frac{1}{{}^{2n+1}C_4}\right) + \ldots\ldots\ldots - \left(\frac{1}{{}^{2n+1}C_{2n-1}} + \frac{1}{{}^{2n+1}C_{2n}}\right)\right)$$

$$= \frac{n(2n+1)}{(2n+2)}\left(-\frac{2}{2n+1}\right) = -\frac{n}{(n+1)}$$

Put $n = 10$, $\sum_{r=1}^{19} \frac{(-1)^r r}{^{20}C_r} = -\frac{10}{11}$

16. $\left(1+i\sqrt{3}\right)^{3n} = ({}^{3n}C_0 - {}^{3n}C_2\, 3 + {}^{3n}C_4\, 3^2 - {}^{3n}C_6\, 3^3 + ...) + i\sqrt{3}\ ({}^{3n}C_1 - {}^{3n}C_3\, 3 + {}^{3n}C_5\, 3^2 - {}^{3n}C_7\, 3^3 + ... + (-3)^{\frac{3n}{2}-1}\ {}^{3n}C_{3n-1})$

$\Rightarrow$ $\sqrt{3}\,({}^{3n}C_1 - {}^{3n}C_3\, 3 + {}^{3n}C_5 3^2 - {}^{3n}C_7\, 3^3 + ...) = I_m((1 + i\sqrt{3}\,)^{3n})$

$$= \text{Im}\left(2e^{i\frac{\pi}{3}}\right)^{3n} = 0$$

17. Coefficient of x^n in ${}^nC_0(1 + x)^{2n} - {}^nC_1(1 + x)^{2n-1} + {}^nC_2(1 + x)^{2n-2} - C_3(1 + x)^{2n-3} + ... + (-1)^n\, {}^nC_n\,(1+x)^n$

= Coefficient of x^n in $(1 + x)^n\,((1 + x) - 1)^n = 1$

18. $(1 - x)^3 = 1 - x^3 - 3x(1 - x)$

$$(1 - x^3)^n = ((1 - x)^3 + 3x(1 - x))^n$$

$$= \sum_{r=0}^{n} {}^nC_r(1-x)^{3(n-r)}(3x(1-x))^r$$

$$= \sum_{r=0}^{n} {}^nC_r(1-x)^{3n-2r}3^r x^r$$

$\Rightarrow$ $a_r = {}^nC_r\, 3^r$

$\Rightarrow$ $\sum_{r=0}^{n} a_r = \sum_{r=0}^{n} {}^nC_r 3^r = 4^n$

19. $T_{r+1} = {}^{10}C_r\, 2^{\frac{10-r}{2}}\, 3^{\frac{r}{5}}$

T_{r+1} is rational for $r = 0, 10$

$\therefore$ Sum of rational terms $= {}^{10}C_0\, 2^5 + {}^{10}C_{10}\, 3^2 = 41$

20. $N = \frac{(7+18)^3}{(3+2)^6} = 1$

21. ${}^mC_1 + {}^mC_3 = 2\,{}^mC_2$

$\Rightarrow \quad m^2 - 9m + 14 = 0$

$\Rightarrow \quad m = 7, m = 2$ (rejected)

$T_6 = 21$

$\Rightarrow \quad {}^7C_5\left(2^{\frac{1}{2}\log_{10}(10-3^x)}\right)^2\left(2^{\frac{1}{5}\log_{10}3^{x-2}}\right)^5 = 21$

$\Rightarrow \quad \log_{10}(10 - 3^x)\,3^{x-2} = 0$

$\Rightarrow \quad 3^{2x} - 10\cdot 3^x + 9 = 0$

$\Rightarrow \quad 3^x = 9, 1$

$\Rightarrow \quad x = 0, 2$

22. $\frac{14}{9}\,{}^mC_2\,{}^mC_4 = ({}^mC_3)^2 \quad \Rightarrow \quad m = 9$

$${}^9C_3\,5^{-\log_{10}(6-\sqrt{8x})}.5^{\frac{\log_{10}(x-1)-2\log_{10}5}{2}} = \frac{84}{5}$$

$\Rightarrow \quad 6\sqrt{2x} - x - 10 = 0$

$\Rightarrow \quad x - 6\sqrt{2x} + 10 = 0$

$\Rightarrow \quad \left(\sqrt{x} - 5\sqrt{2}\right)\left(\sqrt{x} - \sqrt{2}\right) = 0$

$\Rightarrow \quad x = 2, 50$ (rejected)

Hence, $x = 2$

23. $\dfrac{{}^xC_6\,2^{\frac{x-6}{3}}\,3^{-\frac{6}{3}}}{{}^xC_{x-6}\,2^{\frac{6}{3}}\,3^{-\frac{x-6}{3}}} = 2^{\frac{x-12}{3}}\,3^{\frac{x-12}{3}} = 6^{\frac{x-12}{3}} = 6^{-1}$

$\Rightarrow \quad x - 12 = -3$

$\Rightarrow \quad x = 9$

24. $(5 + \sqrt{2})^n + (5 - 2\sqrt{6})^n = 2k, k \in I$

$\Rightarrow \quad [x] + \{x\} + N = 2k, 0 < N < 1, 0 < \{x\} < 1, \{x\} + N \in I$

$\Rightarrow \quad \{x\} + N = 1$

$\therefore \quad x - x^2 + x[x] = x - x(x - [x]) = x(1 - \{x\})$

$\qquad = xN = (5 + 2\sqrt{6})^n(5 - 2\sqrt{6})^n = 1$

25. $(\sqrt{2}+1)6+(\sqrt{2}-1)6 = 2(23 + 6C222 + 6C42 + 6C6) = 198$

$$[x] + \{x\} + N = 198$$

$$\{x\} + N \in I \quad 0 < \{x\} + N < 2$$

$$\Rightarrow \quad \{x\} + N = 1$$

$$\Rightarrow \quad [x] = 197$$

26. $$\left(2+\frac{1}{x}+\frac{1}{x^2}\right)^n = {}^nC_0 2^n + {}^nC_1\, 2^{n-1}\left(\frac{1}{x}\left(1+\frac{1}{x}\right)\right) + {}^nC_2\, 2^{n-2}\,\frac{1}{x^2}\left(1+\frac{1}{x}\right)^2 + \ldots\ldots + {}^nC_r$$

$$\frac{1}{x^r}\left(1+\frac{1}{x}\right)^r + {}^nC_{r+1}\,\frac{1}{x^{r+1}}\left(1+\frac{1}{x}\right)^{r+1} + \ldots\ldots + {}^nC_n\,\frac{1}{x^n}\left(1+\frac{1}{x}\right)^n$$

$\Rightarrow$ Number of terms $= 1+\underbrace{2+2+\ldots\ldots+2}_{\text{n times}} = 2n + 1 = 13$

Sum of coefficients $= (2 + 1 + 1)^6 = 4^6 = 2^{12}$

27. $$\frac{(1-x^2)^{106}}{x^{106}}$$

T_{54} is independent of x

$\therefore$ Total number of terms dependent of x = 107 – 1 = 106

28. ${}^mC_2 + {}^nC_2 - {}^mC_1\,{}^nC_1 + {}^mC_1 - {}^nC_1 = -m$

$$\Rightarrow \quad (m-n)^2 + 3(m-n) = 0$$

$$\Rightarrow \quad n - m = 3$$

Solutions

CHAPTER 5

PROBABILITY

SECTION-1

SINGLE CHOICE QUESTIONS

1. both 'a' both 's' both 'i' both 'n'

$$P = \frac{1 \times 2 + 2 \times 4 + 2 \times 1 + 2 \times 1}{8 \times 8} = \frac{14}{64} = \frac{7}{32}$$

2. selecting 3 out of 7 already drawn (→ $^7C_3$)

$$P = \frac{^7C_3 \times {^3C_2}}{^{10}C_5} = \frac{5}{12}$$

3. Case I F = B

0	0	4	5
1	1	3	4
2	2	2	3
3	3	1	2
4	4	0	1

Case II L = R

$$P = \frac{4\left(\frac{9!}{4!5!} + \frac{9!}{3!4!1!1!} + \frac{9!}{2!2!2!3!} + \frac{9!}{3!3!1!2!} + \frac{9!}{4!4!1!}\right)}{4^9}$$

$$= \frac{3969}{4^7}$$

4. Let he takes 'n' chances

$$P = 1 - P(\text{none correct})$$

[$\because$ Total ways to answer $= 2^5 - 1 = 31$]

$$= 1 - \frac{{}^{30}C_n}{{}^{31}C_n} = 1 - \frac{31-n}{31} = \frac{n}{31}$$

$\Rightarrow \quad \frac{n}{31} > \frac{1}{8} \Rightarrow n \geq 4$ [minimum value of n = 4]

Alternate : $\rightarrow$ P (getting correct in r^{th} trial) $= \frac{{}^{30}C_{r-1}}{{}^{31}C_{r-1}} \times \frac{1}{32-r} = \frac{1}{31}$

$\therefore$ P (correct in 1^{st} trial or 2^{nd} trial or......or n^{th} trial)

$$= \frac{1}{31} + \frac{1}{31} + \frac{1}{31} + \ldots\ldots + \frac{1}{31} = \frac{n}{31}$$

5. $(1-p)^n + {}^nC_1(1-p)^{n-1}p\left((1-p)^m + {}^mC_1(1-p)^{m-1}p\right) + {}^nC_2(1-p)^{n-2}p^2\,(1-p)^m$

- $(1-p)^n$ $\downarrow$ none defective
- ${}^nC_1(1-p)^{n-1}p$ $\downarrow$ 1 defective
- $(1-p)^m$ $\downarrow$ none defective in sample of next 'm' items
- ${}^mC_1(1-p)^{m-1}p$ $\downarrow$ 1 defective in next 'm' items

6. $P = \frac{{}^3C_1\,2^m - {}^3C_2\,(1)^m}{3^m} = \frac{2^m - 1}{3^{m-1}}$

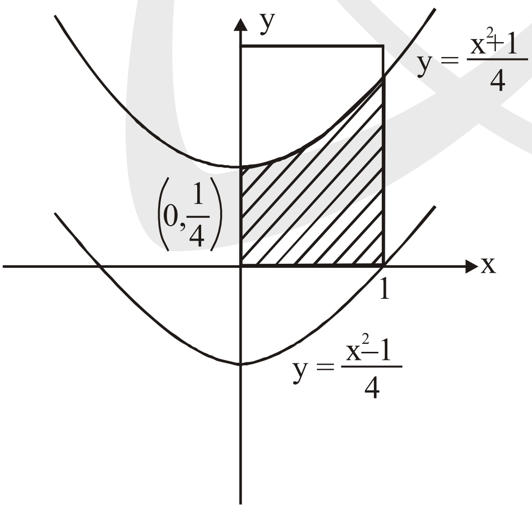
A $\rightarrow$ number of ways s.t. there is no A

B $\rightarrow$ number of ways s.t. there is no B

C $\rightarrow$ number of ways s.t. there is no C

$n(A \cup B \cup C) = S_1 - S_2 + S_3 = {}^3C_1\,2^m - {}^3C_2\,1^m + 0$

7.



$$|x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1x_2} = \sqrt{m^2 - 4n} \le 1$$

$$|m^2 - 4n| \le 1$$

$$\Rightarrow -1 \le x^2 - 4y \le 1$$

$$A = \int_0^1 \frac{x^2 + 1}{4} dx = \frac{1}{3}$$

$$P = \frac{1/3}{1} = \frac{1}{3}$$

8. $$P = \frac{{}^nC_0 x^n y^\circ + {}^nC_2 x^{n-2} y^2 + {}^nC_4 x^{n-4} y^4 + ...}{(x+y)^n}$$

$$= \frac{(x+y)^n + (x-y)^n}{2(x+y)^n}$$

9. 4 steps or 6 steps

$$P = \frac{\frac{4!}{2!2!}}{4^4} + \frac{+2\left(\frac{6!}{3!2!1!} - \frac{4!}{2!2!} \times 2!\right)}{4^6} = \frac{3}{64}$$

10. $\frac{x}{2} - \left\{\frac{x}{2}\right\} + \frac{x}{3} - \left\{\frac{x}{3}\right\} + \frac{x}{5} - \left\{\frac{x}{5}\right\} = \frac{31x}{30}$

$\Rightarrow$ $\left\{\frac{x}{2}\right\} + \left\{\frac{x}{3}\right\} + \left\{\frac{x}{5}\right\} = 0$

$\Rightarrow$ $\frac{x}{2} \in I,\ \frac{x}{3} \in I\ \&\ \frac{x}{5} \in I$

$\Rightarrow$ $x = 30k,\ k \in I$

$\therefore$ $P = \frac{33}{1000}$

11. $P(A \cap B) = P(A) + P(B) - P(A \cup B) \ge x + y - 1$

$\&\ P(A \cap B) \le x$

$\Rightarrow$ $P(A/B) = \frac{P(A \cap B)}{P(B)} \in \left[\frac{x+y-1}{y}, \frac{x}{y}\right]$

12. $$P = \frac{{}^7C_2(2^4 - 2)}{7^4} = \frac{6}{49}$$

13. Number of favourable ways = n!

(distribute n red balls over n white balls)

no. of totals ways = $\frac{(2n)!}{(n!)(2!)^n}$

(divide 2n balls into 'n' equal groups)

$\therefore \quad p = \frac{n!}{\left(\frac{(2n)!}{(n!)(2!)^n}\right)} = \frac{2^n}{{}^{2n}C_n}$

14. Let number s are x_1, $x_1 + 2d$, $x_1 + 4d$

$x_1 + 4d \le 4m + 1$

$\Rightarrow x_1 \le 4(m - d) + 1$, $d = 1, 2, 3, ..., m$

$\therefore$ Number of favourable ways = (4m – 3) + (4m – 7)+.....+1

$= \frac{m}{2}(4m - 2) = m(2m - 1)$

Number of total ways = ${}^{4m+1}C_3 = \frac{(4m+1)4m(4m-1)}{6}$

$$P = \frac{m(2m-1)6}{(4m+1)4m(4m-1)} = \frac{3(2m-1)}{2(16m^2-1)}$$

15. $E_i \rightarrow$ the event that 'i' white are drawn from the first bag

$A \rightarrow$ one ball drawn from second bag is white.

$$P(E_2/A) = \frac{P(A/E_2)P(E_2)}{\sum_{i=1}^{5} P(E_i)P(A/E_i)}$$

$$= \frac{\frac{{}^4C_3\,{}^6C_2}{{}^{10}C_5} \times \frac{2}{5}}{\frac{4}{5} \times \frac{{}^4C_1\,{}^6C_4}{{}^{10}C_5} + \frac{3}{5}\frac{{}^4C_2\,{}^6C_3}{{}^{10}C_5} + \frac{2}{5}\frac{{}^4C_3\,{}^6C_2}{{}^{10}C_5} + \frac{{}^4C_4\,{}^6C_1}{{}^{10}C_5} \times \frac{1}{5}}$$

$$= \frac{4 \times 15 \times 2}{4 \times 4 \times 15 + 3 \times 6 \times 20 + 2 \times 4 \times 15 + 6} = \frac{20}{121}$$

16. Number of favourable ways

$$= \sum_{r=1}^{10} {}^{10}C_r^2 \quad + \quad 2\sum_{r=1}^{9} {}^{10}C_r \, {}^{10}C_{r+1} \quad + \quad 2\sum_{r=1}^{8} {}^{10}C_r \, {}^{10}C_{r+2}$$

(difference 0) (difference 1) (difference 2)

$$= ({}^{20}C_{10} - 1) + 2\,({}^{20}C_{11} - 10) + 2\,({}^{20}C_8 - {}^{10}C_2)$$

$$= {}^{20}C_{10} + 2\,{}^{20}C_{11} + 2\,{}^{20}C_{12} - 111$$

no. of total ways $= (2^{10} - 1)^2$

$$P = \frac{{}^{20}C_{10} + 2({}^{20}C_{11} + {}^{20}C_{12}) - 111}{(2^{10} - 1)^2}$$

17. $\lim_{x \to 0}\left(\frac{a^x + b^x}{2}\right)^{2/x} = e^{\lim_{x \to 0}\left(\frac{a^x + b^x}{2} - 1\right)\frac{2}{x}} = e^{\ln ab} = ab$

$\Rightarrow \qquad ab = 6$

$(a, b) = (1, 6), (6, 1), (2, 3), (3, 2)$

$$P = \frac{4}{36} = \frac{1}{9}$$

18. $$P = \frac{\frac{1}{n} \times 1}{\frac{1}{n} \times 1 + \frac{n-1}{n} \times \frac{1}{n}} = \frac{n}{2n-1}$$

19. $$P = \frac{\frac{2}{10} \times 0.6}{\frac{2}{10} \times 0.6 + \frac{3}{10} \times 0.5 + \frac{5}{10} \times 0.4} = \frac{12}{47}$$

20. Let probability of single bacteria to die = P

$\therefore \qquad P = \frac{1}{4} \times 1 + \frac{1}{2} \times P \times P + \frac{1}{4} \times P \times P \times P$

bacteria die or split

A $\rightarrow$ bacteria do not split

B $\rightarrow$ bacteria split into 2

C → bacteria split into 3

D → bacterial die

$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$

$$P = \frac{1}{4} \times 1 + \frac{1}{2} \times P \times P + \frac{1}{4} \times P \times P \times P$$

$$\Rightarrow \quad P^3 + 2P^2 - 4P + 1 = 0$$

$$\Rightarrow \quad (P - 1)(P^2 + 3P - 1) = 0$$

$$\Rightarrow \quad P = \frac{-3+\sqrt{13}}{2}$$

∴ Probability that bacteria survives

$$= 1 - \frac{\sqrt{13}-3}{2} = \frac{5-\sqrt{13}}{2}$$

21. A → last throw get 1, 2, 3 or 4

B → last throw get 2, 3, or 4

$$P(B/A) = \frac{3}{4}$$

22. (C)

$$P = \frac{\frac{{}^5C_5}{{}^{10}C_5} \times \frac{5}{5}}{\frac{{}^5C_5}{{}^{10}C_5} \times \frac{5}{5} + \frac{{}^5C_4\,{}^5C_1}{{}^{10}C_5}\frac{4}{5} + \frac{{}^5C_3\,{}^5C_2}{{}^{10}C_5}\frac{3}{5} + \frac{{}^5C_2\,{}^5C_3}{{}^{10}C_5}\frac{2}{5} + \frac{{}^5C_1\,{}^5C_4}{{}^{10}C_5} \times \frac{1}{5}}$$

$$= \frac{5}{5+100+300+200+25}$$

$$= \frac{1}{126}$$

23. Favourable cases are (1, 4), (1, 9), (2, 8), (4, 9)

$$\therefore \quad P = \frac{4}{{}^9C_2} = \frac{1}{9}$$

24. $$P = {}^4C_1\left(\frac{3}{9}\right)\left(\frac{6}{9}\right)^3 = \frac{32}{81}$$

25. $$P = 1 - \frac{^{14}C_5}{^{15}C_5} = \frac{1}{3} = \frac{1}{3}$$

26. P_1 get paired with P_2 in 1st round = 2k

where K (2 + 3 + 4) = 1 $\Rightarrow$ $k = \frac{1}{9}$

$\therefore$ P (P_2 reaches second round) = 1 – P(P_2 paired with P_1)

$$= 1 - \frac{2}{9} = \frac{7}{9}$$

27. $x_1 + x_2 + x_3 = 8, 27$

No. of favourable ways = $\dfrac{^{8-3+2}C_2 - 3\times 3}{3!} + \dfrac{^{27-3+2}C_2 - 3\,^{27-18+2}C_2 - (1+3\times 7)}{3!}$

= 25

$$P = \frac{25}{^{15}C_3} = \frac{25}{455} = \frac{5}{91}$$

28. $\underbrace{HH--H}_{m}XX--X$ $\quad P = \frac{1}{2^m} + \frac{m+1}{2^{m+1}} - \frac{1}{2^{2m+1}}$

THH--HXX--X $\quad = \frac{(m+3)2^m - 1}{2^{2m+1}}$

XTHH---HX---X

⋮

XX ---XT$\underbrace{HH--H}_{m}$

29. $$P = \frac{4^4 - \left({}^2C_1 3^4 - 2^4\right)}{6^4} = \frac{55}{648}$$

Favourable number of ways

= Number appearing can be 2, 3, 4, 5 only – at least one of 2 or 5 excluded.

30. 7 digits are distinct and a_3 is the smallest

$\therefore$ Number of favourable ways = $^{10}C_7 \times 1 \times {}^6C_2$

(${}^{10}C_7$ → selecting 7 digit; 1 → selecting a_3; ${}^6C_2$ → selecting a_1, a_2)

$$P = \frac{\left(\frac{10\times9\times8\times6\times5}{3\times2\times2}\right)}{9\times9\times8\times7\times6\times5\times4}$$

$$= \frac{5}{1512}$$

31. A → event that white balls are not among the 10 selected marbles.

B → event that blue balls are not among the 10 selected marbles.

C → event that red balls are not among the 10 selected marbles.

$$P(A \cup B \cup C) = \left(\frac{80}{100}\right)^{10} + \left(\frac{70}{100}\right)^{10} + \left(\frac{50}{100}\right)^{10} - \left(\frac{50}{100}\right)^{10} - \left(\frac{30}{100}\right)^{10} - \left(\frac{20}{100}\right)^{10}$$

$$= \frac{8^{10} + 7^{10} - 3^{10} - 2^{10}}{10^{10}}$$

32. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad \frac{1}{2} = 2P(A) - (P(A))^2$

$\Rightarrow \quad P(A) = 1 - \frac{1}{\sqrt{2}}$

33. $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = \frac{7}{18}$

$\Rightarrow \quad P(A \cup B) = \frac{11}{18}$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = \frac{11}{18}$

$\Rightarrow \quad P(A \cap B) = \frac{1}{18}$

$\Rightarrow \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{6}$

34. Odds against missing card being spade

$$= \frac{P(\text{missing card is not spade \& 2cards drawn are spades})}{P(\text{missing card is spade \& 2cards drawn are spades})}$$

$$= \frac{\frac{3}{4} \times \frac{^{13}C_2}{^{51}C_2}}{\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2}}$$

$$= \frac{3 \times 13 \times 12}{12 \times 11} = \frac{39}{11}$$

35. $|x - y| \geq 6$ holds for (1, 7),....,(1, 10), (2, 8), (2, 9), (2, 10), (3, 9), (3, 10), (4, 10)

Number of ways for which $|x - y| \geq 6 = 4 + 3 + 2 + 1 = 10$

$$P(|x - y| \leq 5) = 1 - \frac{10}{^{10}C_2} = \frac{7}{9}$$

36. Probability $= \frac{^6C_3}{6^3} = \frac{5}{54}$

37. $\underbrace{HH--H}_{7}\underbrace{XX---X}_{\text{H or T}}$ $\qquad P = \frac{1}{2^7}$

T H H -----H X - - X $\qquad P = \frac{1}{2^8}$

X T H -----H X - - X $\qquad P = \frac{1}{2^8}$

⋮

X- - XT$\underbrace{HH--H}_{\text{last7}}$ $\qquad P = \frac{1}{2^8}$

$$\Rightarrow \qquad P = \frac{1}{2^7} + \underbrace{\frac{1}{2^8} + \frac{1}{2^8} + \frac{1}{2^8} + ... + \frac{1}{2^8}}_{5\text{ times}} = \frac{1}{2^7} + \frac{5}{2^8}$$

$$= \frac{7}{2^8}$$

38. $$\frac{2}{7} = \frac{{}^nC_1\, {}^{n-4}C_2}{3\, {}^nC_3}$$

$$\frac{2}{7} = \frac{n(n-4)(n-5)}{n(n-1)(n-2)} = \frac{(n-4)(n-5)}{(n-1)(n-2)}$$

$\Rightarrow 5n^2 - 57n + 136 = 0$

$\Rightarrow \quad n = 8.$

39. $(x - y) = n$ number of $(x, y) = n + 1$

$= n - 1$ number of $(x, y) = n + 2$

$\vdots$

$= 1$ $= 2n$

$= 0$ $= 2n + 1$

$\therefore$ Number of favourable ways $= 2((n + 1) + (n + 2) + (n + 3)+...(2n)) + 2n + 1$

$= 3n^2 + 3n + 1$

$$P = \frac{3n^2 + 3n + 1}{(2n+1)^2}$$

40. $$P = \frac{\frac{1}{2} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{2}{5}} = \frac{12}{17}$$

41. Clearly his car is at one of the crosses

|× × × × .....×|

The number of ways in which the remaining (m – 1) cars can take their places (excluding the car of the man)

$= {}^{n-1}C_{m-1}$

The number of ways in which the remaining (m – 1) cars can take places leaving the two places on two sides of his car $= {}^{n-3}C_{m-1}$

$$\therefore \quad P = \frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}} = \frac{(n-m)(n-m-1)}{(n-1)(n-2)}$$

42. $$P = {}^6C_2\left(\frac{6}{12}\right)^2\left(\frac{6}{12}\right)^4\frac{6}{12} = \frac{15}{128}$$

43. $$\log_a b = \log_{2^m} 2^n = \frac{n}{m}$$

Let	m = 1,	n = 2, 3,......,25	number of (m, n) = 24
	m = 2,	n = 4, 6,......,24	number of (m, n) = 11
	m = 3,	n = 6, 9,......,24	number of (m, n) = 7
	m = 4,	n = 8, 12,......,24	number of (m, n) = 5
	m = 5,	n = 10, 15, 20, 25	number of (m, n) = 4
	m = 6,	n = 12, 18, 24	number of (m, n) = 3
	m = 7,	n = 14, 21	number of (m, n) = 2
	m = 8,	n = 16, 24	number of (m, n) = 2
	m = 9,	n = 18	number of (m, n) = 1
	m = 10,	n = 20	number of (m, n) = 1
	m = 11,	n = 22	number of (m, n) = 1
	m = 12,	n = 24	number of (m, n) = 1

$$P = \frac{62}{25\times 24} = \frac{31}{100}$$

44. A = number of persons going to hotel A = 1

B = number of persons going to hotel A = 0

C = number of persons going to hotel B = 0

D = number of persons going to hotel C = 0

$$P = 1 - \frac{n(A\cup B\cup C\cup D)}{n(s)}$$

$$n(A \cup B \cup C \cup D) = \sum n(A) - \sum n(A\cap B) + \sum n(A\cap B\cap C) - n(A \cap B \cap C \cap D)$$

$$= ({}^{20}C_1\, 2^{19} + 3.2^{20}) - ({}^{20}C_1\, 1^{19} \times 2 + 1 \times 2 + 1)$$

$\therefore$ $$P = 1 - \left(\frac{13.2^{20} - 43}{3^{20}}\right)$$

45. $$P = \frac{\frac{7!}{3!2!2!2!} \times 2!}{7^7} = \frac{30}{7^6}$$

46. $$P = \frac{{}^8C_2 + {}^7C_2}{{}^{15}C_3} = \frac{7}{65}$$

47. Probability of getting prime outcome in any throw = P(2, 3, 5) = $\frac{3}{6} = \frac{1}{2}$

$$\therefore \quad P = \left(\frac{1}{2}\right)^{16} \left({}^8C_0^2 + {}^8C_1^2 + {}^8C_2^2 + \ldots + {}^8C_8^2\right)$$

$$= \frac{{}^{16}C_8}{2^{16}}$$

48. $P(A \cup B \cup C) = 1, P(A \cap B \cap C) = 0$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A) + P(B) = 1$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - (P(A \cap B) + P(B \cap C) + P(C \cap A)) + P(A \cap B \cap C)$$

$$\Rightarrow \quad 1 = 1 + \frac{7}{15} - (0 + P(B \cap C) + \frac{1}{5}) + 0$$

$$\Rightarrow \quad P(B \cap C) = \frac{4}{15}$$

50. $$P = \frac{{}^6C_3 \times 1}{{}^7C_4} = \frac{4}{7}$$

51. 3H ; (2H, 2T) ; (1H, 4T) ; 6T

$$P = \left(\frac{1}{2}\right)^3 + \frac{4!}{2!2!}\left(\frac{1}{2}\right)^4 + \frac{5!}{4!}\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6$$

$$= \frac{8 + 24 + 10 + 1}{64} = \frac{43}{64}$$

52.

x	x^4
$5k$	5λ
$5k+1$	$5\lambda+1$
$5k+2$	$5\lambda+1$
$5k+3$	$5\lambda+1$
$5k+4$	$5\lambda+1$

$$P = \frac{{}^{80}C_2 + {}^{20}C_2}{{}^{100}C_2}$$

$$= \frac{20\times19+80\times79}{100\times99} = \frac{19+316}{5\times99}$$

$$= \frac{335}{5\times99} = \frac{67}{99}$$

53.
$$P = \frac{{}^7C_3\times9}{7!}$$

$$= \frac{1}{16}$$

54.
$$P = \frac{{}^nC_3\times1}{{}^nC_3\times3} = \frac{1}{3}$$

55.
$$P = \frac{1}{2}\left(\frac{6+5}{36}+\frac{2}{11}\right) = \frac{193}{792}$$

SECTION-2

ONE OR MORE THAN ONE CORRECT

1. **(a)** $\dfrac{7\times8+7\times8}{{}^{64}C_2} = \dfrac{7\times8\times2\times2}{64\times63} = \dfrac{1}{18}$

(b) $p = \dfrac{7\times7+7\times7}{{}^{64}C_2} = \dfrac{2\times7\times7\times2}{64\times63} = \dfrac{7}{144}$

Alternate : (a) $\dfrac{4(2) + 24\times 3 + 36\times 4}{64 \times 63}$

- $4(2)$ → Corner squares
- 24×3 → Corner row & column squares excluding 4 corner squares
- 36×4 → remaining squares

$$= \frac{1+9+1}{8\times 63} = \frac{1+9+1}{2\times 9} = \frac{1}{18}$$

(b) $\dfrac{36\times4+24\times2+4\times1}{64\times63} = \dfrac{7}{144}$

2. Let $P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = x = P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = a = P(E_1)P(\bar{E}_2)P(\bar{E}_3)$

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = b = P(\bar{E}_1)P(E_2)P(\bar{E}_3)$

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = x = P(\bar{E}_1)P(\bar{E}_2)P(E_3)$

$$\frac{a}{x} = \frac{P(E_1)}{P(\bar{E}_1)} = \frac{P(E_1)}{1-P(E_1)} \Rightarrow P(E_1) = \frac{a}{a+x}$$

$$P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_2) = x = \frac{x}{a+x}\,\frac{x}{b+x}\,\frac{x}{c+x}$$

$\Rightarrow$ $(a + x)(b + x)(c + x) = x^2$

3. (A)

pool-A	pool-B
P_4	×
×	×
×	×
×	×

$$P = \frac{^4C_3}{^7C_3} = \frac{4}{35}$$

(B) $P = 0$ (obvious)

(C) $P = 1$ (obvious)

(D) Pool-A	Pool-B	Pool -C	Pool-D
P_6	×	×	×
×	×	×	×

$$P = \frac{{}^2C_1}{{}^7C_1} = \frac{2}{7}$$

4. D → event that person keeps driver

E_1 → event that person own sedan

E_1 → event that person own SUV

$$P(D) = P(E_1 \cap \bar{E}_2)P(D/E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2)\,P(D/\bar{E}_1 \cap E_2) + P(E_1 \cap E_2)\,P(D/E_1 \cap E_2)$$

$$= \frac{3}{10}\times\frac{6}{10}\times\frac{6}{10}+\frac{7}{10}\times\frac{4}{10}\times\frac{7}{10}+\frac{3}{10}\times\frac{4}{10}\times\frac{9}{10}$$

$$= \frac{103}{250}$$

$$P(E_1/D) = \frac{P(E_1 \cap \bar{E}_2)P(D/E_1 \cap \bar{E}_2) + P(E_1 \cap E_2)P(D/E_1 \cap E_2)}{P(D)}$$

$$= \frac{\left(\frac{54}{250}\right)}{\left(\frac{103}{250}\right)} = \frac{54}{103}$$

6. A → event that candidate passes in exam A.

B → event that candidate passes in exam B.

C → event that candidate passes in exam C.

$$P(A \cap B \cap C) = \frac{1}{2}\cdot\frac{2}{5} = \frac{1}{10} = abc$$

$$\frac{2}{5} = \Sigma P(A \cap B) - 3P(A \cap B \cap C)$$

$$\Rightarrow \quad \Sigma P(A \cap B) = \frac{7}{10}$$

$$\frac{3}{4} = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

$$\Rightarrow \quad \sum P(A) = \frac{27}{20} = a + b + c$$

7. $P(A_1 \cup A_2 \cup A_3 \cup \ldots\ldots \cup A_n) = 1 - P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap \ldots\ldots \cap \overline{A}_n)$

$$= 1 - P(\overline{A}_1)\,P(\overline{A}_2)\ldots\ldots P(\overline{A}_n)$$

$$= 1 - ((1 - P(A_1))\,(1 - P(A_2))\ldots\ldots(1 - P(A_n)))$$

8. Let $P(A) = P$

$$P(B) = \frac{1}{2}P$$

$$P(C) = \frac{1}{2} \times \frac{1}{2} \times P$$

$$P(A) + P(B) + P(C) = 1$$

$$\Rightarrow \quad \frac{7}{4}P = 1 \quad \Rightarrow P = \frac{4}{7}$$

9. $P(n) = 1 - \dfrac{{}^6C_n\, n!}{6^n} \qquad n \le 6$

$$P(2) = 1 - \frac{{}^6C_2\, 2!}{6^2} = \frac{1}{6}$$

$$P(3) = 1 - \frac{{}^6C_3\, 3!}{6^3} = \frac{4}{9}$$

$$P(4) = 1 - \frac{{}^6C_4\, 4!}{6^4} = \frac{13}{18}$$

$$P(6) = 1 - \frac{6!}{6^6} = \frac{319}{324}$$

10. Probability of getting more even than odd = probability of getting less even than odd outcomes

$$= \frac{1}{2}\,(1 - \text{probability of getting equal odd \& even outcomes})$$

$$= \frac{1}{2}\left(1 - \frac{{}^8C_0^2 + {}^8C_1^2 + {}^8C_2^2 \ldots\ldots + {}^8C_8^2}{2^{16}}\right)$$

$$= \frac{1}{2}\left(1 - \frac{{}^{16}C_8}{2^{16}}\right)$$

11. $P(n) = (1 - P(n-1))\frac{1}{3}$

$P(2) = 0$

$P(3) = \frac{1}{3}$, $P(4) = \frac{2}{9}$, $P(5) = \frac{7}{27}$

$P(6) = \frac{20}{81}$, $P(7) = \frac{61}{243}$

12. $Q = 1 - ({}^4C_1P(1-P)^3 + {}^4C_3P^3(1-P)$

$$= 1 - \frac{1}{2}\left[(P + (1-P))^4 - ((-P + (1-P))^4\right]$$

$$= \frac{1}{2}(1 + (1 - 2P)^4)$$

13. $P(A) = 6$ or $\bar{6}\ \bar{7}\ 6$ or $\bar{6}\ \bar{7}\ \bar{6}\ \bar{7}\ 6$ or ....∞

$$= \frac{5}{36} + \frac{31}{36} \times \frac{30}{36} \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{30}{36}\right)^2 \times \frac{5}{36} + \ldots.\infty$$

$$= \frac{5/36}{1 - \frac{30 \times 31}{(36)^2}} = \frac{5 \times 6}{216 - 155} = \frac{30}{61}$$

$$P(B) = \frac{31}{36} \times \frac{6}{36} + \frac{31}{36} \times \left(\frac{30}{36} \times \frac{31}{36}\right) \times \frac{6}{36} + \frac{31}{36} \times \left(\frac{30}{36} \times \frac{31}{36}\right)^2 \times \frac{6}{36} + \ldots\infty$$

$$= \frac{\frac{31}{36} \times \frac{6}{36}}{1 - \frac{30}{36} \times \frac{31}{36}}$$

$$= \frac{31}{61}$$

14. $ad - bc \neq 0$ for unique soln.

ad	bc	(a, d, b, c)
1	2 or 4	(1, 1, 1, 2), (1, 1, 2, 1), (1, 1, 2, 2)
2	1 or 4	(1, 2, 1, 1), (2, 1, 1, 1), (1, 2, 2, 2), (2, 1, 2, 2)
4	1 or 2	(2, 2, 1, 1), (2, 2, 1, 2), (2, 2, 2, 1)

$$P = \frac{10}{16} = \frac{5}{8}$$

$\therefore$ for non trivial soln.

$$P = \frac{3}{8}$$

15. $$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.25$$

$$0.75 \leq P(A \cup B \cup C) = P(A) + P(B) + P(C) - (P(A\cap B) + P(B\cap C) + P(C\cap A)) + P(A\cap B\cap C) \leq 1$$

$\Rightarrow 0.1 \leq P(B \cap C) \leq 0.35$

17. (b) $$P = \frac{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{5}}{\frac{1}{3}\times\frac{1}{3}\times\frac{2}{5}+\frac{2}{3}\times\frac{2}{3}\times\frac{2}{5}+\frac{2}{3}\times\frac{1}{3}\times\frac{3}{5}+\frac{2}{3}\times\frac{1}{3}\times\frac{2}{5}} = \frac{8}{2+8+6+4} = \frac{2}{5}$$

(d) $$P = \frac{8+6}{2+8+6+4} = \frac{7}{10}$$

SECTION-3

COMPREHENSION BASED QUESTIONS

Comprehension (Q.1 To Q.3):

1. $$\frac{(n-1)!2!}{n!} = \frac{2}{n}$$

2. 

$$\frac{{}^{n-2}C_m(n-m-1)!2!\times m!}{n!} = \frac{2(n-2)!(n-m-1)}{n!} = \frac{2(n-m-1)}{n(n-1)}$$

alt $\rightarrow$ $$\frac{(n-m-1)2!(n-2)!}{n!} = \frac{2(n-m-1)}{n(n-1)}$$

3. $\frac{2}{n(n-1)}((n-1)-0)+((n-1)-1)+((n-1)-2)+....((n-1)-m)$

$$= \frac{2}{n(n-1)}\left[\frac{(m+1)}{2}(2n-2-m)\right]$$

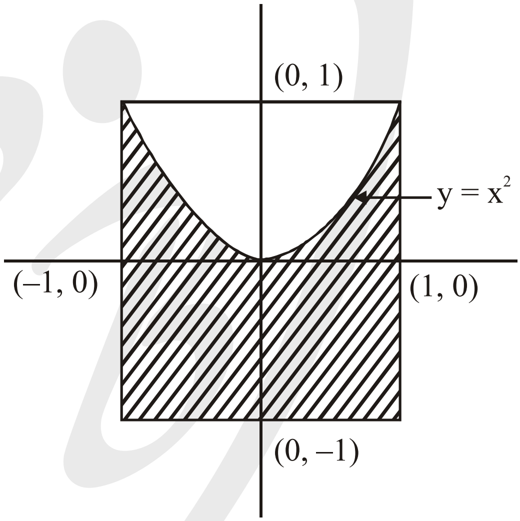
$$= \frac{(m+1)(2n-m-2)}{n(n-1)}$$

Comprehension (Q.4 to Q.5)

4.

$4(a^2 - b) \geq 0$

$b \leq a^2$



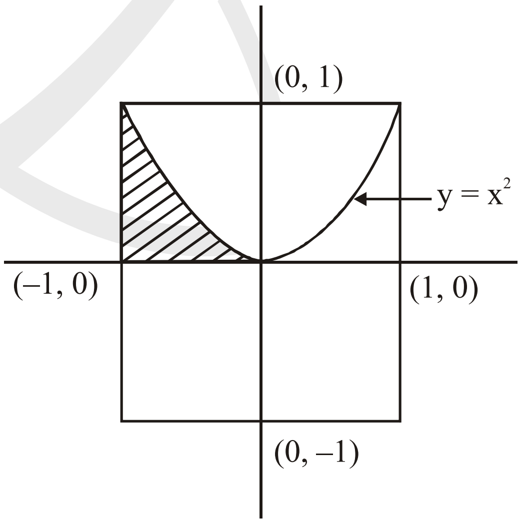
5. $D \geq 0 \Rightarrow b \leq a^2$

$-a > 0 \Rightarrow a < 0$

$\&\ f(0) > 0 \Rightarrow b > 0$

$$P(B) = \frac{\frac{1}{3}}{(2)^2} = \frac{1}{12}$$

$$P(A) = \frac{2\left(\frac{1}{3}\right)+2(1)}{2(2)} = \frac{\frac{8}{3}}{4} = \frac{2}{3}$$



Comprehension (Q.6 to Q.7)

6. P (r or (1, r – 1) or (1, 1, r – 2),......., $\underbrace{(1,1.....1,2)}_{r-2\text{ times}}$)

$$= \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + ... + \frac{1}{6^{r-1}}$$

$$= \frac{\frac{1}{6}\left(1 - \frac{1}{6^{r-1}}\right)}{1 - \frac{1}{6}} = \frac{1}{5}\left(1 - \frac{1}{6^{r-1}}\right)$$

7. $P\underbrace{(1,1,......,1,2)}_{(r-2)\text{times}}$ or $\underbrace{(1,1,......,1,3)}_{(r-3)}$ .... or $\underbrace{(1,1,......,1,6)}_{(r-6)\text{times}}$

$$= \left(\frac{1}{6}\right)^{r-1} + \left(\frac{1}{6}\right)^{r-2} + \left(\frac{1}{6}\right)^{r-3} + ... + \left(\frac{1}{6}\right)^{r-5}$$

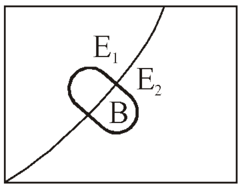
$$= \frac{\left(\frac{1}{6}\right)^{r-1}(6^5 - 1)}{5} = \frac{1}{5}\left(\left(\frac{1}{6}\right)^{r-6} - \left(\frac{1}{6}\right)^{r-1}\right)$$

Comprehension (Q.8 to Q.9)

8. P (i) = Ki   i = 1, 2, 3, 4, 5, 6

where P(i) = prob. of obtaining no. equal to i

$$\sum_{i=1}^{6} P(i) = 1 \Rightarrow k\left(\frac{6 \times 7}{2}\right) = 1 \Rightarrow k = \frac{1}{21}$$



P(B) = Probability that drawn ball is black

$E_1 \Rightarrow$ ball is drawn from urn A

$E_2 \Rightarrow$ ball is drawn from urn B

$$P(E_1) = \frac{1}{21}(2 + 3 + 5) = \frac{10}{21}$$

$$P(E_2) = \frac{1}{21}(1 + 4 + 6) = \frac{11}{21}$$

$$P(B) = P(E_1 \cap B) + P(E_2 \cap B) = P(E_1)P(B/E_1) + P(E_2)P(B/E_2)$$

$$= \frac{10}{21} \times \frac{3}{5} + \frac{11}{21} \times \frac{2}{5} = \frac{52}{105}$$

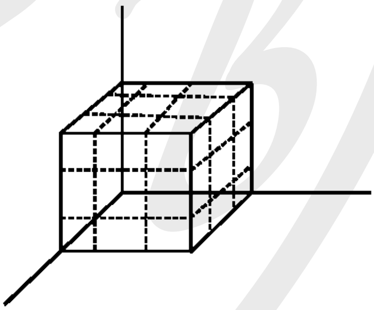
9. P(W) = probability that drawn ball is white

$$P(E_2/W) = \frac{P(E_2 \cap W)}{P(W)} = \frac{P(E_2)P(W/E_2)}{P(E_1)P(W/E_1) + P(E_2)P(W/E_2)}$$

$$= \frac{\frac{11}{21} \times \frac{3}{5}}{\frac{10}{21} \times \frac{2}{5} + \frac{11}{21} \times \frac{3}{5}} = \frac{33}{53}$$

Comprehension (Q.12 to Q.14)

Observing from fig.



12. $P = \frac{1}{27}$

13. $P = \frac{12}{27} = \frac{4}{9}$

14. $P = \frac{6}{27} = \frac{2}{9}$

Comprehension (Q.15 to Q.16)

$$P(2) = \frac{3}{6} \times \frac{1}{6} = \frac{3}{36}$$

$$P(3) = \frac{3}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{1}{6} = \frac{8}{36} = P(1, 2 \text{ or } 2, 1)$$

$$P(4) = \frac{3}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{2}{6} \times \frac{2}{6} = \frac{14}{36} = P(1, 3 \text{ or } 3, 1 \text{ or } 2, 2)$$

$$P(5) = \frac{2}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{2}{6} = \frac{8}{36} = P(2, 3 \text{ or } 3, 2)$$

$$P(6) = \frac{1}{6} \times \frac{3}{6} = \frac{3}{36}$$

COMPREHENSION (Q.17 TO Q.19) :

17. Let W → Win, D → Draw, L → Lose for X

P(X wins in n games) = P ((2W, (n – 2)D) or (2W,, 1 L, (n – 3)D))

$= {}^{n-1}C_1\, p^2 q^{n-2} + (n-2)(n-1)\, p^2 r\, q^{n-3}$

$= (n-1)\, p^2 q^{n-2} + (n-1)(n-2)\, p^2 q^{n-3} r$

$= (n-1)\, p^2 q^{n-3} (q + (n-2)r)$

18. P(Y wins) = P(2L, (4)D) or (2L, 1W, 3D)

$= {}^5C_1\, r^2 q^4 + 5 \times 4\, (r^2 p\, q^3)$

$= 5q^3r^2(q + 4p)$

19. $P(Y \text{ wins}) = \dfrac{r^2}{(1-q)^2} + \dfrac{{}^2C_1\, pr^2}{(1-q)^3}$

$= \dfrac{r^2((p+r)+2p)}{(1-q)^3} = \dfrac{r^2(3p+r)}{(1-q)^3}$

COMPREHENSION (Q.20 TO Q.22)

20. Game ends with last two tosses resulting in 2 heads or 2 tails.

⇒ P (ends with 2 heads/ends with 2 heads or 2 tails)

$$= \frac{\frac{2}{3} \times \frac{2}{3}}{\frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3}} = \frac{4}{5}$$

Alternate : Let P = probability that no. of heads exceed no. of tails by 2.

⇒ P = P(H) P(H) + P(H) P(T) P + P(T)P (H) P

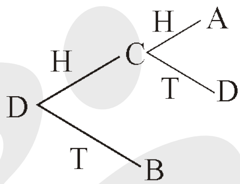
$P = \frac{4}{9} + \frac{2}{9}P + \frac{2}{9}P \Rightarrow P = \frac{4}{5}$

21. P(min. throws/ends with head) = $\dfrac{\dfrac{2}{3}\times\dfrac{2}{3}}{\dfrac{4}{5}} = \dfrac{5}{9}$

22. Obvious

Comprehension (Q.23 to Q.24)

23. Let P = I start in field D & win

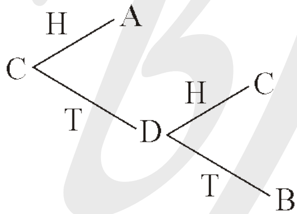


$$P = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}P\right)$$

$$\Rightarrow \quad \frac{3}{4}P = \frac{1}{4}$$

$$\Rightarrow \quad P = \frac{1}{3}$$

24. Let P = I start in field C & win



$$P = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\times P\right)$$

$$\Rightarrow \quad P = \frac{2}{3}$$

Comprehension (Q.25 to Q.26) :

25.

x	x^2	Number of x
7k	7μ	3
7k + 1	$7\mu + 1$	4
7k + 2	$7\mu + 4$	4
7k + 3	$7\mu + 2$	4
7k + 4	$7\mu + 2$	4
7k + 5	$7\mu + 4$	3
7k + 6	$7\mu + 1$	3

$$P = \frac{{}^3C_2 + {}^7C_2 \times 2 + {}^8C_2}{{}^{25}C_2} = \frac{73}{300}$$

26.

x	x^2	Number of x
5k	5μ	5
5k + 1	$5\mu + 1$	5
5k + 2	$5\mu + 4$	5
5k + 3	$5\mu + 4$	5
5k + 4	$5\mu + 1$	5

$$P = \frac{{}^{5}C_2 + {}^{10}C_2 \times 2}{{}^{25}C_2} = \frac{1}{3}$$

Comprehension (Q.27 to Q.29)

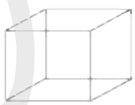
27. $P = \dfrac{\dfrac{4!}{2!2!}}{4^4} = \dfrac{3}{128}$

28. $P = \dfrac{2\left(\dfrac{6!}{3!2!} - \dfrac{4!}{2!2!} \times 2!\right)}{4^6} = \dfrac{3}{128}$

29. P = 0

Comprehension (Q.30 to Q.31)

30. $P = 1 \times \dfrac{2}{3} \times \dfrac{1}{3} \times \dfrac{1}{3} \times \dfrac{1}{3} \times \dfrac{1}{3}$

Alternate: For this to happen each pair of opposite faces  must be painted with the same colour. probability that first pair of opp. faces are coloured with same colour = $1 \times \dfrac{1}{3}$

2nd = $\dfrac{2}{3} \times \dfrac{1}{3}$

3rd = $\dfrac{1}{3} \times \dfrac{1}{3}$

31. $P = 1 - \dfrac{{}^3C_2(2!)^3}{\left({}^3C_2\,2!\right)^3} = \dfrac{8}{9}$

Probability that only two colours are used given that opp. faces have different colours.

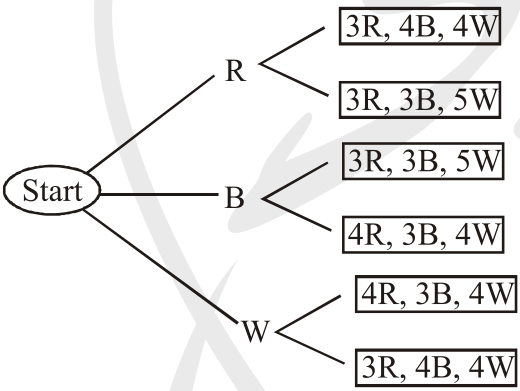
Alternate : P = 1 – P (exactly two colours are used given that each pair of opposite faces are painted with a different colour)

$$= 1 - 1 \times \frac{1}{3} \times \frac{1}{3} = \frac{8}{9}$$

($\because$ Prob. that a definite pair is selected to paint opposite faces $= \frac{1}{3}$)

Comprehension (Q.32 to Q.33)

32.



$$P = \frac{3}{10}\times 1 + \frac{3}{10}\times\frac{1}{2} + \frac{4}{10}\times\frac{1}{2} = \frac{13}{20}$$

33. $$P = \frac{\frac{3}{10}\times\left(\frac{3}{11}+\frac{3}{11}\right)}{\frac{3}{10}\times\left(\frac{3}{11}+\frac{3}{11}\right)+\frac{3}{10}\times\left(\frac{3}{11}+\frac{4}{11}\right)+\frac{4}{10}\times\left(\frac{3}{11}+\frac{4}{11}\right)}$$

$$= \frac{18}{67}$$

Comprehension (Q.34 to Q.36)

Either both a & b are divisible by p or both not divisible by p

$$\therefore \quad P_n(p) = \frac{\left[\frac{n}{p}\right]^2 + \left(n - \left[\frac{n}{p}\right]\right)^2}{n^2}$$

$$= 1 - \frac{2}{n}\left[\frac{n}{p}\right] + \frac{2}{n^2}\left[\frac{n}{p}\right]^2$$

$$\lim_{n\to\infty} P_n(p) = 1 - \frac{2}{p} + \frac{2}{p^2}$$

$$P_{25}(3) = \frac{8^2 + 17^2}{(25)^2} = \frac{353}{625}$$

COMPREHENSION (Q.37 TO Q.39)

37. $P = \frac{{}^{52}C_2 \times {}^{2}C_1 \times {}^{50}C_1}{({}^{52}C_2)^2} = \frac{50}{663}$

38. $P = \frac{{}^{13}C_1 \times 2 \times {}^{39}C_1 \times {}^{39}C_2}{({}^{52}C_2)^2} = \frac{247}{578}$

39. $P = \frac{{}^{52}C_3 \times 2}{{}^{104}C_3} = \frac{25}{103}$

COMPREHENSION (Q.40 TO Q.41)

40. A(n) = P(BB or BWB or WBB)

$$= \frac{n}{n+2}\frac{n-1}{n+1} + \frac{n}{n+2}\frac{2}{n+1}\frac{n-1}{n} + \frac{2}{n+2}\frac{n}{n+1}\frac{n-1}{n}$$

$$= \frac{(n+4)(n-1)}{(n+2)(n+1)}$$

$$\lim_{n\to\infty}\prod_{r=2}^{n} A(r) = \lim_{n\to\infty}\left(\prod_{r=2}^{n}\frac{r+4}{r+2}\prod_{r=2}^{n}\frac{r-1}{r+1}\right)$$

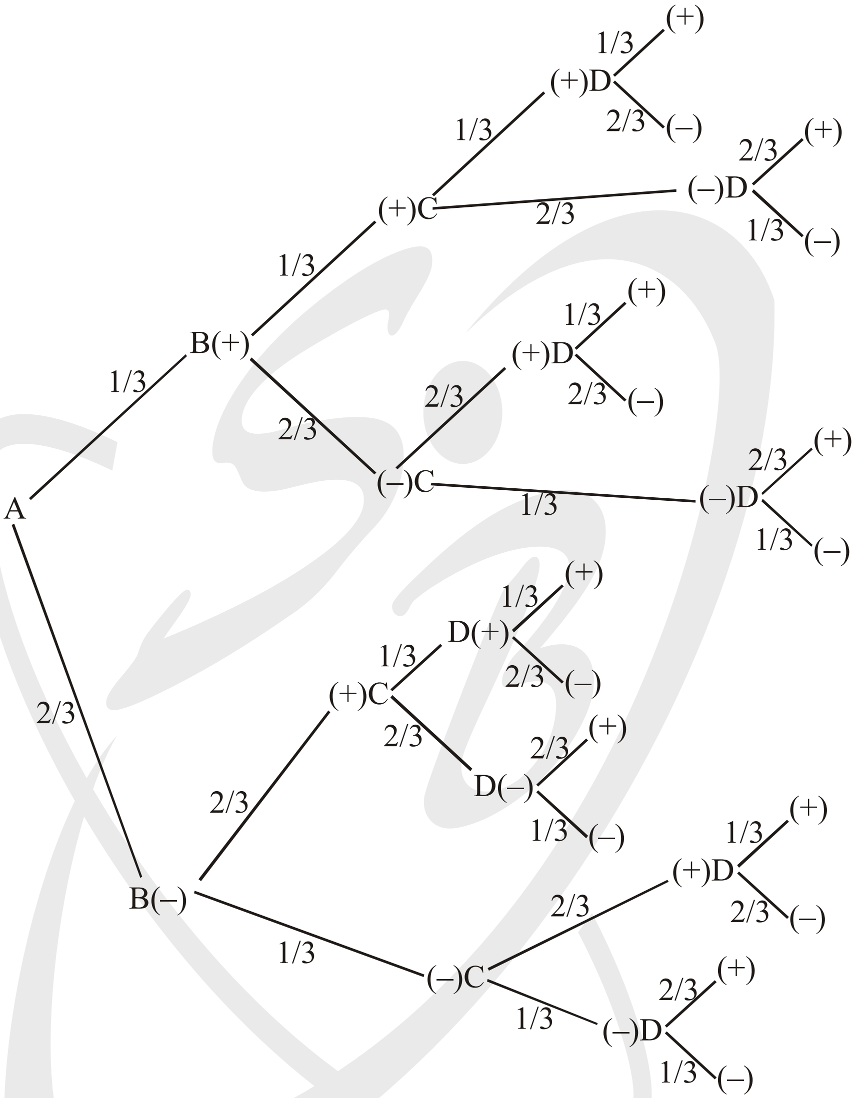
$$= \lim_{n\to\infty}\left(\frac{(n+4)(n+3)}{4\times 5}\frac{1\times 2}{(n+1)(n)}\right) = \frac{1}{10}$$

41. $B(n) = 1 - \frac{n^2 + 3n - 4}{n^2 + 3n + 2} = \frac{6}{(n+2)(n+1)}$

$$\lim_{n\to\infty}\sum_{r=1}^{n} B(r) = \lim_{n\to\infty} 6\left(\frac{1}{2} - \frac{1}{n+2}\right) = 3$$

COMPREHENSION (Q.42 TO Q.43)

Sol.



42. $P = \left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right) + \frac{1\times1\times2\times2}{3^4} + \frac{1\times2\times(2\times1+1\times2)}{3^4} + \frac{2\times2\times(1\times1+2\times2)}{3^4} + \frac{2\times1\times(2\times1+1\times2)}{3^4}$

$P = \frac{5+8+20+8}{81} = \frac{41}{81}$

43. $P = \dfrac{\frac{1}{3}\times\frac{1}{3}\times\left(\frac{1}{3}\times\frac{1}{3}+\frac{2}{3}\times\frac{2}{3}\right)+\frac{1}{3}\times\frac{2}{3}\times\left(\frac{2}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\right)}{\frac{41}{81}}$

$= \dfrac{5+2(4)}{41} = \dfrac{13}{41}$

Comprehension (Q.44 to Q.46)

44. P = P(W or BBW or BBBBW)

$= \dfrac{3}{10} + \dfrac{5}{10}\times\dfrac{4}{9}\times\dfrac{3}{8} + \dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{3}{6}$

$= \dfrac{83}{210}$

45. P = P(BW or BBBW or BBBBBW)

$= \dfrac{5}{10}\dfrac{3}{9} + \dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{3}{7} + \dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{1}{6}\dfrac{3}{5} = \dfrac{43}{210}$

46. P = P(R or BR or BBR or BBBR or BBBBR or BBBBBR)

$= \dfrac{2}{10} + \dfrac{5}{10}\dfrac{2}{9} + \dfrac{5}{10}\dfrac{4}{9}\dfrac{2}{8} + \dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7} + \dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{2}{6} + \dfrac{5}{10}\dfrac{4}{9}\dfrac{3}{8}\dfrac{2}{7}\dfrac{1}{6}\dfrac{2}{5}$

$= \dfrac{2}{5}$

SECTION-4

MATCH THE COLUMN

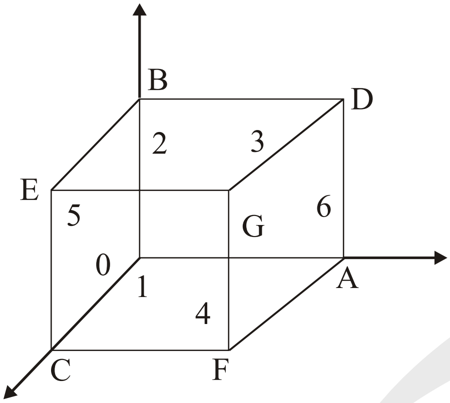
1. (A) $P = \dfrac{1\times {}^{11}C_5}{{}^{12}C_6} = \dfrac{1}{2}$

(B) $P = \dfrac{{}^{2}C_1\times {}^{10}C_5}{{}^{12}C_6} = \dfrac{6}{11}$

(C) $P = \dfrac{{}^{10}C_4}{{}^{12}C_6} = \dfrac{5}{22}$

(D) $P = \dfrac{10}{11}$

2.

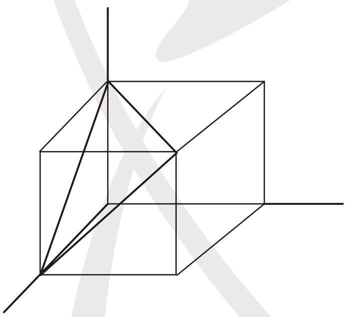


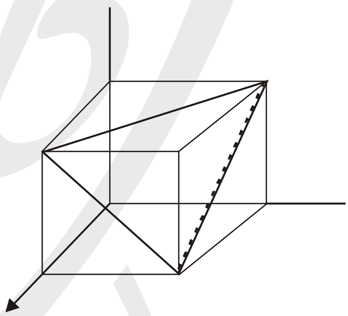
(A) P = $\dfrac{{}^4C_3 \times 6 + 4 \times 2}{{}^8C_3} = \dfrac{4}{7}$

(3 vertices in single phase) + (selecting adjacent faces 1-2 or 2-3 or 3-4 or 4-5 and one side 5 or 6 is selected automatically)

$1 \to$ CEGF, $2 \to$ EGDB, $3 \to$ OADB, $4 \to$ OCFA, $5 \to$ OCEB, $6 \to$ AFGD

e.g. $\to$





(B) $P = \dfrac{{}^4C_3 \times 6}{{}^8C_3} = \dfrac{3}{7}$

(C) $P = 1 - \dfrac{4}{7} = \dfrac{3}{7}$

(D) $P = \dfrac{{}^4C_3 \times 6 + 6 \times 4}{{}^8C_3} = \dfrac{48}{56} = \dfrac{6}{7}$

3. (A) $P = \dfrac{{}^2C_1 \times {}^{14}C_3}{{}^{16}C_4} = \dfrac{2}{5}$

(B) $P = \dfrac{{}^{14}C_2}{{}^{15}C_2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{20}$

(C) $P = \frac{1}{^{15}C_1} \times \frac{1}{2} = \frac{1}{30}$

(D) $P = \frac{^{14}C_6}{^{15}C_7} \times \frac{1}{2} = \frac{7}{30}$

4. (A) 7 digits not containing

(1, 8), (2, 7), (3, 6), (4, 5)

$\Rightarrow P = \frac{4 \times 7!}{^9C_7 \times 7!} = \frac{1}{9}$

(B) $P = 1 - \frac{^{12}C_2}{^{16}C_2} = \frac{9}{20}$

(C) 7^m ends with 7, 9, 3, 1

3^n ends with 3, 9, 7, 1

$(7^m, 3^n)$ ends with (7, 3), (9, 1), (3, 7), (1, 9)

$$P = \frac{25}{98} \times \frac{25}{98} + \frac{25}{98} \times \frac{24}{98} + \frac{24}{98} \times \frac{24}{98} + \frac{24}{98} \times \frac{25}{98} = \frac{(25+24)^2}{(98)^2} = \frac{1}{4}$$

(D) favourable ways are HHHHT or TTTTH $\Rightarrow P = \frac{2}{32} = \frac{1}{16}$

5. **(A)** $P = \frac{6![6! - (^6C_1 5! - ^6C_2 4! + ^6C_3 3! - ^6C_4 2! + ^6C_5 1! - ^6C_6)]}{6!6!}$

$= 1 - \frac{6! - 3 \times 5! + 20 \times 3! - 30 + 6 - 1}{6!}$

$= \frac{265}{720} = \frac{53}{144}$

(B) $P = \frac{6! \times 1}{6!6!} = \frac{1}{6!}$

(C) $P = \frac{6!\left[^6C_1 \times 1 \times (5! - ^5C_1 4! + ^5C_2 3! - ^5C_2 2! + ^5C_4 1! - ^5C_5)\right]}{6!6!}$

$= \frac{6 \times (44)}{6!} = \frac{44}{120} = \frac{11}{30}$

(D) $P = 1 - \left(\frac{53}{144} + \frac{11}{30}\right) = \frac{691}{720}$

6. **(A)** $P = \dfrac{{}^3C_2 + {}^3C_2}{{}^6C_3} = \dfrac{3}{10}$

(B) $b^2 = ac$

$(a, b, c) = (1, 2, 4)$

$\Rightarrow p = \dfrac{1}{{}^6C_3} = \dfrac{1}{20}$

(C) $\dfrac{1}{a} + \dfrac{1}{c} = \dfrac{2}{b}$

$b = 3, (a, c) = (2, 6)$

$b = 4, (a, c) = (3, 6)$

$\Rightarrow p = \dfrac{2}{20} = \dfrac{1}{10}$

(D) $a + b > c, \quad a < b < c$

$(a, b, c) = (2, 3, 4), (2, 4, 5), (3, 4, 5), (2, 5, 6)$

$(3, 4, 6), (3, 5, 6), (4, 5, 6)$

$\Rightarrow P = \dfrac{7}{20}$

7. (A) $P = \dfrac{1}{13}\left(\dfrac{{}^4C_4 + {}^5C_4 + \ldots.. + {}^{12}C_4}{{}^{12}C_4}\right) = \dfrac{1}{13}\dfrac{{}^{13}C_5}{{}^{12}C_4} = \dfrac{1}{5}$

(B) $P = \dfrac{{}^{10}C_4}{{}^{12}C_4} = \dfrac{14}{33}$

(C) $P = \dfrac{1}{13}\left(\dfrac{{}^2C_2\,{}^{10}C_2 + {}^3C_2\,{}^9C_2 + {}^4C_2\,{}^8C_2 + \ldots.. + {}^{10}C_2\,{}^2C_2}{{}^{12}C_4}\right)$

$= \dfrac{1}{13}\dfrac{2({}^2C_2\,{}^{10}C_2 + {}^3C_2\,{}^9C_2 + {}^4C_2\,{}^8C_2 + {}^5C_2\,{}^7C_2) + {}^6C_2\,{}^6C_2}{{}^{12}C_4} = \dfrac{1}{5}$

(D) $P = \dfrac{{}^{10}C_4}{{}^4C_4 + {}^5C_4 + \ldots. + {}^{12}C_4} = \dfrac{{}^{10}C_4}{{}^{13}C_5} = \dfrac{70}{429}$

8. (A) $P(m = 3) = \dfrac{\text{Throw resulting in}(3,4,5,6)-(4,5,6)}{\text{Total ways}} = \dfrac{4^5-3^5}{6^5}\left(\dfrac{2}{5}\right)^5 - \left(\dfrac{1}{2}\right)^5$

(B) $P(n = 4) = \dfrac{\text{Throw resulting in}(1,2,3,4)-(1,2,3)}{\text{Total ways}} = \dfrac{4^5-3^5}{6^5}$

(C) $P(m = 2, n = 5)$ = Throw resulting 2, 3, 4, 5 – atleast one of 2 or 5 not attained

$$= \frac{4^5-(3^5+3^5-2^5)}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^4 + \left(\frac{1}{3}\right)^5$$

(D) Smallest number is 2 – number are 5 or 6

$$P = \frac{5^5-2^5}{6^5} = \left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$$

9. (A) $P = 1 - P$ (all numbers are ≥ 10)

$$= 1 - \frac{{}^{16}C_4}{{}^{25}C_4} = 1 - \frac{182}{1265} = \frac{1083}{1265}$$

(B) $P = \dfrac{{}^{9}C_1 \times 1 \times {}^{15}C_2}{{}^{25}C_4} = \dfrac{189}{2530}$

(C) $P = P(1 \text{ even}, 3 \text{ odd}) + P(3 \text{ even}, 1 \text{ odd})$

$$= \frac{{}^{13}C_3\,{}^{12}C_1 + {}^{13}C_1\,{}^{12}C_3}{{}^{25}C_4} = \frac{286}{575}$$

(D) $P = 1 - P(abcd \text{ is odd})$

$$= 1 - \frac{{}^{13}C_4}{{}^{25}C_4} = \frac{217}{230}$$

SECTİON-5

SUBJECTIVE TYPE PROBLEMS

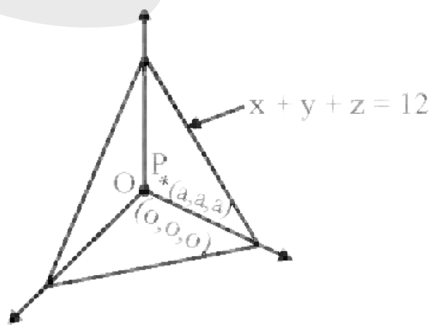
1. $((a-b)^2 + (b-c)^2 + (c-a)^2) \leq 0$

$\Rightarrow a = b = c$

$\because$ Points O (0, 0, 0) and P(a, a, a)

lie on name side of plane $x + y + z = 12$

$\Rightarrow \quad -12\,(a + a + a - 12) > 0$



$\Rightarrow \quad 0 < a < 4$

$\Rightarrow \quad a = \{1, 2, 3\}$

$$P = \frac{3}{6} = \frac{1}{2}$$

2. $f'(x) = 3x^2 + 2ax + b \geq 0 \quad \forall\, x \in R$

$\Rightarrow D \leq 0 \Rightarrow a^2 \leq 3b$

a	b
1	1
1, 2	2
1, 2, 3	3
1, 2, 3	4
1, 2, 3	5
1, 2, 3, 4	6

No. of favourable ways = 6 (1 + 2 + 3 (3) + 4) = 6 × 16

Total no. of ways = 6^3

$$P = \frac{16 \times 6}{6^3} = \frac{4}{9}$$

3. $$P = 7\left[\left(\frac{1}{7}\times\frac{1}{2}\right)\times\frac{1}{2}\times\frac{1}{2} + \left(\frac{6}{7}\times\frac{1}{2}\times\frac{1}{2}\right)\times\left(\frac{1}{3}\times\frac{1}{2}\right)\frac{1}{2} + \left(\frac{6}{7}\times\frac{1}{2}\times\frac{1}{2}\right)\left(\frac{2}{3}\times\frac{1}{2}\times\frac{1}{2}\right)\times\frac{1}{2}\right]$$

$$= \frac{3}{8}$$

= (paired in 1st round) + (paired in 2nd round) + (paired in 3rd round)

4. $$P = \frac{\dfrac{3}{13}\times\dfrac{{}^{11}C_1\,{}^{40}C_3}{{}^{51}C_4}}{\dfrac{3}{13}\times\dfrac{{}^{11}C_1\,{}^{40}C_3}{{}^{51}C_4} + \dfrac{10}{13}\times\dfrac{{}^{12}C_1\,{}^{39}C_3}{{}^{51}C_4}}$$

$$= \frac{11}{48}$$

5. __ __ __ $\underline{5}$ __ __ __ __

$$P = 1 - (\text{all } 2, 4, 6 \text{ in one side of } 5)$$

$$= 1 - \frac{3!\,3! \times 2}{7!}$$

$$= \frac{69}{70}$$

6. $E_1 \rightarrow$ boy watching doordarshan

$E_2 \rightarrow$ boy watching ten sports

$E \rightarrow$ boy fell asleep

$$P(E_1/E) = \frac{\frac{1}{5} \times \frac{3}{4}}{\frac{1}{5} \times \frac{3}{4} + \frac{4}{5} \times \frac{1}{4}} = \frac{3}{7}$$

7.

$$P(E_i) = \frac{1}{2}$$

$$P(E_m) = {}^{10}C_m \left(\frac{1}{2}\right)^{10}$$

$$P(E_i \cap E_m) = {}^{9}C_{m-1} \left(\frac{1}{2}\right)^{10}$$

$$P(E_i \cap E_m) = P(E_i)\,P(E_m)$$

$$\Rightarrow \quad {}^{9}C_{m-1} \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right) {}^{10}C_m \left(\frac{1}{2}\right)^{10}$$

$$\Rightarrow \quad m = 5$$

8.

$$P = \frac{26}{39} = \frac{2}{3}$$

$$p + q = 5$$

9. Let number of men = m

number of women = w

$$\Rightarrow \quad P = {}^{\alpha}C_1\left(\frac{m}{m+w}\right)^1\left(\frac{w}{m+w}\right)^{\alpha-1} + {}^{\alpha}C_3\left(\frac{m}{m+w}\right)^3\left(\frac{w}{m+w}\right)^{\alpha-3} + \ldots$$

$$= \frac{1}{2}\left(\left(\frac{m+w}{m+w}\right)^{\alpha} - \left(\frac{w-m}{w+m}\right)^{\alpha}\right)$$

$$\Rightarrow \quad \frac{1}{2}-\left(\frac{1}{2}\right)^{\alpha+1} = \frac{1}{2}\left(1-\left(\frac{\mu-1}{\mu+1}\right)^{\alpha}\right)$$

$$\Rightarrow \quad \frac{\mu-1}{\mu+1} = \frac{1}{2}$$

$$\Rightarrow \quad \mu = 3$$

10. Put $\quad x = -w, -w^2$

$$(-w)^{n+1} - (-w)^n + 1 = 0$$

$$\Rightarrow \quad (-w)^n w^2 + 1 = 0$$

$$\Rightarrow \quad (-1)^n w^{n+2} + 1 = 0$$

$$\Rightarrow \quad n = 6\lambda + 1$$

Also $n = 6\lambda + 1$ satisfy for $x = -w^2$

$$\Rightarrow \quad P = \frac{1}{6}$$

11. $p^2 \geq 4q$

q	**p**	**Number of (p, q)**
1	2,3,....,10	9
2	3,4,....,10	8
3	4,5,....,10	7
4	4,5,....,10	7
5	5,6,....,10	6
6	5,6,....,10	6

7	6,7,....,10	5
8	6,7,....,10	5
9	6,7,....,10	5
10	7,8,9,10	4

$$P = \frac{62}{10\times 10} = \frac{31}{50}$$

12. $$P = \frac{\left(\frac{11!}{2!2!2!5!}\right)}{\left(\frac{11!}{2!2!2!}\right)} = \frac{1}{120}$$

13. $$P = \frac{{}^{10}C_1\,{}^{10}C_1 + {}^{10}C_2}{{}^{30}C_2} = \frac{1}{3}$$

14. A $\rightarrow$ there is one undefeated team

B $\rightarrow$ there is one winless team

$P = 1 - \frac{n(A\cup B)}{2^{10}}$ ($\because$ total ways = 2^{10}, number of games = 10)

$$= 1 - \frac{{}^5C_1 2^6 + {}^5C_1 2^6 - {}^5C_2 \times 2! \times 2^3}{2^{10}}$$

$$= 1 - \frac{5\times 2^7 - 5\times 2^5}{2^{10}} = \frac{17}{32}$$

15. $x_1Rx_2Rx_3Rx_4\,Wx_5\,Wx_6\,Wx_7\,Wx_8$

$x_1 + x_2 + ... + x_8 = 3$

$$P = \frac{{}^{3+7}C_7}{\frac{10!}{3!4!3!}} = \frac{1}{35}$$

Alternate : observe that the position of blue balls is irrelevant for success. Thus we worry only about permutations of R R R W W W W. .

$$\Rightarrow P = \frac{4!3!}{7!}$$

16. Each person moves along 8 line segments. In order to meet, the persons must meet at diagonal points (0, 4), (1, 3), (2, 2), (3, 1) or (4, 0)

$$P = \frac{{}^4C_0\,{}^4C_4 + {}^4C_1\,{}^4C_3 + {}^4C_2\,{}^4C_2 + {}^4C_3\,{}^4C_1 + {}^4C_4\,{}^4C_0}{\sum\sum_{0\le i\le j\le 4} {}^4C_i\,{}^4C_j} = \frac{{}^8C_4}{\left(\sum_{i=0}^{4} {}^4C_i\right)\left(\sum_{j=0}^{4} C_j\right)}$$

$$= \frac{{}^8C_4}{2^8} = \frac{35}{128}$$

17. The generating function for throwing both the dice is $(3x^1 + 2x^2 + x^3)(x^1 + 2x^2 + 3x^3)$

$$= 3x^2 + 8x^2 + 14x^4 + 8x^5 + 3x^6$$

$\therefore$ 4 is the most likely sum, with probability of it occurring is $\frac{14}{36} = \frac{7}{18}$

18. $x_1 + x_2 + x_3 + x_4 = 10,\ x_i \in \{1, 2, 3, 4\}$

$$P = \frac{{}^{10-4+3}C_3 - {}^4C_1\,{}^{10-8+3}C_3}{(4)^4} = \frac{{}^9C_3 - 4\,{}^5C_3}{(4)^4} = \frac{11}{64}$$

Alternate : Favourable case = (1, 1, 4, 4) ; (1, 2, 3, 4), (1, 3, 3, 3), (2, 2, 2, 4), (2, 2, 3, 3)

$$P = \frac{\frac{4!}{2!2!} + 4! + \frac{4!}{3!} + \frac{4!}{3!} + \frac{4!}{2!2!}}{4^4} = \frac{44}{256} = \frac{11}{64}$$

19. The experiment consists in observing, among all ${}^{m+n}C_n$ configurations of heads and tails, the number of configurations of the form

$$y_1x_1y_2x_2y_3x_3y_4x_4y_5$$

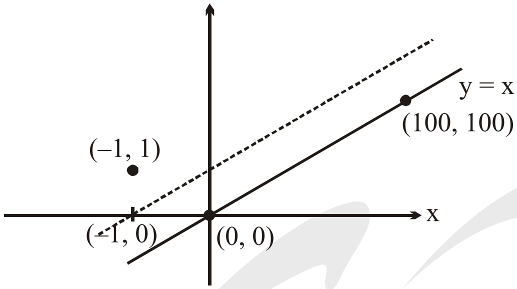
where x_k are filled with tails and the y_k are filled with tails. We need integral solutions to

$y_1 + y_2 + y_3 + y_4 + y_5 = 5,\quad y_1 \ge 0, y_5 \ge 0, y_k > 0, 2 \le k \le 4$

and $x_1 + x_2 + x_3 + x_4 = 10,\quad x_k > 0, 1 \le k \le 4$

$$\Rightarrow P = \frac{{}^{5-3+4}C_4 \times {}^{10-4+3}C_3}{{}^{15}C_{10}} = \frac{{}^6C_4\,{}^9C_3}{{}^{15}C_{10}} = \frac{60}{143}$$

20. Let till any point of time, there are 'x' 100 Re notes and 'y' 200 Re notes. Then for not having any problem at any time $x \geq y$.



Shift origin to (–1, 0) and reflect (0, 0) about y = x + 1.

Favourable ways = Total – going from (–1, 0) to (100, 100)

Favourable ways = ${}^{200}C_{100} - {}^{101+99}C_{101} = \frac{{}^{200}C_{100}}{101}$

$$P = \frac{\left(\frac{{}^{200}C_{100}}{101}\right)}{{}^{200}C_{100}} = \frac{1}{101}$$

21. $P = \frac{{}^{6}C_{2}\,{}^{5}C_{3}}{{}^{10}C_{5}} = \frac{5}{21}$

22. $P = \frac{{}^{30}C_{15}}{{}^{31}C_{15}} = \frac{16}{31}$

Alternate : $P = \frac{30}{31} \times \frac{14}{15} \times \frac{6}{7} \times \frac{2}{3} = \frac{16}{31}$

Solutions

CHAPTER 6

MATRICES

SECTION-1

SINGLE CHOICE QUESTIONS

1. $(PQ)^T = Q^TP^T = ((AB)^T - (BA)^T)((AB)^T + (BA)^T)$

$= (B^T.A^T - A^TB^T)(B^TA^T + A^TB^T)$

$= (BA - AB)(BA + AB)$

$= -QP$

2. $|A - \lambda I| = 0 \quad \Rightarrow \begin{vmatrix} 1-\lambda & -1 & 0 \\ -1 & 1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad (1-\lambda)((1-\lambda)^2 - 1) + 1(\lambda - 1) = 0$

$\Rightarrow \quad -\lambda^3 + 3\lambda^2 - \lambda - 1 = 0$

$\Rightarrow \quad A^3 - 3A^2 + A + I = O$

$\Rightarrow \quad A^2 - 3A + I + A^{-1} = O$

3. $|adj(adj2A)| = |adj\ 2A|^2 = |2A|^4 = (2^3|A|)^4$

$= 2^{12}|A|^4 = 2^{12}$

4. $|A_\lambda| = 4\lambda - 2$

$$\sum_{\lambda=1}^{300} |A_\lambda| = 4\left(\frac{300 \times 301}{2}\right) - 2(300) = 2(300)^2$$

5. $|A| = \begin{vmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{vmatrix} = xyz - 8x - 3z + 24 + 4 - 4y = 48$

$\therefore \quad A(adjA) = |A|\,I = 48\,I$

6. $M^2N^2N^{-1}(M^T)^{-1}(N^T)^{-1}M^T$

$$= M^2N^2N^{-1}M^{-1}N^{-1}M^T = M^2N(NM)^{-1}M$$

$$= M^2N(MN)^{-1}M = M^2NN^{-1}M^{-1}M = M^2$$

7. $AB = A \Rightarrow ABA = A^2 \Rightarrow AB = A^2 \Rightarrow A^2 = A$

$BA = B \Rightarrow BAB = B^2 \Rightarrow BA = B^2 \Rightarrow B = B^2$

$\therefore \quad A^2B^2 = AB = A$

8. $A + I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$(A + I)^2 = 2(A + I)$$

$\Rightarrow \quad A^2 = I, A^3 = A, A^4 = I, A^5 = A, \ldots\ldots\ldots.$

$\therefore \quad \sum_{r=1}^{2017} A^r = \left(\underbrace{A + A + \ldots.A}_{1009 \text{ times}}\right) + \left(\underbrace{I + I + \ldots.. + I}_{1008 \text{ times}}\right)$

$$= 1009\,A + 1008\,I$$

$$= \begin{bmatrix} 1008 & 1009 \\ 1009 & 1008 \end{bmatrix}$$

$|B| = (1008)^2 - (1009)^2 = -2017$

9. $(AB - BA)^T = (AB)^T - (BA)^T = B^TA^T - A^TB^T$

$$= BA - AB = -(AB - BA)$$

10. $AB = A, \quad BA = B$

$\Rightarrow \quad ABA = A^2 \quad \Rightarrow \quad AB = A^2 \quad \Rightarrow \quad A^2 = A$

$BA = B \quad \Rightarrow \quad BAB = B^2 \quad \Rightarrow \quad BA = B^2 \quad \Rightarrow \quad B = B^2$

$(A + B)^2 = A^2 + B^2 + AB + BA = 2(A + B)$

$(A + B)^4 = 4(A + B)^2 = 8(A + B)$

$(A + B)^6 = 8(A + B)\,(A + B)^2 = 16(A + B)^2 = 32(A + B)$

11. $(A^{-1}BA)^n = (A^{-1}BA)(A^{-1}BA)(A^{-1}BA)..........(A^{-1}BA)(A^{-1}BA)$

$= A^{-1}B(AA^{-1})B(AA^{-1})B(AA^{-1}).........B(AA^{-1})BA$

$= A^{-1}B^nA$

12. $AB = A - A^2 = O$

$BA = A - A^2 = O$

$AB + BA + I - (I - A)^2 = I - (I - 2A + A^2) = A$

13. $(I + A)^n = I + {}^nC_1A + {}^nC_2A^2 + ... + {}^nC_nA^n$

$= I + ({}^nC_1 + {}^nC_2 + {}^nC_3 + ...+ {}^nC_n)A$

$= I + (2^n - 1)A$

14. A must be a scalar matrix

$\Rightarrow \quad A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \Rightarrow \quad |A| = 6^3 = 216$

15. $A^2 + B = I \;\Rightarrow\; AAB + B^2 = B \;\Rightarrow\; B^2 = B$

$\therefore \quad A^2 + B^2 = A^2 + B = I$

16. $2x + 2 - 2y = 0$

$x + 4 + 2y = 0 \quad \Rightarrow \quad x = -2, y = -1$

17. $A^T_{1\times3}B_{3\times3}A_{3\times1} = (A^TBA)_{1\times1} = [0]$

18. $|(A - 5I)A^{99}| = |A - 5I|\,|A|^{99} = -109 \times 1 = -109$

19. $A = I + B, \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B^2 = O$

$A^n = (I + B)^n = I + {}^nC_1B = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

$$A + A^2 + A^3 + A^4 + A^5 = \begin{bmatrix} 5 & \sum_{r=1}^{5} r \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 0 & 5 \end{bmatrix}$$

$$|A + A^2 + A^3 + A^4 + A^5| = 25$$

20. $A^3 - 8A = O \Rightarrow A^4 - 8A^2 = O \Rightarrow (A^2 + I)(A^2 - 9I) = -9I$

$$\Rightarrow \quad (A^2 + I)\left(\frac{1}{9}(9I - A^2)\right) = I$$

$$\Rightarrow \quad (A^2 + I)^{-1} = \frac{1}{9}(9I - A^2)$$

21. $\begin{bmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \\ \lambda x_3 \end{bmatrix} \Rightarrow \begin{matrix} \lambda x_1 - \alpha x_2 - \beta x_3 = 0 \\ \alpha x_1 + \lambda x_2 - \gamma x_3 = 0 \\ \beta x_1 + \gamma x_2 + \lambda x_3 = 0 \end{matrix}$

For non trivial solution

$$\begin{vmatrix} \lambda & -\alpha & -\beta \\ \alpha & \lambda & -\gamma \\ \beta & \gamma & \lambda \end{vmatrix} = 0 \Rightarrow (\lambda^2 + \alpha^2 + \beta^2 + \gamma^2)\lambda = 0 \Rightarrow \lambda^2 = -(\alpha^2 + \beta^2 + \gamma^2)$$

22. $(A + B)^2 = A^2 + B^2 + AB + BA = A^2 + B^2 + 2AB$

$\Rightarrow \quad AB = BA$

23. $AA^T = I = A^TA$

$\Rightarrow A^TC^nA = A^T(ABA^T)(ABA^T) \ldots\ldots (ABA^T)A = B^n$

Let $\quad B = I + D \quad D = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

$$D^2 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B^n = (I + D)^n = I + nD = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$$

24. $BB^T = (I - A)(I + A)^{-1}((I + A)^T)^{-1}(I - A)^T$

$$= (I - A)(I + A)^{-1}(I - A)^{-1}(I + A)$$

$$\left[\begin{array}{l} \because (I+A)(I-A) = (I-A)(I+A) \\ \Rightarrow (I-A)^{-1}(I+A)^{-1} = (I+A)^{-1}(I-A)^{-1} \end{array}\right]$$

$\therefore$ $BB^T = (I - A)(I - A)^{-1}(I + A)^{-1}(I + A) = I$

25. $A + A^T = I \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a = \frac{1}{2}, d = \frac{1}{2}, b + c = 0 \qquad A = \begin{bmatrix} \frac{1}{2} & b \\ -b & \frac{1}{2} \end{bmatrix}$

$AA^T = I \Rightarrow \frac{1}{4} + b^2 = 0 \qquad \Rightarrow \qquad (b, c) = \left(\frac{1}{2}, -\frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right)$

SECTION-2

ONE OR MORE THAN ONE

1. $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$$a_{11}^2 + a_{12}^2 = a_{21}^2 + a_{22}^2 = 1 \ \& \ a_{11}a_{21} + a_{12}a_{22} = 0$$

If $a_{11} = 0, a_{12} = \pm 1, a_{22} = 0, a_{21} = \pm 1$

If $a_{12} = 0, a_{11} = \pm 1, a_{21} = 0, a_{22} = \pm 1$

$\therefore$ Number of matrices in set B = 2 × 2 + 2 × 2 = 8

$$|A - I| = |A| - (a_{11} + a_{22}) + 1$$

If $\quad a_{11} = a_{22} = 0, |A| = 1, a_{12}a_{21} = -1$

$\Rightarrow \quad A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

If $\quad a_{12} = 0 = a_{21}$

$\Rightarrow \quad |A - I| = a_{11}a_{22} - a_{11} - a_{22} + 1 = (a_{11} - 1)(a_{22} - 1)$

$\Rightarrow \quad a_{11} = -1, a_{22} = -1$

$\Rightarrow \quad A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\Rightarrow$ Number of matrices A = 3

2. $(A - \mu I)(B - \lambda I) = \lambda\mu I$

$\Rightarrow \quad |A - \mu I| \neq 0, |B - \lambda I| \neq 0$

$$\left(\frac{A}{\mu} - I\right)\left(\frac{B}{\lambda} - I\right) = I$$

$$\Rightarrow \quad \left(\frac{B}{\lambda} - I\right)\left(\frac{A}{\mu} - I\right) = I$$

$\Rightarrow \quad (B - \lambda I)(A - \mu I) = \lambda\mu I$

$\Rightarrow \quad BA = \lambda A + \mu B$

$\therefore \quad AB = BA$

3. $(AB)^n((AB)^n)^T = (AB)(AB)......(AB)\ (AB)^T(AB)^T........(AB)^T$

$= (AB)\ (AB).......(AB)(B^TA^T)\ (B^TA^T).......(B^TA^T)$

$= I \quad \forall\ n \in N$

$|AB| = \pm 1 \qquad [\because\ |A| = \pm 1, |B| = \pm 1]$

$|\ |AB|\ B| = |\pm B| = \pm 1$

$|\ |AB|\ A| = |\pm A| = \pm 1$

4. $A^T = A - B \quad \Rightarrow \quad A = A^T - B^T$

$B^T = B - C$

$\therefore \quad A = A^T - B^T = (A - B) - (B - C) = A - 2B + C$

$\Rightarrow \quad C = 2B \quad \Rightarrow \quad |C| = 2^n|B|$

$\therefore \quad B^T = B - C = B - 2B = -B$

$\Rightarrow \quad B^T = -B \quad \Rightarrow \quad |B| = 0$

$|A + B| = |A^T + B^T| = |(A - B) + (B - C)|$

$= |A - C| = |A - 2B|$

$\Rightarrow \quad |A + B| = |A - 2B|$

5. $\begin{bmatrix} a & 1 \\ b & c \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \Rightarrow \quad a = -1, b + 2c = 2$

$\begin{bmatrix} -1 & 1 \\ b & c \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \Rightarrow \quad x_1 = -1, y_1 = b$

$\begin{bmatrix} -1 & 1 \\ b & c \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad \Rightarrow \quad x_2 = 1, y_2 = c$

$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = \pm\frac{1}{2} \quad \Rightarrow \quad b + c = \pm 1$

$b + c = 1$ (rejected)

$b + c = -1, \quad \Rightarrow \quad b = -4, c = 3$

6. (a) tr(A) may be equal to 0

(b) $A^2 = I \quad \Rightarrow \quad A^2 + A^4 + ... + A^{50} = 25I$

$\Rightarrow \quad |A^2 + A^4 + ... + A^{50}| = (25)^n$

(c) $\text{adj}(|A|A) = |A| \text{ adj}(A)$

$|\text{adj}(|A|A)| = |A|^2 |\text{adj}(A)| = |A|^2 |A| = |A|^3$

7. $AB = A \quad \Rightarrow \quad ABA = A^2 \quad \Rightarrow \quad AB = A^2$

$BA = B \quad \Rightarrow \quad BAB = B \quad \Rightarrow \quad BA = B^2 \Rightarrow B = B^2$

$A^2 - AB + B^2 = B^2 = B$

8. $$A = \begin{bmatrix} 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & \frac{1}{2} & 0 \end{bmatrix}$$

$$|A - \lambda I| = 0 \quad \Rightarrow \quad \begin{bmatrix} -\lambda & \frac{1}{2} & 1 \\ \frac{1}{2} & -\lambda & \frac{1}{2} \\ 1 & \frac{1}{2} & -\lambda \end{bmatrix} = 0$$

$\Rightarrow \quad \lambda^3 - \frac{3}{2}\lambda - \frac{1}{2} = 0$

$\Rightarrow \quad 2A^3 - 3A - I = O$

9. $AB = B \Rightarrow BAB = B^2 \Rightarrow AB = B^2 \Rightarrow B = B^2$

$BA = A \Rightarrow ABA = A^2 \Rightarrow BA = A^2 \Rightarrow A = A^2$

10. $A = I + C \Rightarrow C = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 2 & 0 \end{bmatrix}$

$$(I + C)^{100} = I + {}^{100}C_1 C + {}^{100}C_2 C^2 + {}^{100}C_3 C^3 + \ldots\ldots + {}^{100}C_{100} C^{100}$$

$$C^2 = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$

$$C^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad A^{100} = I + 100C + 4950C^2$

$$B = A^{100} - I = \begin{bmatrix} 0 & 0 & 0 \\ 200 & 0 & 0 \\ 20200 & 200 & 0 \end{bmatrix}$$

11. $N = 6 \times 6 = 2^2 3^2 \Rightarrow$ Sum of divisors $= (1 + 2 + 2^2)(1 + 3 + 3^2) = 91$

$|A| = 1$ or -1

$|\text{adj } A| = |A|^2 = 1$

12. $AB = BA$

$(A + B)(A - B) = A^2 - B^2 = (A - B)(A + B)$

$\therefore \quad C = (A^T + B^T)(A + B)(A - B)^{-1} = (A - B)(A + B)(A - B)^{-1}$

$= (A + B)(A - B)(A - B)^{-1}$

$C = A + B$

$\Rightarrow \quad C^T = A - B = 2A - (A + B) = 2A - C$

13. $A = \begin{bmatrix} 2-\pi & 3-\pi & 4-\pi \\ 3-\pi & 4-\pi & 5-2\pi \\ 4-\pi & 5-2\pi & 6-2\pi \end{bmatrix}$

$\Rightarrow \qquad A^T = A$

$\Rightarrow \qquad (\text{adj } A)^T = \text{adj } A$

$\Rightarrow \qquad ((\text{adj } A)^T)^3 = (\text{adj } A)^3$

$\Rightarrow \qquad ((\text{adj } A)^3)^T = (\text{adj } A)^3$

$\text{Tr}(A) = 12 - 4\pi$

14. $|\text{adj } (\text{adj } A)| = |\text{adj } A|^2 = |A|^4$

$|\text{adj } 3A| = |3A|^2 = (3^3|A|)^2 = 3^6 \times 4$

$|\text{adj } (\text{adj } 3A)| = |\text{adj } 3A|^2 = 3^{12}2^4$

$|\text{adj } 2A| = |2A|^2 = (2^3|A|)^2 = 2^6 2^2 = 2^8$

15. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = I + B$

$B^2 = -I, (I + B)^n = ({}^nC_0 - {}^nC_2 + {}^nC_4 - {}^nC_6 + \ldots.) I + ({}^nC_1 - {}^nC_3 + {}^nC_5 - {}^nC_7 + \ldots.) B$

$\Rightarrow \qquad A^n = (2^{n/2}\cos\frac{n\pi}{4})I + (2^{n/2}\sin\frac{n\pi}{4})B$

$$A^n = \begin{bmatrix} 2^{n/2}\cos\frac{n\pi}{4} & 2^{n/2}\sin\frac{n\pi}{4} \\ -2^{n/2}\sin\frac{n\pi}{4} & 2^{n/2}\cos\frac{n\pi}{4} \end{bmatrix}$$

16. $x^2 - 5x + 7 = 0$

$x^5 = (x^2 - 5x + 7)(x^3 + 5x^2 + 18x + 55) + 149x - 385$

$x^5 = (A^2 - 5A + 7I)(A^3 + 5A^2 + 18A + 55I) + 149A - 385I$

$\Rightarrow \qquad A^5 = 149A - 385\,I$

17. Since x occurs in exactly 2 places, D(x) can atmost be a polynomial of degree 2.

$\Rightarrow \qquad D(x) = ax^2 + bx + c$

$c = 1, a - b = 0 \qquad 4a + 2a + 1 = 7$

$a = b = c = 1$

$D(x) = x^2 + x + 1$

18. $A^T = -A \quad \Rightarrow \quad AA^T = -A^2 = I$

19. $(\text{adj } B)\,\text{adj}(\text{adj}B) = |\text{adj } B|\, I$

$(B\,\text{adj } B)\,\text{adj}\,(\text{adj}B) = |\text{adj } B|\, B$

$\Rightarrow \quad |B|\,\text{adj}\,(\text{adj } B) = |B|^{n-1}B$

$\Rightarrow \quad \text{adj}(\text{adj}B) = |B|^{n-2}B$

$(\text{adj } A)^T = (|A|A^{-1})|^T = |A|(A^T)^{-1} = |A|A^{-1} = \text{adj } A$

20. $PQ = QP \quad \Rightarrow \quad (PQ)^T = Q^TP^T = QP = PQ$

$(P^{-1}Q)^T = Q^T(P^T)^{-1} = QP^{-1} = P^{-1}Q$

$\because \quad Q = QP \quad \Rightarrow \quad Q = P^{-1}QP \quad \Rightarrow \quad QP^{-1} = P^{-1}Q$

$(Q^{-1}P)^T = P^T(Q^T)^{-1} = PQ^{-1} = Q^{-1}P$

$[\because PQ = QP \Rightarrow Q^{-1}PQ = P \Rightarrow Q^{-1}P = PQ^{-1}]$

21. $(2A + 3BB^T)^T = I^T \quad \Rightarrow \quad 2A^T + 3BB^T = I \quad \Rightarrow \quad A = A^T$

$(B^{-1})^T = A \quad \Rightarrow \quad (B^T)^{-1} = A = A^T = B^{-1} \quad \Rightarrow \quad B^T = B$

Also, $B^{-1} = A \quad \Rightarrow \quad AB = BA = I$

$\therefore \quad 2A + 3B^2 = I \quad \Rightarrow \quad 2AB + 3B^3 = B \quad \Rightarrow \quad 2I + 3B^3 = B$

$\therefore \quad A^{-1} + I - AB - 3B^3 = B + I - I - 3B^3 = 2I$

22. $(A(\alpha))A(-\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = I$

$\Rightarrow \quad (A(\alpha))^{-1} = A(-\alpha)$

$$A(\alpha)\,A(\beta) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

$$\Rightarrow \quad A(\pi)\,A\left(\frac{3\pi}{2}\right)A\left(\frac{5\pi}{4}\right)A\left(\frac{7\pi}{8}\right)A\left(\frac{9\pi}{16}\right)....\infty = A\left(\pi + \frac{3\pi}{2} + \frac{5\pi}{4} + \frac{7\pi}{8} + ...\infty\right)$$

$$= A(6\pi) = I$$

23. $A = AB - BA$

$\Rightarrow \quad AA^{-1} = ABA^{-1} - BAA^{-1} \quad \Rightarrow \quad I + B = ABA^{-1}$

$A^{-1}A = A^{-1}AB - A^{-1}BA \quad \Rightarrow \quad B - I = A^{-1}BA$

$|I + B| = |B - I| = |B|$

24. (A) $|adj(kA)| = |kA|^{n-1} = (k^n|A|)^{n-1} = k^{n^2-n}\,|A|^{n-1}$

(C) $AB = 0 \quad \Rightarrow \quad A^{-1}(AB) = 0 \quad \Rightarrow \quad B = 0$

25. M is 1×1 matrix

$M^T = (A^TBA)^T = A^TB^TA = - \quad A^TBA = -M \quad \Rightarrow \quad M = -M \quad \Rightarrow \quad M = O$

30. $A^2 + A + I = 0$

$\Rightarrow \quad (A - I)(A^2 + A + I) = 0 \qquad [\because A - I \neq 0]$

$\Rightarrow \quad A^3 = I \quad \Rightarrow \quad A^{3k} = I, A^{3k+1} = A, A^{3k+2} = A^2$

$A^4 + A^9 = A + I = -(A^{-1})$

$A^6 = I \quad \Rightarrow \quad A^{-1} = A^5$

$A^7 + A^8 = A + A^2 = -I$

31. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|A + A^T| = 0 \quad \Rightarrow (b + c)^2 = 4ad \quad \Rightarrow (b - c)^2 = 4(ad - bc) > 0$

$|A - A^T| = (b - c)^2 > 0$

32. $C^3(A - B) = 0 \quad \Rightarrow \quad C^3 = 0 \qquad [\because |A - B| \neq 0]$

$A^4 - B^4 = 0 \quad \& \quad B^3A - A^3B = 0$

$\Rightarrow \quad (A^3 + B^3)(A - B) = 0$

$\Rightarrow \quad A^3 + B^3 = 0 \qquad [\because (A - B) \neq 0]$

$\Rightarrow \quad A^3 + B^3 + C^3 = 0$

33. $3ABA^{-1} + 3A = 2A^{-1}BA + 2A$

$$\Rightarrow \quad 3A(BA^{-1} + I) = 2(A^{-1}B + I)A$$

$$\Rightarrow \quad 3A(B + A)A^{-1} = 2A^{-1}(B + A)A$$

$$\Rightarrow \quad 3^n|A|\,|B + A|\,|A^{-1}| = 2^n\,|A^{-1}|\,|B + A|\,|A|$$

$$\Rightarrow \quad |B + A| = 0$$

Again, $\quad 2(A^{-1}BA - ABA^{-1}) = ABA^{-1} + A = A(BA^{-1} + I)$

$$\Rightarrow \quad 2(A^{-1}BA - ABA^{-1}) = A(B + A)\,A^{-1}$$

$$\Rightarrow \quad 2^n|A^{-1}\,BA - ABA^{-1}| = |A|\,|B + A|\,|A^{-1}| = 0$$

34. Check options

(B) $(A + B)\,(A^2 - AB - B^2) = A^3 - AAB - ABB + BAA - BAB - B^3$

$$= A^3 + ABA + BAA - B^3 = A^3 - BAA + BAA - B^3$$

$$= A^3 - B^3$$

(C) $(A + B)\,(A^2 + AB - B^2) = A^3 + AAB - ABB + BAA + BAB - B^3$

$$= A^3 - 2ABB + ABA + BAA - B^3$$

$$= A^3 - 2AB^2 - B^3$$

(A) $(A - B)\,(A^2 + AB + B^2) = A^3 + AAB + ABB - BAA - BAB - B^3$

$$= A^3 - B^3 + 2AB^2$$

SECTION-3

Comprehension (Q.1 To Q.3)

$(A - \lambda I)X = 0$

If $(A - \lambda I)^{-1}$ exist, then $X = 0$ which is not true

$$\Rightarrow \quad |A - \lambda I| = 0 \quad \Rightarrow \quad \begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad -\lambda^3 + (a_{11} + a_{22} + a_{33})\lambda^2 + (b)\lambda + c = 0$$

$\therefore \quad \lambda_1 + \lambda_2 + \lambda_3 = \text{Tr}(A)$

$\lambda_1\lambda_2\lambda_3 = c = |A|$

$AX = \lambda X \Rightarrow X = \lambda A^{-1}X \Rightarrow A^{-1}X = \frac{1}{\lambda}X$

$\text{Tr}(A^{-1}) = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$

$AX = \lambda X \Rightarrow A^2X = \lambda AX = \lambda^2 X$

$\Rightarrow \quad A^3X = \lambda^2 AX = \lambda^3 X$

$A^3X = \lambda^3 X$

$\text{Tr}(A^3) = \lambda_1^3 + \lambda_2^3 + \lambda_3^3$

$$\begin{vmatrix} 1-\lambda & 1 & 2 \\ 2 & 2-\lambda & 1 \\ 2 & 1 & 2-\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad -\lambda^3 + 5\lambda^2 - \lambda - 3 = 0$$

$(\lambda - 1)(\lambda^2 - 4\lambda - 3) = 0 \quad \Rightarrow \quad \lambda = 1, 2 + \sqrt{7}, 2 - \sqrt{7}$

$$\sum_{i=1}^{3} \frac{1}{\lambda_i} = \frac{1}{-3} = -\frac{1}{3}$$

$$\sum_{i=1}^{3} \lambda_i^3 = (\lambda_1 + \lambda_2 + \lambda_3)((\lambda_1 + \lambda_2 + \lambda_3)^2 - 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)) + 3\lambda_1\lambda_2\lambda_3$$

$= 5(25 - 3(1)) + 3(-3)$

$= 101$

Comprehension (Q.4 to Q.5)

4. For Δ to be odd 'a' must be odd

$\Rightarrow \quad P = \frac{5}{9}$

5. If a = 3, b = 5, then Δ is odd

$\Rightarrow \quad k + \Delta$ is even

Comprehension (Q.6 to Q.7)

$$A + B^T = \text{adj } B$$

$\Rightarrow$ $A^T + B = (\text{adj } B)^T$

and $B + A^T = \text{adj } A$

$\Rightarrow$ $(\text{adj } B)^T = \text{adj } A$

$\Rightarrow$ $|B|^2 = |A|^2 \quad \Rightarrow \quad |B| = \pm |A|$

$$(|B|\, B^{-1})^T = |A|\, A^{-1}$$

If $|B| = |A|$

$\Rightarrow$ $(B^{-1})^T = A^{-1} = (B^T)^{-1}$

$\Rightarrow$ $A = B^T$

$\therefore$ $\text{adj } B = 2B^T \quad \Rightarrow \quad |B|^2 = 2^3|B|$

$\Rightarrow$ $|B| = 8,\ |A| = 8$

If $|B| = -|A|$

$\Rightarrow$ $(-B^{-1})^T = A^{-1} = (-B^T)^{-1}$

$\Rightarrow$ $A = -B^T$

$\therefore$ $\text{adj } B = O$ which is impossible

Now, $A = B^T$

$$\text{adj } B = 2A$$

$$(\text{adj } B)\, B = 2AB = |B|\, I$$

$\Rightarrow$ $AB = 4\, I$

Also $B\text{adj } B = 2BA = |B|\, I$

$\Rightarrow$ $BA = 4I$

Comprehension (Q.8 to Q.10)

9. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} = \begin{bmatrix} a_1 - b_1 + 2c_1 & a_2 - b_2 + 2c_2 \\ 2a_1 - b_1 + c_1 & 2a_2 - b_2 + c_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$c_1 - a_1 = 1$; $b_1 = 1 + 3a_1$, $c_2 = -1 + a_2$, $b_2 = 3a_2 - 2$

10. $\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} a_1 + 4b_1 & a_2 + 4b_2 & a_3 + 4b_3 \\ 2a_1 - 3b_1 & 2a_2 - 3b_2 & 2a_3 - 3b_3 \\ 5a_1 + 4b_1 & 5a_2 + 4b_2 & 5a_3 + 4b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Impossible

$\Rightarrow$ No right matrix

$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} a_1 + 2a_2 + 5a_3 & 4a_1 - 3a_2 + 4a_3 \\ b_1 + 2b_2 + 5b_3 & 4b_1 - 3b_2 + 4b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

has infinite left inverse matrices

In option (d)

$\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} 3a_1 + 3b_1 & 3a_2 + 3b_2 & 3a_3 + 3b_3 \\ a_1 + b_1 & a_2 + b_2 & a_3 + b_3 \\ 4a_1 + 4b_1 & 4a_2 + 4b_2 & 4a_3 + 4b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ No right inverse matrix

$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 3a_1 + a_2 + 4a_3 & 3a_1 + a_2 + 4a_3 \\ 3b_1 + b_2 + 4b_3 & 3b_1 + b_2 + 4b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ No left inverse matrix.

Comprehension (Q.11 to Q.13)

11. Exactly one zero $= 4 \times ({}^{4-3+2}C_2) = 12$

Exactly two zeroes $= 2 \times ({}^{4-2+1}C_1) = 6$

Total number of invertible matrices A = 12 + 6 = 18

12. $|A|_{max} = 4$, $|A|_{min} = -4$

13. $\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

Comprehension (Q.14 to Q.15)

$$AX = \lambda X,\ X \neq 0 \Rightarrow \quad |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 11-\lambda & -4 & -7 \\ 7 & -2-\lambda & -5 \\ 10 & -4 & -6-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 0, 1, 2 \Rightarrow \lambda = 0, \mu = 1, \nu = 2$$

$$\left.\begin{aligned} 11x - 4y - 7z = 0 \\ 7x - 2y - 5z = 0 \\ 10x - 4y - 6z = 0 \end{aligned}\right\} \Rightarrow \quad x = y = z = \pm\frac{1}{\sqrt{3}} \Rightarrow \quad U = \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$\left.\begin{aligned} 10x - 4y - 7z = 0 \\ 7x - 3y - 5z = 0 \\ 10x - 4y - 7z = 0 \end{aligned}\right\} \Rightarrow \quad x = \frac{z}{2},\ y = -\frac{z}{2} \Rightarrow (x, y, z) = \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

$$\Rightarrow \quad V = \left(\frac{1}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}\right)$$

$$\left.\begin{aligned} 9x - 4y - 7z = 0 \\ 7x - 4y - 5z = 0 \\ 10x - 4y - 8z = 0 \end{aligned}\right\} \Rightarrow \quad x = z,\ y = \frac{z}{2} \Rightarrow \quad W = \left(\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$$

Comprehension (Q.19 to Q.21)

$$BB^T = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad B^TAB = [k]$$

$$M(x)\,M(y) = (I + xABB^T)\,(I + yABB^T) = I + (x + y)\,ABB^T + xyABB^TABB^T$$

$$= I + (x + y + kxy)ABB^T$$

$$= M(x + y + kxy)$$

$$M(x)\,M(y) = M(0) = I \quad \Rightarrow \quad x + y + kxy = 0 \quad \Rightarrow \quad y = \frac{-x}{1 + kx}$$

$$(M(x))^{-1} = M(y) = M\left(\frac{-x}{1 + kx}\right)$$

$$R = \begin{bmatrix} 1 & P & P \\ P & 1 & P \\ P & P & 1 \end{bmatrix} = \begin{bmatrix} (1-P)+P & P & P \\ P & (1-P)+P & P \\ P & P & (1-P)+P \end{bmatrix}$$

$$= \begin{bmatrix} 1-P & 0 & 0 \\ 0 & 1-P & 0 \\ 0 & 0 & 1-P \end{bmatrix} + \begin{bmatrix} P & P & P \\ P & P & P \\ P & P & P \end{bmatrix}$$

$$= (1 - P)\, I + PBB^T$$

Comprehension (Q.22 to Q.24)

$$|A - nI| = 0 \quad \begin{vmatrix} 1-n & 2 & 0 \\ 2 & 1-n & 0 \\ 0 & 0 & 1-n \end{vmatrix} = (1 - n)^3 - 4(1 - n)$$

$\Rightarrow$ $n = -1, 1, 3,\ n_1 = -1,\ n_2 = 1,\ n_3 = 3$

$\Rightarrow$ $A^3 - 3A^2 - A + 3I = O$

$$|A|\,|B|\,|A|^T = |N| = n_1n_2n_3 = -3 = |A|^2\,|B|$$

$$|B| = -\frac{1}{3}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ B^3 = 8\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Tr(A^k) = 3 + 2({}^kC_2 2^2 + {}^kC_4 2^4 + {}^kC_6 2^6 + \ldots\ldots.)$$

$$= 1 + 2\,({}^kC_0 + {}^kC_2 2^2 + {}^kC_4 2^4 + \ldots..) = 1 + 3^k + (-1)^k$$

Comprehension (Q.27 to Q.29)

$$R_1\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}=[1\ \ 0\ \ 0] \quad\Rightarrow\quad R_1=[1\ \ 0\ \ 0]$$

$$A^{-1}=\begin{bmatrix}1&0&0\\-2&1&0\\1&-2&1\end{bmatrix}$$

$R_1=[1\ \ 0\ \ 0]$

$R_2=[-4\ \ 3\ \ 0]$

$R_3=[-3\ \ 1\ \ 1]$

$$B=\begin{bmatrix}1&0&0\\-4&3&0\\-3&1&1\end{bmatrix} \qquad |B|=3$$

$$|(2A^{100}-A^{99}B)\,B^3|=|A^{99}(2A-B)B^3|=1\times(-1)\times 27=-27$$

$$2A-B=\begin{bmatrix}1&0&0\\8&-1&0\\9&3&1\end{bmatrix}$$

$$B^{-1}=\frac{1}{3}\begin{bmatrix}3&0&0\\4&1&0\\5&-1&3\end{bmatrix}$$

SECTION-4

MATCH THE COLUMN

1. $$A=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}\Rightarrow |A|=2,\ |A-I|=\begin{vmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{vmatrix}$$

$$A+I=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\Rightarrow |A+I|=0$$

$$\Rightarrow\quad (A+I)^2=\begin{bmatrix}3&3&3\\3&3&3\\3&3&3\end{bmatrix}=3(A+I)$$

$\Rightarrow \quad A^2 = A + 2I$

$$A^3 = A^2 + 2A = 3A + 2I$$

$$A^4 = 3A^2 + 2A = 5A + 6I$$

$$A = I + 2A^{-1}$$

$$2A^{-1} = A - I$$

$$A^{-1} + I = \frac{1}{2}(A - I) + I = \frac{1}{2}(A + I)$$

$$A^3 - A^2 + 2A = 4A$$

$\Rightarrow \quad |A^3 - A^2 + 2A| = 4^3|A| = 128$

$$A^4 - 4A - 7I = A - I$$

$$|A^4 - 4A - 7I| = |A - I| = 4$$

$$A^4 - 8A^{-1} - A^2 = 8I$$

$\Rightarrow \quad A^4 - 8A^{-1} - A^2 - 6I = 2I$

2. $$A(t) = \begin{bmatrix} 2\cos t & 1 & 0 \\ 1 & 2\cos t & 1 \\ 0 & 1 & 2\cos t \end{bmatrix}$$

$$|A(t)| = 2\cos t(4\cos^2 t - 1) - 2\cos t = 8\cos^3 t - 4\cos t$$

$$|A(t)| = 4\cos t \cos 2t$$

(A) $|A(t)| = 4 \Rightarrow t = 2n\pi, n \in I$

$\Rightarrow t = -2\pi, 0, 2\pi, 4\pi$

(B) $$\left|A\left(\frac{\pi}{17}\right)\right|\left|A\left(\frac{4\pi}{17}\right)\right| = 16\cos\frac{\pi}{17}\cos\frac{2\pi}{17}\cos\frac{4\pi}{17}\cos\frac{8\pi}{17}$$

$$= \frac{\sin\frac{16\pi}{17}}{\sin\frac{\pi}{17}} = 1$$

(C) $|A(t)| + |A(2t)| = 4\cos t\cos 2t + 4\cos 2t \cos 4t \le 8$

(D) $\int_0^{\pi} 16\cos t\cos 2t\cos 4t\cos 8t\,dt = \int_0^{\pi} \frac{\sin 16t\,dt}{\sin t}$

$$= \int_0^{\pi/2} \left(\frac{\sin 16t}{\sin t} + \frac{\sin(16\pi - 16t)}{\sin(\pi - t)} \right) dt$$

$= 0$

3. (A) $AB = O \Rightarrow A^{-1}AB = O \Rightarrow B = O$ and

(B) If $|A| \neq 0 \Rightarrow B = O$ and if $|B| \neq 0 \Rightarrow A = O$

$\Rightarrow |A| = 0$ and $|B| = 0$

(C) $A^n = O \Rightarrow |A| = 0$

(D) $AB = O \Rightarrow ABB^{-1} = O \Rightarrow A = O$

4. (A) $AB = BA \Rightarrow B^TA^T = A^TB^T \Rightarrow AB^TA^T = B^T$

$\Rightarrow AB^T = B^TA \Rightarrow BA^T = A^TB$

(B) $(X^TCX)^T = X^TC^TX = -X^TCX$

$\Rightarrow |X^TCX| = -|X^TCX| \Rightarrow |X^TCX| = 0$

$X^TCX = [0]$

(C) $A + I = \begin{bmatrix} 1 & 1......1 \\ 1 & 1......1 \\ 1 & 1......1 \end{bmatrix} \Rightarrow (A + I)^2 = n(A + I)$

$\Rightarrow A^2 + (2 - n)A + (1 - n)\,I = 0$

$\Rightarrow A + (2 - n)\,I = (n - 1)A^{-1}$

$\Rightarrow A^{-1} = \frac{A}{(n-1)} + \frac{2-n}{n-1} I$

$\lambda_1 = 2, \lambda_2 = 1$

5. $A^TD = DA^T$

$DD^T = D^TD = I$

$\Rightarrow \quad DD^T A^TD = DA^T \quad \Rightarrow \quad DA^{-1} = DA^T \Rightarrow A^{-1} = A^T$

$\Rightarrow \quad BC^2 = B^T \quad \Rightarrow \quad B = B^T$

(A) $B = B^{-1} \quad \Rightarrow \quad B^2 = I$

$\Rightarrow \quad |B^{2018}C| = |C| = \pm 1$

(B) $(ACA^T)^{2019} = (ACA^T)(ACA^T).....(ACA^T) = AC^{2019}A^T = AC^kA^T$

$\Rightarrow$ k is odd

(C) $|ACA^T| = |A|^2|C| = |C| = \pm 1$

(D) $|BB^T(B^{-1})^2| + |(AA^T)^{2018}| = 1 + 1 = 2$

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. $B^4 = (A^T)^2 - A^2$

$(B^4)^T = ((A^T)^2 - A^2)^T$

$\Rightarrow \quad (B^T)^4 = A^2 - (A^T)^2$

$\Rightarrow \quad (B^T)^4 = -B^4$

$\Rightarrow \quad |B^T|^4 = -|B|^4$

$\Rightarrow \quad |B| = 0$

2. $$Tr(A_n) = \frac{\sum_{i=1}^{3} i}{3^{2n-1}} = \frac{6}{3^{2n-1}}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} Tr(3^r A_r) = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{6}{3^{r-1}} = 6\left(\frac{1}{1-\frac{1}{3}}\right) = 9$$

3. $(AB)^2 = A^2B^2$

$(AB)^{-1}(AB)^2 = (AB)^{-1}A^2B^2$

$\Rightarrow \quad AB = B^{-1}A^{-1}A^2B^2 = B^{-1}AB^2$

$\Rightarrow \quad ABB^{-1} = B^{-1}AB^2B^{-1}$

$\Rightarrow \quad A = B^{-1}AB$

$\Rightarrow \quad BA = BB^{-1}AB = AB$

$BA^2B^{-1} = (AB)\,AB^{-1} = A(AB)B^{-1} = A^2$

4. $R = P^T(PAP^T)\,(PAP^T)......(PAP^T)P = A^8$

$$R = (I + B)^8,\ B = \begin{bmatrix} \sqrt{3}-1 & -2 \\ 0 & 0 \end{bmatrix}$$

$$R = {}^8C_0 I + {}^8C_1 B + {}^8C_2 B^2 + {}^8C_3 B^3 + ... + {}^8C_8 B^8$$

$$B^2 = \begin{bmatrix} \sqrt{3}-1 & -2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \sqrt{3}-1 & -2 \\ 0 & 0 \end{bmatrix} = (\sqrt{3}-1)B$$

$$R = I + {}^8C_1 B + {}^8C_2(\sqrt{3}-1)B + {}^8C_3(\sqrt{3}-1)^2 B + ...\ {}^8C_8(\sqrt{3}-1)^7 B$$

$$= I + \frac{B}{(\sqrt{3}-1)}\left({}^8C_1(\sqrt{3}-1) + {}^8C_2(\sqrt{3}-1)^2 + ... + {}^8C_8(\sqrt{3}-1)^8\right)$$

$$= I + \left(\frac{(1+\sqrt{3}-1)^8 - 1}{\sqrt{3}-1}\right)B = I + \frac{\left((\sqrt{3})^8 - 1\right)}{(\sqrt{3}-1)}B$$

$r_{11} = 81$

5. $$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$$

$$\text{adj}(A) = \begin{pmatrix} a_4 & -a_2 \\ -a_3 & a_1 \end{pmatrix}$$

$$A + |A|\ \text{adj}(A) = \begin{bmatrix} a_1 + (a_1a_4 - a_2a_3)a_4 & a_2 - a_2(a_1a_4 - a_2a_3) \\ a_3 - (a_1a_4 - a_2a_3)a_3 & a_4 + (a_1a_4 - a_2a_3)a_1 \end{bmatrix}$$

$|A + |A|\ \text{adj}\ A| = 0$

$\Rightarrow a_1a_4+(a_1a_4-a_2a_3)(a_4^2+a_1^2)+a_1a_4(a_1a_4-a_2a_3)^2-a_2a_3-a_3a_2(a_1a_4-a_2a_3)^2+2(a_1a_4-a_2a_3)a_2a_3 = 0$

$\Rightarrow \quad (a_1a_4 - a_2a_3)[1 + a_4^2 + a_1^2 + (a_1a_4 - a_2a_3)^2 + 2a_2a_3] = 0$

$\Rightarrow \quad (a_4 + a_1)^2 + 1 + (a_1a_4 - a_2a_3)^2 - 2(a_1a_4 - a_2a_3) = 0$

$\Rightarrow \quad a_1a_4 - a_2a_3 = 1 \ \& \ a_4 + a_1 = 0$

$$|A - |A|\text{adj } A| = \begin{vmatrix} a_1-(a_1a_4-a_2a_3)a_4 & a_2+a_2(a_1a_4-a_2a_3) \\ a_3+(a_1a_4-a_2a_3)a_3 & a_4-(a_1a_4-a_2a_3)a_1 \end{vmatrix} = 0$$

$$= a_1a_4 + a_1a_4(a_1a_4 - a_2a_3)^2 - (a_1a_4 - a_2a_3)(a_1^2 + a_4^2) - a_2a_3$$

$$- a_2a_3(a_1a_4 - a_2a_3)^2 - (a_1a_4 - a_2a_3)2a_2a_3$$

$$= (a_1a_4 - a_2a_3)[1 + (a_1a_4 - a_2a_3)^2 - a_1^2 - a_4^2 - 2a_2a_3]$$

$$= (a_1a_4 - a_2a_3)(2 + 2(a_1a_4 - a_2a_3)^2) = 1(2 + 2) = 4$$

6. $|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = a_1a_5a_9 - a_1a_6a_8 - a_2a_4a_9 + a_2a_6a_7 + a_3a_4a_8 - a_3a_5a_7$

$|A| \neq 0$ if a_1, a_5, a_9 all are equal to 1 and remaining 1's in any of remaining siz places in 6 ways.

Same process is repeated for other terms of expansion

$\therefore \quad N = 6 \times 6 = 36$

7. $A^2 - 2AB + BA - 2B^2 = O$

$\Rightarrow \quad (A + B)(A - 2B) = O$

$\Rightarrow \quad A - 2B = O$

$AB^{-1} = 2I$

$|AB^{-1}| = 2^3(1) = 8$

8. $\text{Tr}(ABC) = -43 - x^2 + 5bx$

$\therefore \quad -43 - x^2 + 5bx \leq -18 \quad \forall\, x \in R$

$\Rightarrow \quad x^2 - 5bx + 25 \geq 0 \quad \forall\, x \in R$

$\Rightarrow \quad 25b^2 - 100 \leq 0 \quad \Rightarrow \quad b \in [-2, 2]$

9. $|B| = |A \text{adj} A| = ||A|\, I| = |A|^3$

$$|A| = \begin{vmatrix} 1 & -1 & -2 \\ 1 & 4 & -1 \\ 2 & 1 & 9 \end{vmatrix} = 62$$

$$\Rightarrow \quad \left[\frac{\sqrt[3]{|B|}}{8}\right] = \left[\frac{62}{8}\right] = 7$$

10. (adj A) adj(adj A) = |adj A| I = $|A|^2$ I

$\Rightarrow$ (A adjA) adj (adj A) = $|A|^2A$

$\Rightarrow$ (|A| I) adj (adj A) = $|A|^2A$

$\Rightarrow$ adj (adj A) = |A|A = 3A

11. $AB^2 = BA \Rightarrow A^6B^2 = A^5BA \Rightarrow B^2 = A^5BA$

$B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$

$B^4 = A^4BA^2$

$B^8 = A^4BA^2A^4BA^2 = A^4B^2A^2 = A^4(A^5BA)A^2$

$\Rightarrow B^8 = A^3BA^3$

$B^{16} = A^3BA^3A^3BA^3 = A^3B^2A^3 = A^3A^5BAA^3$

$\Rightarrow B^{16} = A^2BA^4$

Similarly $B^{32} = ABA^5$ & $B^{64} = BA^6 = B$

$\therefore B^{64} = B$

$B^{-1}B^{64} = B^{-1}B$

$\Rightarrow B^{63} = I$

12. $AB = \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix}, (AB)^2 = \begin{bmatrix} 25x^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 50x^2-2 & 25x^2 \end{bmatrix} = I$

$= 25x^2 = 1 \Rightarrow 5x = -1$

$Tr(AB + (AB)^2 + ... + (AB)^6) = 2(5x + (5x)^2 + (5x)^3 + ... (5x)^6) + 6$

$= 2(-1 + 1 - 1 + 1 - 1 + 1) + 6 = 6$

13. $A^{-1} + B^{-1} = (A + B)^{-1} \Rightarrow (A^{-1} + B^{-1})(A + B) = (A + B)^{-1}(A + B)$

$\Rightarrow 2I + A^{-1}B + B^{-1}A = I \quad \Rightarrow A^{-1}B + B^{-1}A = -I$

Let $A^{-1}B = P$

$P + P^{-1} = -I \Rightarrow P^2 + I = -P \Rightarrow P^2 + P + I = O$

$\Rightarrow (P - I)(P^2 + P + I) = O \Rightarrow P^3 = I$

$\Rightarrow |P| = 1 \Rightarrow |A^{-1}|\,|B| = 1 \Rightarrow |A| = |B|$

14. $A^2 + A\,\text{adj}A = AA^{-1} \Rightarrow A^2 = (1 - |A|)\,I$

$\Rightarrow |A|^2 = (1 - |A|)^2 \Rightarrow |A| = \frac{1}{2}$

$|2A^{-1}| = 4|A^{-1}| = \frac{4}{|A|} = 8$

15. $N = (3! + 1)\,3 \times 3 \times 3 = 189$

16. $A(\text{adj}A + \text{adj}B) = B \Rightarrow |A|\,I + A\,\text{adj}B = B$

$\Rightarrow B + A(\text{adj}B)\,B = B^2$

$\Rightarrow B + A = B^2$

$\Rightarrow |A + B| = |B|^2 = 1$

17. 1st element of $A_{10} = a_{1^2+2^2+...+9^2+1} = a_{286}$

last element of $A_{10} = a_{1^2+2^2+....+10^2} = a_{385}$

$Tr(A_{10}) = [\log_2 286] + [\log_2 297] + [\log_2 308] + ... [\log_2 385]$

$= 8 + 8 + ... + 8 = 80$

18. $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\Rightarrow a - b = -1,\ c - d = 2$

$A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = A\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Rightarrow -a + 2b = 1,\ -c + 2d = 0$

$\Rightarrow d = 2,\ c = 4,\ a = -1,\ b = 0$

19. $PP^T = P^TP = I \quad \Rightarrow X = A^{50} = (I + B)^{50}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$B^2 = 0 \qquad \Rightarrow A^{50} = I + 50B = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

22. $$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = 2\sqrt{2}\begin{bmatrix} \cos\frac{\pi}{12} & -\sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix}$$

$$= \left(2\sqrt{2}\right)^2\begin{bmatrix} \cos\frac{\pi}{12} & -\sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}^2\begin{bmatrix} a_{n-2} \\ b_{n-2} \end{bmatrix}$$

$$= \left(2\sqrt{2}\right)^{n-1}\begin{bmatrix} \cos\frac{\pi}{12} & -\sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}^{n-1}\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$\because \quad \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} a_{22} \\ b_{22} \end{bmatrix} = \left(2\sqrt{2}\right)^{21}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2^{31}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$= 2^{32}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

23. $$|A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 - a_1 & a_5 - a_2 & a_6 - a_3 \\ a_7 - a_1 & a_8 - a_2 & a_9 - a_3 \end{vmatrix}$$

$\because$ a_i's are all odd

$\Rightarrow$ All of elements of 2nd and 3rd row are even

$\Rightarrow$ $|A|$ is divisible by 4

$\therefore$ $|A|$ can be equal to –4, 0, 4 only

24. $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & (a+d)b \\ c(a+d) & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

If $a + d = 0, a^2 + bc = 1 \quad b = 0, c = 0, d = 1, -1, \quad a = 1, -1$

or $\quad c = 0, b = 1, -1, a = 1, -1$

If $\quad b = 0, c = 0, a = 1, -1, d = 1, -1$

Number of quadruples (a, b, c, d) = 16

25. $|A - \lambda I| = 0 \quad \Rightarrow \quad \begin{vmatrix} \frac{1}{2}-\lambda & \frac{1}{2} \\ a & b-\lambda \end{vmatrix} = 0$

$$\Rightarrow \lambda^2 - \left(b + \frac{1}{2}\right)\lambda + \frac{b-a}{2} = 0$$

$A^2 = xA - yI \quad ; \quad x = b + \frac{1}{2}, y = \frac{b-a}{2}$

$A^3 = xA^2 - yA = x(xA - yI) - yA$

$A^3 = (x^2 - y)A - xyI$

$A^3 = A \quad \Rightarrow \quad (x^2 - y - 1)A = xyI$

$\therefore \quad \frac{x^2-y-1}{2} = xy \quad \frac{x^2-y-1}{2} = 0 \quad$ and $\quad xy = 0$

If $x = 0, y = -1$ or $y = 0, x = \pm 1$

$b + \frac{1}{2} = 0, \frac{b-a}{2} = -1 \Rightarrow \quad b = -\frac{1}{2}, a = \frac{3}{2}$

$b - a = 0, b + \frac{1}{2} = 1 \Rightarrow \quad b = \frac{1}{2}, a = \frac{1}{2}$

$b - a = 0, b + \frac{1}{2} = -1 \Rightarrow \quad b = -\frac{3}{2}, a = -\frac{3}{2}$

$(a, b) = \left(\frac{3}{2}, -\frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(-\frac{3}{2}, -\frac{3}{2}\right)$

Solutions

CHAPTER 7

COMPLEX NUMBERS

SECTION-1

SINGLE CHOICE QUESTIONS

1. $z_r - z_0 = (z_1 - z_0)e^{i\frac{2\pi}{n}(r-1)}$

$$\sum_{r=1}^{n} z_r^k = (z_0 + (z_1 - z_0)) + \left(z_0 + (z_1 - z_0)e^{i\frac{2\pi}{n}}\right)^k + \left(z_0 + (z_1 - z_0)e^{i\frac{4\pi}{n}}\right)^k$$

$$+ \ldots + \left(z_0 + (z_1 - z_0)e^{i\frac{2\pi(n-1)}{n}}\right)^k$$

$$= nz_0^k \sum_{r=1}^{k} {}^kC_r (z_1 - z_0)^r z_0^{k-r}\left(1 + e^{i\frac{2\pi}{n}r + i\frac{4\pi}{n}r} + \ldots + e^{i\frac{2\pi(n-1)}{n}r}\right)$$

$$= nz_0^k + \sum_{r=1}^{k} {}^kC_r (z_1 - z_0)^r z_0^{k-r}\left(\frac{1 - e^{i2\pi r}}{1 - e^{i\frac{2\pi}{n}r}}\right)$$

$$= nz_0^k$$

2. $\left(\left|\alpha - \sqrt{\alpha^2 - \beta^2}\right| + \left|\alpha + \sqrt{\alpha^2 - \beta^2}\right|\right)^2$

$$= \left|\alpha - \sqrt{\alpha^2 - \beta^2}\right|^2 + \left|\alpha + \sqrt{\alpha^2 - \beta^2}\right|^2 + 2\left|(\alpha - \sqrt{\alpha^2 - \beta^2})(\alpha + \sqrt{\alpha^2 - \beta^2})\right|$$

$$= 2\left(|\alpha|^2 + \left|\sqrt{\alpha^2 - \beta^2}\right|^2\right) + 2\left|\alpha^2 - (\alpha^2 - \beta^2)\right|$$

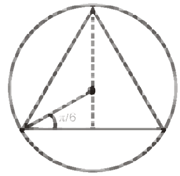
$$= 2\left(|\alpha|^2 + |\beta|^2\right) + 2|\alpha - \beta||\alpha - \beta|$$

$$= (|\alpha + \beta|^2 + |\alpha - \beta|^2 + 2|\alpha - \beta|\,|\alpha + \beta|$$

$$= (|\alpha + \beta| + |\alpha - \beta|)^2$$

3. Roots are vertices of equilateral triangle lying on circle with radius $|\alpha|$

$$\text{Length of side} = \left(|\alpha|\cos\frac{\pi}{6}\right) = \sqrt{3}\,|\alpha|$$



4. $$\frac{a^2}{|z_2 - z_3|^2} = \frac{b^2}{|z_3 - z_1|^2} = \frac{c^2}{|z_1 - z_2|^2} = \lambda$$

$$\Rightarrow \quad \frac{a^2}{(z_2 - z_3)} + \frac{b^2}{(z_3 - z_1)} + \frac{c^2}{(z_1 - z_2)} = \lambda(\bar{z}_2 - \bar{z}_3 + \bar{z}_3 - \bar{z}_1 + \bar{z}_1 - \bar{z}_2) = 0$$

5. $|\bar{z}_1 z_2| = \sqrt{2}$

$$\arg z_1 - \arg z_2 = \frac{\pi}{4} = -\arg(\bar{z}_1) - \arg z_2$$

$$\Rightarrow \quad \arg(\bar{z}_1 z_2) = -\frac{\pi}{4}$$

$$\Rightarrow \quad \bar{z}_1 z_2 = \sqrt{2}e^{-i\frac{\pi}{4}} = 1 - i$$

6. $z_1 + z_2 + z_3 = 1 + i$

$$\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$$

$$\Rightarrow \quad z_1z_2 + z_2z_3 + z_3z_1 = 0$$

$$z_1^2 + z_2^2 + z_3^2 + 2\Sigma z_1z_2 = 2i$$

$$\Rightarrow \quad z_1^2 + z_2^2 + z_3^2 = 2i$$

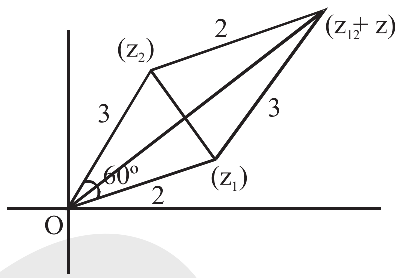
7. $\bar{z} - z = |z|(e^{-i\theta} - e^{i\theta}) = -2|z|i\sin\theta = ki,\ k > 0$

$$\therefore \quad \arg\left(\frac{\bar{z} - z}{2019}\right) = \frac{\pi}{2}$$

8. $|z_1 + z_2| = \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos 120^{o}}$

$$= \sqrt{2^2 + 3^2 - 2(2)(3)\left(-\frac{1}{2}\right)} = \sqrt{19}$$

$$|z_1 - z_2| = \sqrt{2^2 + 3^2 - 2(2)(3)\frac{1}{2}} = \sqrt{7}$$



9. A and B sets represents two non parallel lines. Hence, no. of intersection points = 1.

10. $|3z_1z_2 + z_2z_3 + 2z_3z_1| = ||z_3|^2 z_1z_2 + |z_1|^2 z_2z_3 + |z_2|^2 z_3z_1||$

$$= |z_1z_2z_3(\bar{z}_3 + \bar{z}_1 + \bar{z}_2)| = |z_1z_2z_3||z_1 + z_2 + z_3|$$

$$= 2\sqrt{6}$$

11. $z = \left(\frac{1}{z_1} + \frac{1}{z_2}\right)\left(\frac{1}{z_2} + \frac{1}{z_3}\right)\left(\frac{1}{z_3} + \frac{1}{z_4}\right)\ldots\left(\frac{1}{z_{n-1}} + \frac{1}{z_n}\right)\left(\frac{1}{z_n} + \frac{1}{z_1}\right)z_1z_2\ldots z_n$

$$= \frac{(\bar{z}_1 + \bar{z}_2)(\bar{z}_2 + \bar{z}_3)\ldots(\bar{z}_{n-1} + \bar{z}_n)(\bar{z}_n + \bar{z}_1)}{\bar{z}_1\bar{z}_2\ldots\bar{z}_n}$$

$$= \bar{z}$$

12. $|z|^{2017} = |p|\,|\bar{z}|$

$\Rightarrow$ $|z| = 0$ or $|z| = |p|^{\frac{1}{2016}}$

$\therefore$ $z^{2018} = p|z|^2$

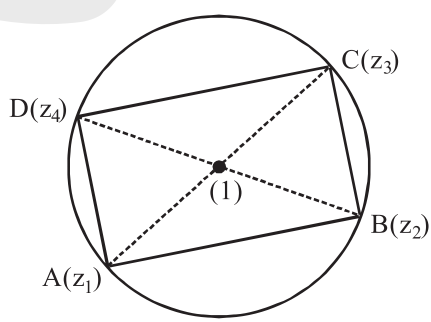
$\Rightarrow$ $z^{2018} = 0$ or $|p|^{\frac{1}{1008}}p$

$\Rightarrow$ No. of solutions = 1 + 2018 = 2019.

13. $\frac{z_1 + z_3}{2} = 1$

$$\frac{z_2 + z_4}{2} = 1$$

$\Rightarrow$ $z_1 + z_2 + z_3 + z_4 = 4$



14. $\frac{7z_2}{5z_1} = ik \qquad k \in R$

$\Rightarrow \qquad \frac{z_2}{z_1} = \left(\frac{5}{7}k\right)i$

$\therefore \qquad \left|\frac{2+3\frac{z_2}{z_1}}{2-3\frac{z_2}{z_1}}\right| = \left|\frac{2+\left(\frac{15}{7}k\right)i}{2-\left(\frac{15}{7}k\right)i}\right| = 1$

15. $z = \left(\frac{1-i}{1+i}\right)^n = \left(\frac{-2i}{2}\right)^n = (-i)^n$

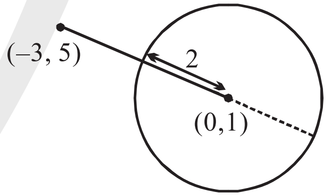
$I_m(z) < 0 \quad \text{for} \quad n = 4k + 1 \qquad k \in I$

Sum = 1 + 5 + 9 +......97 = 1225

16. $|z - iz_0| = |z + 3 - 5i|$

$5 - 2 \le |z + 3 - 5i| \le 5 + 2$

$\Rightarrow \qquad 3 \le |z + 3 - 5i| \le 7$

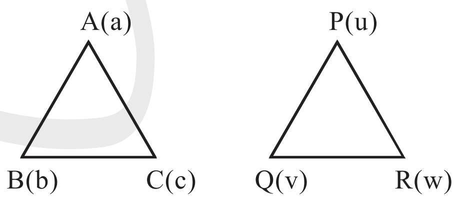


17. $z_1 = -2 + 4z$

$\Rightarrow \qquad |z_1 + 2| = 4|z|$

$\Rightarrow \qquad |z_1 + 2| = 4$

18. $\frac{c-a}{b-a} = \frac{w-u}{v-u}$



$\Rightarrow \qquad \frac{AC}{AB} = \frac{PR}{PQ}$ and $\angle A = \angle P$

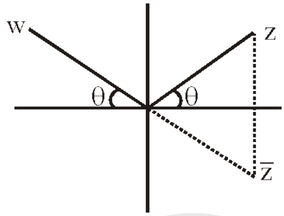
$\Rightarrow \qquad \Delta BAC \sim \Delta QPR$

19. sinx = cosx and cos2x = sin2x

$\Rightarrow \qquad x \in \phi$

20. $(1 + \omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

21. $w = -\overline{z}$



$\Rightarrow$ $z = -\overline{w}$

22. z lies on ⊥ar bisector of two conjugate complex numbers –iw and $i\overline{w}$

$\Rightarrow$ z is purely real.

$\therefore$ $z = k,\ k \in R$

$$|z + iw| \le |z| + |iw| \le 2$$

$\Rightarrow$ $|z| = 1$

$\Rightarrow$ $z = 1$ or -1

23. $(1+i)^{n_1} + (1-i)^{n_1} = z_1 + \overline{z}_1 = 2\,\text{Re}(z_1)$

$$(1+i)^{n_2} + (1-i)^{n_2} = z_2 + \overline{z}_2 = 2\,\text{Re}(z_2)$$

$\Rightarrow$ $(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2} \in R\ \ \forall\ n_1, n_2 > 0$

24. $w = \dfrac{(\cos\theta - 1) + i\sin\theta}{(\cos\theta + 1) + i\sin\theta}$

$$= w = \frac{\left((\cos\theta - 1) + i\sin\theta\right)\left((\cos\theta + 1) - i\sin\theta\right)}{(\cos\theta + 1)^2 + \sin^2\theta}$$

$$\text{Re}(w) = \frac{(\cos^2\theta - 1) + \sin^2\theta}{(\cos\theta + 1)^2 + \sin^2\theta} = 0$$

25. $z^n - 1 = (z - 1)(z - a_1)(z - a_2)...(z - a_{n-1})$

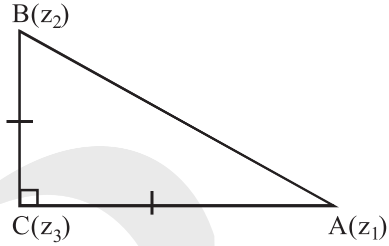
$\Rightarrow$ $1 + z + z^2 + ... + z^{n-1} = (z - a_1)(z - a_2)\ ...\ (z - a_{n-1})$

Put $z = 1$, $n = (1 - a_1)(1 - a_2)\ ...\ (1 - a_{n-1})$

26. Put $x = 0, \omega, \omega^2$

Expression reduces to zero.

27. $z_2 - z_3 = (z_1 - z_3)e^{i\frac{\pi}{2}}$



$$(z_2 - z_3)^2 = -(z_1 - z_3)^2$$

$$z_2^2 + 2z_3^2 + z_1^2 - 2z_1z_3 - 2z_2z_3 = 0$$

$$(z_1 - z_2)^2 + 2z_3^2 + 2z_1z_2 - 2z_2z_3 - 2z_1z_3 = 0$$

$$(z_1 - z_2)^2 + 2(z_3 - z_2)(z_3 - z_1) = 0$$

$$\Rightarrow \quad (z_1 - z_3)(z_3 - z_2) = \frac{1}{2}(z_1 - z_2)^2$$

28. $xyz = (a + b)(a\omega + b\omega^2)(a\omega^2 + b\omega)$

$= (a + b)(a\omega + b\omega^2)(a\omega^2 + b\omega^4)$

$= (a + b)(a + b\omega)(a + b\omega^2)$

$= a^3 + b^3$

29. $((1 + i)x - 2i)(3 - i) + ((2 - 3i)y + i)(3 + i) = 10i$

$(x + (x - 2)i)(3 - i) + (2y + (1 - 3y)i)(3 + i) = 10i$

$\Rightarrow \quad 4x + 9y = 3$ and $2x - 7y = 13$

$\Rightarrow \quad x = 3, y = -1$

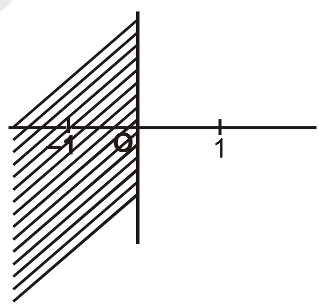
30. $\sum_{r=1}^{n}(r-1)(r-\omega)(r-\omega^2) = \sum_{r=1}^{n}(r-1)(r^2+r+1) = \sum_{r=1}^{n}(r^3-1)$

$$= \frac{n^2(n+1)^2}{4} - n$$

$$= \frac{n(n-1)(n^2+3n+4)}{4}$$

31. $e^{i(\alpha-\beta)} + e^{i(\beta-\gamma)} + e^{i(\gamma-\alpha)} = 1$

$\Rightarrow \cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = 1$



32. $|z + 1| < |z - 1|$

$\Rightarrow \quad \text{Re}(z) < 0$

33. Put $z = k$, $k \in R$

$$k^2 + pk + r = 0 \text{ and } qk + s = 0 \quad \Rightarrow \quad k = -\frac{s}{q}$$

$$\therefore \quad \frac{s^2}{q^2} - \frac{ps}{q} + r = 0 \quad \Rightarrow \quad s^2 + rq^2 = pqs$$

34. $(az_1 - bz_2)(a\bar{z}_1 - b\bar{z}_2) + (bz_1 + az_2)(b\bar{z}_1 + a\bar{z}_2)$

$$= a^2|z_1|^2 + b^2|z_2|^2 + b^2|z_1|^2 + a^2|z_2|^2$$

$$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$$

35. $-\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) 2\sin\frac{x}{2} + \tan x = 0$

$$\Rightarrow \quad (\tan x - 1)(1 - \cos x) = 0$$

$$\Rightarrow \quad x = 0, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}$$

SECTION-2

ONE OR MORE THAN ONE

1. Let $z = x + iy$, $x \in R, y \in R$

$$\Rightarrow \quad x\sqrt{x^2 + y^2} + ax = 0 \text{ and } y\sqrt{x^2 + y^2} + ay + 1 = 0$$

$$x = 0, \ \sqrt{x^2 + y^2} + a = 0 \text{ doesn't satisfy as } a > 0$$

$$\therefore \quad x = 0, \ y|y| + ay + 1 = 0$$

If $y \geq 0$, $y^2 + ay + 1 = 0$ doesn't satisfy as $a > 0$

If $y < 0$, $y^2 - ay - 1 = 0 \quad \Rightarrow \quad y = \frac{a \pm \sqrt{a^2 + 4}}{2}$

$\frac{a + \sqrt{a^2 + 4}}{2} > 0$, Hence rejected

$$\therefore \quad \boxed{z = i\left(\frac{a - \sqrt{a^2 + 4}}{2}\right)}$$

2. (a) $z^2 + \alpha z + \beta = 0$ ...(1)

$$\bar{z}^2 + \bar{\alpha}\bar{z} + \bar{\beta} = 0$$

$$z^2 + \bar{\alpha} z + \bar{\beta} = 0 \quad ...(2)$$

(1) – (2) $\Rightarrow$ $z = \dfrac{\bar{\beta} - \beta}{\alpha - \bar{\alpha}}$

Put in (1) we get $(\beta - \bar{\beta})^2 = (\bar{\alpha} - \alpha)(\alpha\bar{\beta} - \bar{\alpha}\beta)$

(c) $z^2 + \alpha z + \beta = 0$ ...(1)

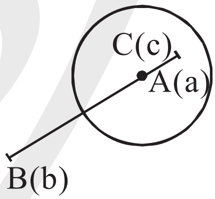
$$z^2 - \bar{\alpha} z + \bar{\beta} = 0 \quad ...(2)$$

(1) – (2) $\Rightarrow$ $z = \dfrac{\bar{\beta} - \beta}{\alpha + \bar{\alpha}}$

Put in (1) $(\bar{\beta} - \beta)^2 + \alpha(\bar{\beta} - \beta)(\alpha + \bar{\alpha}) + \beta(\alpha + \bar{\alpha})^2 = 0$

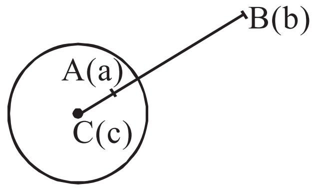
$\Rightarrow$ $(\beta - \bar{\beta})^2 + (\alpha + \bar{\alpha})(\alpha\bar{\beta} + \bar{\alpha}\beta) = 0$

3. (a) $\dfrac{c-b}{|c-b|} = \dfrac{a-c}{|a-c|}$



$\Rightarrow$ $c - b = \dfrac{(a-c)r^2}{|a-c|^2} = \dfrac{r^2}{\bar{a} - \bar{c}}$

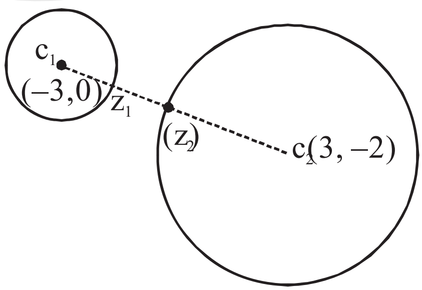
(d) $\dfrac{b-c}{|b-c|} = \dfrac{a-c}{|a-c|}$



$\Rightarrow$ $b - c = \dfrac{(a-c)r^2}{|a-c|^2}$

$$= \frac{r^2}{\bar{a} - \bar{c}}$$

4. Slope of $c_1c_2 = -\dfrac{1}{3}$



$$z_1 = \left(-3 - 1\left(-\frac{3}{\sqrt{10}}\right), 0 - 1\left(\frac{1}{\sqrt{10}}\right)\right)$$

$$z_2 = \left(3 + 4\left(\frac{-3}{\sqrt{10}}\right), -2 + 4\left(\frac{1}{\sqrt{10}}\right)\right)$$

5. $\operatorname{cosec}^{-1}\left(\frac{(x+iy)(1-i)}{2}\right) = \operatorname{cosec}^{-1}\left(\frac{(x+y)+i(y-x)}{2}\right)$ is defined

If $\quad y - x = 0$ and $\frac{x+y}{2} \geq 1$ or ≤ -1

$\Rightarrow \quad x \geq 1$ or $x \leq -1$

6. $z_r = \frac{1-b^r}{1-b} \Rightarrow \left|z_r - \frac{1}{1-b}\right| = \left|\frac{b^r}{1-b}\right| \leq \left|\frac{b}{1-b}\right| < \left|\frac{1}{1-b}\right|$

7. If n is even,

$$z = \left((1+\omega)(1+\omega^2)\right)\left((1+\omega)(1+\omega^2)\right)\ldots\ldots\ldots\left((1+\omega)(1+\omega^2)\right) = 1$$

If n is odd

$$z = \left((1+\omega)(1+\omega^2)\right)\left((1+\omega)(1+\omega^2)\right)\ldots\left((1+\omega)(1+\omega^2)\right) = 1 = -\omega^2$$

8. $ax^4 + bx^3 + cx^2 + dx + e = (x^2 + x + 1)(ax^2 + (b-a)x + e)$

Equating coefficients of x^2 and x, we get

$$c = b + e$$
$$d = b - a + e$$

$\Rightarrow \quad 2(b + e) = a + c + d$

9. $\bar{z}_1 = -i\bar{z}_2 \quad \Rightarrow \quad z_1 = iz_2$

$$\arg z_1 - \arg z_2 = \frac{\pi}{2}$$

$$\arg z_1 + \arg z_2 = \pi$$

$\Rightarrow \quad \arg(z_1) = \frac{3\pi}{4}, \quad \arg(z_2) = \frac{\pi}{4}$

10. $Z = \frac{z-1}{e^{i\theta}}$

$$Z + \frac{1}{Z} = \bar{Z} + \frac{1}{\bar{Z}}$$

$\Rightarrow \quad (Z - \bar{Z})\left(1 - \frac{1}{|Z|^2}\right) = 0$

$\Rightarrow \quad |Z| = 1$

$\Rightarrow \quad \left|\frac{z-1}{e^{i\theta}}\right| = 1$

$\Rightarrow \quad |z - 1| = 1$

11. $\frac{\alpha_1}{\alpha_2}$ is purely imaginary

$$\Rightarrow \quad \frac{\alpha_1}{\alpha_2} + \frac{\bar{\alpha}_1}{\bar{\alpha}_2} = 0 \;\Rightarrow\; \alpha_1\bar{\alpha}_2 + \bar{\alpha}_1\alpha_2 = 0$$

12. $3|z_4 - z_1| = |z_4 - z_1| + |z_4 - z_2| + |z_4 - z_3|$

Similarly $\quad 3|z_4 - z_2| = |z_4 - z_1| + |z_4 - z_2| + |z_4 - z_3|$

$$3|z_4 - z_3| = |z_4 - z_1| + |z_4 - z_2| + |z_4 - z_3|$$

$$\Rightarrow \quad |z_4 - z_1| = |z_4 - z_2| = |z_4 - z_3|$$

and $\frac{z_4 - z_1}{z_3 - z_2}$ is purely imaginary.

$\Rightarrow$ z_4 represents orthocentre and circumcentre of ΔABC

$\Rightarrow$ ΔABC is equilateral

13. $\alpha^3 + \beta^3 = 10, \qquad (\alpha\beta)^3 = 27$

$$\Rightarrow \quad \alpha\beta = 3, 3\omega, 3\omega^2$$

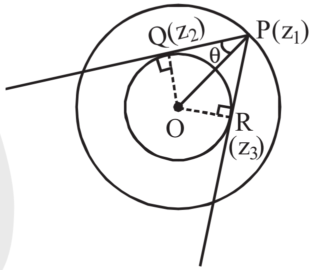
$$(\alpha + \beta)^3 - 3(\alpha + \beta)\alpha\beta = 10$$

Let $\alpha + \beta = t$

$$\Rightarrow \quad t^3 - 9t - 10 = 0 = (t + 2)(t^2 - 2t - 5)$$

$$\Rightarrow \quad \alpha + \beta = -2,\ 1 \pm \sqrt{6}$$

14. $\sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{6} \;\Rightarrow\; \angle QPR = \frac{\pi}{3}$



$$\Rightarrow \quad \arg\left(\frac{z_2}{z_3}\right) = \pm\frac{2\pi}{3}$$

$\Rightarrow$ ΔPQR is equilateral.

Circumcircle of ΔPQR has OP as diameter.

$\Rightarrow$ Circumcentre is $\left(\frac{z_1}{2}\right)$ and $\left|\frac{z_1}{2}\right| = 1$

$\because$ Circumcentre and centroid coincide for equilateral triangle

$$\Rightarrow \quad \left|\frac{z_1 + z_2 + z_3}{3}\right| = 1$$

$$\left(\frac{4}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right)\left(\frac{4}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3}\right) = (\bar{z}_1 + \bar{z}_2 + \bar{z}_3)(z_1 + z_2 + z_3) = |z_1 + z_2 + z_3|^2 = 9$$

15. $z = 4\omega$

$$\Rightarrow \quad z^{2n} + 2^{2n} z^n + 2^{4n} = 2^{4n}(\omega^{2n} + \omega^n + 1) = \begin{cases} 0 & n \neq 3k \\ 3.2^{4n} & n = 3k \end{cases}, \ k \in I$$

16. $iz^3 + z^3 - z + i = (z - i)(iz^2 - 1) = 0$

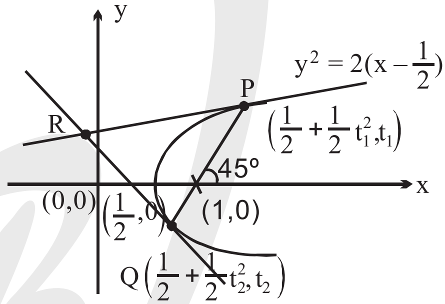
$$\Rightarrow \quad z = i,\ z^2 = -i$$

$$\Rightarrow \quad z = i,\ \frac{1}{\sqrt{2}}(1-i),\ \frac{1}{\sqrt{2}}(-1+i)$$

17. $x^2 = (x-1)^2 + y^2$

$$\frac{2}{t_1 + t_2} = 1$$

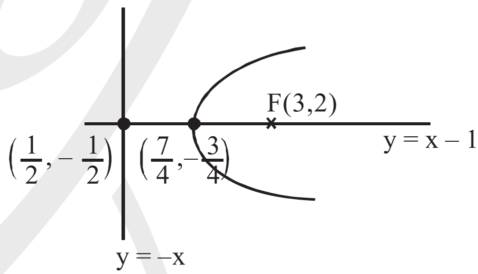
$$\Rightarrow \quad t_1 + t_2 = 2$$



$$R \equiv \left(\frac{1}{2}t_1t_2 + \frac{1}{2}, \frac{1}{2}(t_1 + t_2)\right) = \left(\frac{1}{2} + \frac{1}{2}t_1t_2, 1\right)$$

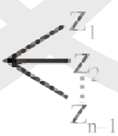
$\Rightarrow$ R lies on Im(z) = 1

18. $\sqrt{(x-3)^2 + (y-2)^2} = \left|\frac{x+y}{\sqrt{2}}\right|$



19. $z^n - 1 = (z-1)(z-z_1)(z-z_2)\ldots(z-z_{n-1}) = 0$

$$\Rightarrow \quad 1 + z + z^2 + \ldots + z^{n-1} = 0$$



Replace $\quad \dfrac{1}{3-z} = y \quad \Rightarrow \quad z = \dfrac{3y-1}{y}$

$\Rightarrow y^{n-1} + y^{n-2}(3y-1) + y^{n-3}(3y-1)^2 + \ldots + (3y-1)^{n-1} = 0$

$\Rightarrow y^{n-1}(1 + 3 + 3^2 + \ldots 3^{n-1}) - y^{n-2}(1 + 2.3 + 3.3^2 + 4.3^3 + \ldots + (n-1)3^{n-2})$

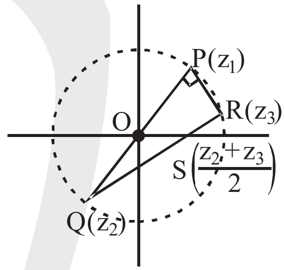
$+ \ldots + (-1)^{n-1} = 0$

$$\Rightarrow \quad \frac{1}{3-z_1}+\frac{1}{3-z_2}+\ldots+\frac{1}{3-z_{n-1}}=\frac{1+2.3+3.3^2+\ldots+(n-1)3^{n-2}}{1+3+3^2+\ldots+3^{n-1}}$$

$$=\frac{n\,3^{n-1}}{(3^n-1)}-\frac{1}{2}$$

20. $|z_1+z_2| = |z_1| - |z_2|$

$$\Rightarrow \quad \arg z_1 - \arg z_2 = \pi$$



$$|z_1+i(z_3-z_1)|=|z_1|+|i(z_3-z_1)|$$

$$\Rightarrow \quad \arg z_1 = \arg\left(i(z_3-z_1)\right)$$

Circumcentre of $\Delta PQR = \frac{z_2+z_3}{2}$

$$\Rightarrow \quad SP = SR = SQ$$

$$\Rightarrow \quad \left|\frac{z_3+z_2}{2}-z_1\right|=\left|\frac{z_3-z_2}{2}\right|$$

21. $(2z+5)(2\overline{z}+5)=(6z-9)(6\overline{z}-9)$

$$\Rightarrow 32|z|^2-64(z+\overline{z})+56=0$$

$$\Rightarrow \quad |z|^2 = 4\,\text{Re}(z) - \frac{7}{4}$$

$$a = 4, \quad b = -\frac{7}{4}$$

22. $\arg\left((-1+i)^{50}\right)=\arg\left(e^{i\frac{3\pi}{4}\times 50}\right)=-\frac{\pi}{2}$

$$\Rightarrow \quad n = -1, \quad a = 1 + i$$

Let $z^2 - (1+i)z + b + 2i = 0$ has root $z = k$, $k \in R$

$$\Rightarrow \quad k^2 - k + b = 0 \text{ and } -k + 2 = 0$$

$$\Rightarrow \quad k = 2, \quad b = -2$$

Roots are 2 and $-1 + i$

23. $z_2 = \lambda z_3 + (1-\lambda)z_1$

$\lambda \in (0, 1)$

$|z_3 - z_2| = (1-\lambda)\,|z_3 - z_1|$ and $|z_2 - z_1| = \lambda|z_3 - z_1|$

$$\frac{|z_3|-|z_2|}{|z_3 - z_2|} = \frac{|z_3| - |\lambda z_3 + (1-\lambda)z_1|}{(1-\lambda)|z_3 - z_1|} \geq \frac{|z_3| - \lambda|z_3| - (1-\lambda)|z_1|}{(1-\lambda)|z_3 - z_1|} = \frac{|z_3|-|z_1|}{|z_3 - z_1|}$$

$$\frac{|z_2|-|z_1|}{|z_2 - z_1|} = \frac{|\lambda z_3 + (1-\lambda)z_1| - |z_1|}{\lambda|z_3 - z_1|} \leq \frac{\lambda|z_3| + (1-\lambda)|z_1| - |z_1|}{\lambda|z_3 - z_1|} = \frac{|z_3|-|z_1|}{|z_3 - z_1|}$$

24. $az^2 + bz + c = 0$ (roots z_1, z_2) with $|z_1| = 1$

$$|z_1 z_2| = \left|\frac{c}{a}\right| = 1 \quad \Rightarrow \quad |z_2| = 1$$

$\therefore$ $$|z_1 + z_2| = \left|-\frac{b}{a}\right| = 1$$

$\Rightarrow$ $$(z_1 + z_2)\left(\frac{1}{z_1} + \frac{1}{z_2}\right) = 1$$

$\Rightarrow$ $$b^2 = ac$$

Similarly for $bz^2 + cz + a = 0$ (roots z_3, z_4) with $|z_3| = 1$

We get $c^2 = ab$

$\Rightarrow$ $$b^2c^2 = a^2bc$$

$\Rightarrow$ $$bc = a^2$$

$\therefore$ $$a^2 + b^2 + c^2 = ab + bc + ca$$

$\therefore$ $a = b = c$ or a, b, c are vertices of equilateral triangle.

$$|a - b| = |b - c| = |c - a|$$

25. $z_1 + z_2 + z_3 = 0$

$$\Rightarrow \quad \bar{z}_1 + \bar{z}_2 + \bar{z}_3 = 0$$

$$\Rightarrow \quad \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$$

$$\Rightarrow \quad z_1z_2 + z_2z_3 + z_3z_1 = 0$$

$$\Rightarrow \quad (z_1 + z_2 + z_3)^2 = 0 = z_1^2 + z_2^2 + z_3^2 + 2(z_1z_2 + z_2z_3 + z_3z_1)$$

$$\Rightarrow \quad z_1^2 + z_2^2 + z_3^2 = 0$$

Also $\quad z_1^3 + z_2^3 + z_3^3 = 3z_1z_2z_3$

$$\Rightarrow \quad \frac{z_1^2}{z_2z_3} + \frac{z_2^2}{z_3z_1} + \frac{z_3^2}{z_1z_2} = 3$$

26. **Case-I** Triangle is right angled at z.

$\Rightarrow \quad \dfrac{z^2 - z}{z^3 - z}$ is purely imaginary

$$\Rightarrow \quad \frac{1}{z+1} = ki \quad \Rightarrow \quad z+1-\frac{1}{k}i \quad \Rightarrow \quad z = -1+\lambda i$$

z lies on line $x = -1$, $z \neq -1$

Case-II Triangle is right angled at z^2

$\Rightarrow \quad \dfrac{z - z^2}{z^3 - z^2}$ is purely imaginary

$$\Rightarrow \quad \frac{1}{z} = ki \quad \Rightarrow \quad x = 0$$

Case-III Triangle is right angled at z^3

$\Rightarrow \quad \dfrac{z - z^3}{z^2 - z^3}$ is purely imaginary

$\Rightarrow \quad \dfrac{1+z}{z}$ is purely imaginary

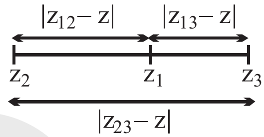
$$\Rightarrow \quad 1 + \frac{x}{x^2 + y^2} = 0$$

$$\Rightarrow \quad x(x + 1) + y^2 = 0$$

27. $(p + ip')^2 = 4(q + ip')$

$\Rightarrow \quad p^2 - (p')^2 = 4q$ and $pp' = 2p'$

28. If A lies between B and C



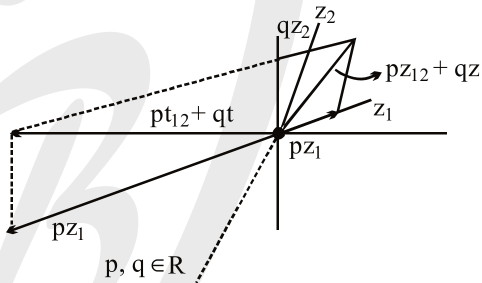
$$z_1 = \frac{|z_1 - z_2|z_3 + |z_1 - z_3|z_2}{|z_2 - z_3|}$$

If A, B, C are non collinear excentre of ΔABC

$$I_1 = \frac{-|z_2 - z_3|z_1 + |z_1 - z_2|z_3 + |z_1 - z_3|z_2}{-|z_2 - z_3| + |z_1 - z_2| + |z_1 - z_3|} = 0$$

29. (a) Using ||gm law any complex no. z_3 in complex plane can be expressed as

$$z_3 = pz_1 + pz_2$$

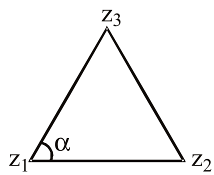


(b) If z_1, z_2, z_3 are collinear, then

$$z_3 = \frac{pz_1 + qz_2}{p + q}$$

$\Rightarrow \quad pz_1 + pz_2 - (p + q)z_3 = 0$ and $p + q + (-p - q) = 0$

(c) $$\frac{z_2 - z_1}{|z_2 - z_1|}e^{i\alpha} = \frac{z_3 - z_1}{|z_3 - z_1|}$$



$$z_1\left(|z_3 - z_1|e^{i\alpha} - |z_2 - z_1|\right) + z_2\left(-|z_3 - z_1|e^{i\alpha}\right) + z_3|z_2 - z_1| = 0$$

and $|z_3 - z_1|e^{i\alpha} - |z_2 - z_1| - |z_3 - z_1|e^{i\alpha} + |z_2 - z_1| = 0$

30. (b) $3z^2 - 2(z_1 + z_2 + z_3)z + z_1z_2 + z_2z_3 + z_3z_1 = 0$

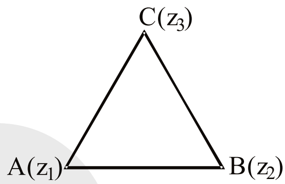
$\Rightarrow -(z_1 + z_2 + z_3)^2 + 3(z_1z_2 + z_2z_3 + z_3z_1) = 0$

$\Rightarrow \quad z_1z_2 + z_2z_3 + z_3z_1 = z_1^2 + z_2^2 + z_3^2$

All other options also results in $\sum z_1^2 = \sum z_1 z_2$

31. $$z_3 - z_1 = (z_2 - z_1) e^{\pm i\frac{\pi}{3}}$$

$$z_3 = z_1\left(1 - e^{i\frac{\pi}{3}}\right) + z_2\, e^{i\frac{\pi}{3}}$$



or $$z_3 = z_1\left(1 - e^{-i\frac{\pi}{3}}\right) + z_2\, e^{-i\frac{\pi}{3}}$$

$$z_3 = (1+i)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) + (2-i)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) \text{ T}$$

or $$(1+i)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) + (2-i)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$$

$$= \frac{3+2\sqrt{3}}{2} + \frac{i\sqrt{3}}{2} \quad \text{or} \quad \frac{3-2\sqrt{3}}{2} - \frac{i\sqrt{3}}{2}$$

$\Rightarrow$ Centroid $$= \frac{z_1 + z_2 + z_3}{3} = \frac{9+2\sqrt{3}}{6} + \frac{i\sqrt{3}}{6} \quad \text{or} \quad \frac{9-2\sqrt{3}}{6} - \frac{i\sqrt{3}}{6}$$

32. $$\left|z + \frac{1}{z}\right| \le |z| + \frac{1}{|z|} \le 4 + \frac{1}{4} = \frac{17}{4}$$ $\left[\because f(x) = x + \frac{1}{x} \text{ is increasing } \forall\, x \in [2, 4]\right]$

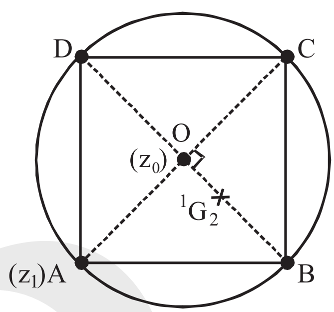
$$\left|z + \frac{1}{z}\right| \ge \left||z| - \frac{1}{|z|}\right| \ge 2 - \frac{1}{2} = \frac{3}{2}$$ $\left[\because f(x) = \left|x - \frac{1}{x}\right| \text{ is increasing } \forall\, x \in [2, 4]\right]$

33. $$-\frac{z-i}{z+i} = -\frac{b + ic - (1+a)i}{b + ic + (1+a)i} = -\frac{\big(b + i(c-a-1)\big)\big(b - i(c+a+1)\big)}{b^2 + (c+a+1)^2}$$

$$= -\frac{\left(b^2 + c^2 - (a+1)^2 + i2b(-a-1)\right)}{2 + 2ca + 2c + 2a}$$

$$= -\frac{(1+a)(-2a - 2ib)}{2(1+c)(1+a)}$$

$$= \frac{a+ib}{1+c}$$



34. $\frac{z_G - z_0}{OG} = \frac{z_1 - z_0}{OA} e^{\pm i\frac{\pi}{2}}$

$$OA = 3OG$$

$\Rightarrow$ $$z_G - z_0 = \pm i\left(\frac{z_1 - z_0}{3}\right)$$

$\Rightarrow$ $$z_G = z_0\left(1 - \frac{i}{3}\right) + \frac{i}{3}z_1 \quad \text{or} \quad z_0\left(1 + \frac{i}{3}\right) - \frac{i}{3}z_1$$

35. If k < 0, then

$$\text{Sum} = \sum_{r=0}^{n-1} \frac{\pi + 2r\pi}{n} = \frac{n\pi + (n-1)n\pi}{n} = n\pi$$

If k > 0, then

$$\text{Sum} = \sum_{r=0}^{n-1} \frac{2r\pi}{n} = (n-1)\pi$$

36. $z_1^2 + z_2^2 + z_1 z_2 = 0$

$\Rightarrow$ $$\frac{z_1}{z_2} = \omega, \omega^2$$

$\Rightarrow$ $$|z_1| = |z_2|$$

37. $z_1 = e^{i\theta_1}, \ z_2 = e^{i\theta_2}$

$\text{Re}(z_1\bar{z}_2) = 0 \quad \Rightarrow \quad \cos(\theta_1 - \theta_2) = 0$

$\Rightarrow$ $$\theta_1 - \theta_2 = (2n+1)\frac{\pi}{2}, \ n \in I$$

$$|\omega_1|^2 = a^2 + c^2 = \cos^2\theta_1 + \cos^2\theta_2$$

$$= \cos^2\theta_1 + \cos^2\left(\theta_1 - (2n+1)\frac{\pi}{2}\right)$$

$$= \cos^2\theta_1 + \sin^2\theta_1 = 1$$

$$|\omega_2|^2 = b^2 + d^2 = \sin^2\theta_1 + \sin^2\theta_2$$

$$= \sin^2\theta_1 + \sin^2\left(\theta_1 - (2n+1)\frac{\pi}{2}\right)$$

$$= \sin^2\theta_1 + \cos^2\theta_1 = 1$$

$$\text{Re}(\omega_1\bar{\omega}_2) = ab + cd = \frac{1}{2}(\sin 2\theta_1 + \sin 2\theta_2)$$

$$= \frac{1}{2}\left(\sin 2\theta_1 + \sin\left(2\theta_1 - (2n+1)\pi\right)\right) = 0$$

$$\text{Im}(\omega_1\bar{\omega}_2) = bc - ad = \sin(\theta_1 - \theta_2) = \sin\left((2n+1)\frac{\pi}{2}\right)$$

SECTION-3

Comprehension (Q.1 To Q.3):

$$z_1 + z_2 + z_3 = p \quad ...(1)$$

$$z_1 + z_2\omega + z_3\omega^2 = q \quad ...(2)$$

$$z_1 + z_2\omega^2 + z_3\omega = r \quad ...(3)$$

Applying (1) + (2) × ω^2 + (3) × ω

$\Rightarrow$ $3z_2 = p + q\omega^2 + r\omega$

Applying (1) + (2) × ω + (3) × ω^2

$\Rightarrow$ $3z_3 = p + q\omega + r\omega^2$

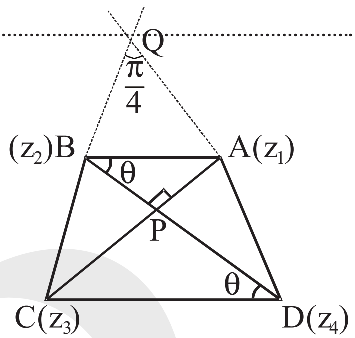
$$|p|^2 + |q|^2 + |r|^2 = (z_1 + z_2 + z_3)(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) + (z_1 + z_2\omega + z_3\omega^2)(\bar{z}_1 + \bar{z}_2\omega^2 + \bar{z}_3\omega)$$

$$+ (z_1 + z_2\omega^2 + z_3 w)(\bar{z}_1 + \bar{z}_2\omega + \bar{z}_3\omega^2)$$

$$= 3(|z_1|^2 + |z_2|^2 + |z_3|^2) + (1 + \omega + \omega^2)(z_1\bar{z}_2 + \bar{z}_1 z_2 + z_1\bar{z}_3 + \bar{z}_1 z_3 + z_2\bar{z}_3 + \bar{z}_2 z_3)$$

$$= 3(|z_1|^2 + |z_2|^2 + |z_3|^2)$$

COMPREHENSION (Q.4 TO Q.6) :



Let QB = x, QA = y

$$\frac{QB}{QC} = \frac{QA}{QD} = \frac{4}{10} = \frac{2}{5}$$

$\Rightarrow \quad QC = \frac{5}{2}x, \; QD = \frac{5}{2}y$

$\therefore \quad x^2 + y^2 - 16 = 2xy\cos\frac{\pi}{4} = \sqrt{2}\,xy$ ...(1)

$BC^2 = PB^2 + PC^2$

$\Rightarrow \quad \left(\frac{3}{2}x\right)^2 = (4\cos\theta)^2 + (10\sin\theta)^2$ ...(2)

and $\quad AD^2 = PA^2 + PD^2$

$\Rightarrow \quad \left(\frac{3}{2}y\right)^2 = (4\sin\theta)^2 + (10\cos\theta)^2$ ...(3)

(2) + (3) $\Rightarrow \quad \frac{9}{4}(x^2 + y^2) = 116$

$\Rightarrow \quad x^2 + y^2 = \frac{464}{9}$

$\Rightarrow \quad xy = \frac{160\sqrt{2}}{9}$

Area of trapezium ABCD $= \frac{1}{2}\left(\frac{5}{2}x\right)\left(\frac{5}{2}y\right)\sin\frac{\pi}{4} - \frac{1}{2}xy\sin\frac{\pi}{4} = \frac{140}{3}$

Also area of trapezium ABCD $= \frac{1}{2} \times (14\cos\theta)(14\sin\theta)$

$\Rightarrow \quad 49\sin 2\theta = \frac{140}{3} \quad \Rightarrow \sin 2\theta = \frac{20}{21}$

Area of $\Delta PCB = \frac{1}{2} \times (4\cos\theta)(10\sin\theta) = 10\sin 2\theta = \frac{200}{21}$

$|CP - DP| = 10|\sin\theta - \cos\theta|$

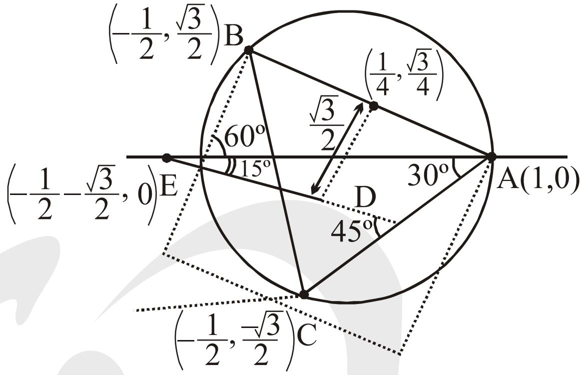
$\therefore \quad (CP - DP)^2 = 100(1 - \sin 2\theta) = \frac{100}{21}$

$\Rightarrow \quad |CP - DP| = \frac{10}{\sqrt{21}}$

Comprehension (Q.7 to Q.9) :

$$D \equiv \left(\frac{1}{4} - \frac{\sqrt{3}}{2}\frac{1}{2}, \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}\right)$$

$$= \left(\frac{1-\sqrt{3}}{4}, \frac{\sqrt{3}-3}{4}\right)$$



$$\text{Slope of DE} = \frac{\frac{\sqrt{3}-3}{4}}{\frac{3}{4}+\frac{\sqrt{3}}{4}} = \sqrt{3}-2$$

$\Rightarrow$ Angle between AC and DE $= \frac{\pi}{4}$

$$DE^2 = \left(\frac{\sqrt{3}-3}{4}\right)^2 + \left(\frac{\sqrt{3}+3}{4}\right)^2 = \frac{2}{16}(3+9) = \frac{3}{2}$$

$$AE = 1 + \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{3+\sqrt{3}}{2}$$

Comprehension (Q.10 to Q.12) :

$$|z^2+1|^2 = 4|z|^2$$

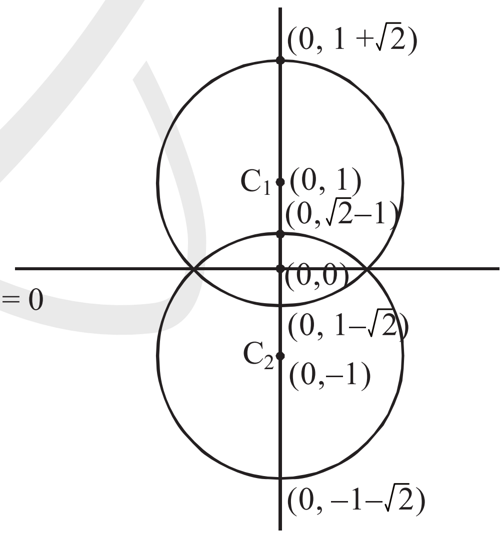
$$\Rightarrow \quad (z^2+1)(\bar{z}^2+1) = 4|z|^2$$

$$\Rightarrow \quad (|z|^4 - 2|z|^2 + 1) + (z^2 + \bar{z}^2 - 2|z|^2) = 0$$

$$\Rightarrow \quad (|z|^2-1)^2 - (i(z-\bar{z}))^2 = 0$$

$$\Rightarrow \quad (z\bar{z} - i(z-\bar{z}) - 1)(z\bar{z} + i(z-\bar{z}) - 1) = 0$$

$$|z|_{max} = 1 + \sqrt{2}$$



If $z = re^{i\frac{\pi}{6}}$, then

From $z\bar{z} - i(z-\bar{z}) - 1 = 0$

$$r^2 + r - 1 = 0 \quad \Rightarrow r = \frac{-1+\sqrt{5}}{2}$$

and from $z\bar{z} + i(z - \bar{z}) - 1 = 0$

$$r^2 - r - 1 = 0 \quad \Rightarrow \quad r = \frac{1+\sqrt{5}}{2}$$

Also $\quad C_1C_2^2 = 4 = r_1^2 + r_2^2 = (\sqrt{2})^2 + (\sqrt{2})^2$

$\Rightarrow$ Circles are orthogonal

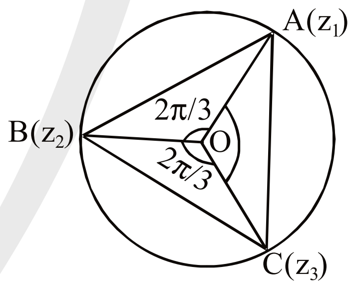
COMPREHENSION (Q.13 TO Q.15) :

Clearly ΔABC is equilateral

$$AP^2 + PB^2 + CP^2 = |z - z_1|^2 + |z - z_2|^2 + |z - z_3|^2$$

$$= 3|z|^2 + |z_1|^2 + |z_2|^2 + |z_3|^2 - z(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) - \bar{z}(z_1 + z_2 + z_3)$$

$$= 3(1^2 + 2^2) = 15$$



$$\frac{1}{z_D} + \frac{1}{z_E} + \frac{1}{z_F} = \bar{z}_D + \bar{z}_E + \bar{z}_F = 0$$

$$z_Q = -\frac{z_2 z_3}{z_1} = -\frac{4e^{i\pi/2}}{2e^{i\pi/4}}$$

$$= -2e^{i\pi/4} = -\sqrt{2}\,(1 + i)'$$

COMPREHENSION (Q.16 TO Q.18) :

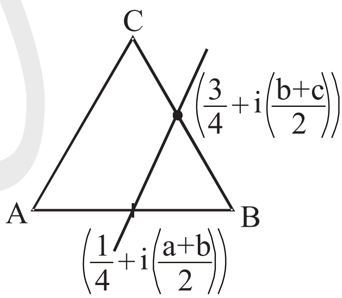
Curve 'C' is

$$x + iy = (a\cos^4 t)i + (1 + 2bi)\cos^2 t \sin^2 t + (1 + ci)\sin^4 t$$

$$x = \sin^4 t + \sin^2 t \cos^2 t = \sin^2 t$$

$$y = \sin^4 t + 2b\sin^2 t\cos^2 t + c\sin^4 t$$

$$\Rightarrow \quad y = a(1 - x)^2 + 2bx(1 - x) + cx^2$$



which represents parabola

Equation of line through mid point of AB and parallel to AC is

$$z = \lambda\left(\frac{1}{4} + i\frac{a+b}{2}\right) + (1 - \lambda)\left(\frac{3}{4} + i\left(\frac{b+c}{2}\right)\right)$$

For point of intersection 'P'

$$\frac{3-2\lambda}{4}+i\left(\frac{b+c}{2}+\lambda\left(\frac{a-c}{2}\right)\right) = i(a\cos^4 t + 2b\sin^2 t\cos^2 t + c\sin^4 t) + \sin^2 t$$

$$\Rightarrow \quad \frac{3-2\lambda}{4} = \sin^2 t$$

$$\Rightarrow \quad \frac{3}{2} - 2\sin^2 t = \lambda$$

and $$\frac{(b+c)}{2}+\lambda\left(\frac{a-c}{2}\right) = a\cos^4 t + 2b\sin^2 t\cos^2 t + c\sin^4 t$$

$$\frac{b+c}{2}+\left(\frac{3}{2}-2\sin^2 t\right)\left(\frac{a-c}{2}\right) = a\cos^4 t + 2b\sin^2 t\cos^2 t + c\sin^4 t$$

$$\frac{3a+2b-c}{4} + (c-a)\sin^2 t = (2b-a-c)\sin^2 t\cos^2 t + a\cos^2 t + c\sin^2 t$$

$$\Rightarrow \quad \frac{3a+2b-c}{4} + (c-a)\sin^2 t = (2b-a-c)\sin^2 t\cos^2 t + (c-a)\sin^2 t + a$$

$$\Rightarrow \quad 0 = (2b-a-c)(4\sin^2 t\cos^2 t - 1)$$

$$\Rightarrow \quad 4\sin^4 t - 4\sin^2 t + 1 = 0$$

$$\Rightarrow \quad \sin^2 t = \frac{1}{2} \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

$$P \equiv \left(\frac{1}{2}, \frac{2b+a+c}{4}\right)$$

Comprehension (Q.19 to Q.20) :

$$p + q\omega + r\omega^2 = a\big(x(-2+\omega+\omega^2) + y(1+\omega^2-2\omega) + z(1+\omega-2\omega^2)\big)$$
$$= -3a(x + y\omega + z\omega^2)$$

$$p + q\omega^2 + r\omega = a(x(-2+\omega^2+\omega) + y(1-2\omega^2+\omega) + z(1+\omega^2-2\omega))$$
$$= -3a(x + y\omega^2 + z\omega)$$

$$\Rightarrow \quad p^2 + q^2 + r^2 - pq - qr - rp = 9a^2(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$p + q + r = 3(x + y + z)$$

$$\Rightarrow \quad p^3 + q^3 + r^3 - 3pqr = 27a^2(x^3 + y^3 + z^3 - 3xyz)$$

Comprehension (Q.21 to Q.23) :

$$\frac{1}{a+x}+\frac{1}{b+x}+\frac{1}{c+x}+\frac{1}{d+x}=\frac{2}{x} \begin{matrix} \nearrow \omega \\ \searrow \omega^2 \end{matrix}$$

$\Rightarrow \quad 2x^4 + (a + b + c + d)x^3 – (abc + abd + acd + bcd)x – 2abcd = 0$

Put $x = \omega$ $\quad (2 – \Sigma abc)\omega + \Sigma a – 2abcd = 0$

Put $x = \omega^2$ $\quad (2 – \Sigma abc)\omega^2 + \Sigma a – 2abcd = 0$

Subtracting, we get

$$(2 – \Sigma abc)(\omega – \omega^2) = 0$$

$\Rightarrow \quad \Sigma abc = 2$

$\Rightarrow \quad \Sigma a = 2abcd$

Also x = 1, satisfy the equation

$$\therefore \quad \frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}+\frac{1}{d+1}=2$$

Comprehension (Q.24 to Q.26) :

$$(z_1z_2z_3)^2 = –80(1 + 4i) \times 60 \times 24(–4 + i)$$

$$z_1^2=\frac{(z_1z_2z_3)^2}{(z_2z_3)^2}=\frac{-80(1+4i)\times 24(-4+i)}{60}$$

$$= 32(8 + 15i)$$

$$z_2^2=\frac{(z_1z_2z_3)^2}{(z_1z_3)^2}=\frac{-80(1+4i)\times 60}{24(-4+i)}=\frac{200}{17}(17i)=200i$$

$$z_3^2=\frac{(z_1z_2z_3)^2}{(z_1z_2)^2}=\frac{60\times 24(-4+i)}{-80(1+4i)}=-18i$$

$$(z_1 + z_2 + z_3)^2 = z_1^2 + z_2^2 + z_3^2 + 2(z_1z_2 + z_2z_3 + z_3z_1)$$

$$= 662i + 256 + 2(–116 – 296i)$$

$$= 24 + 70i$$

$$|z_1 + z_2 + z_3|^2 = \sqrt{(24)^2+(70)^2} = 74$$

$$z_2 = 10\sqrt{2}\,e^{i\frac{\pi}{4}} \quad \text{or} \quad 10\sqrt{2}\,e^{i\frac{5\pi}{4}}$$

$$z_2 = 10(1 + i) \text{ or } -10(1 + i)$$

$$z_1 = \frac{-80(1+4i)}{10(1+i)} \text{ or } \frac{-80(1+4i)}{-10(1+i)}$$

$$= -4(5 + 3i) \quad \text{or} \quad 4(5 + 3i)$$

$$z_3 = \frac{60}{10(1+i)} \quad \text{or} \quad \frac{60}{-10(1+i)}$$

$$= 3(1 - i) \quad \text{or} \quad 3(-1 + i)$$

$$(z_1, z_2, z_3) = (-4(5 + 3i), 10(1 + i), 3(1 - i)) \text{ or } (4(5 + 3i), -10(1 + i), 3(-1 + i)$$

SECTION-4

MATCH THE COLUMN :

1. z_1, z_2, z_3, z_4 satisfy $z^5 = 1$

$\Rightarrow \quad 1^4 + z_1^4 + z_2^4 + z_3^4 + z_4^4 = 0$

$\Rightarrow \quad z_1^4 + z_2^4 + z_3^4 + z_4^4 = -1$

$z^4 + z^3 + z^2 + z + 1 = (z - z_1)(z - z_2)(z - z_3)(z - z_4)$

Put $z = 1$, $5 = (1 - z_1)(1 - z_2)(1 - z_3)(1 - z_4)$

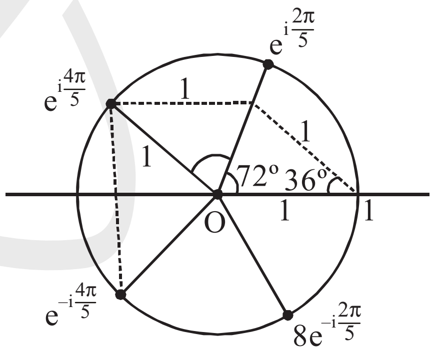
Put $z = -1$, $1 = (1 + z_1)(1 + z_2)(1 + z_3)(1 + z_4)$

$\Rightarrow \quad (z_1^2 - 1)(z_2^2 - 1)(z_3^2 - 1)(z_4^2 - 1) = 5$

Put $z = i$, $\quad 1 = (z_1 - i)(z_2 - i)(z_3 - i)(z_4 - i)$

Put $z = -i$, $\quad 1 = (z_1 + i)(z_2 + i)(z_3 + i)(z_4 + i)$

$\Rightarrow \quad (z_1^2 + 1)(z_2^2 + 1)(z_3^2 + 1)(z_4^2 + 1) = 1$



$$|z_1 + z_2|_{min} = \sqrt{1+1-2\cos 36^\circ} = 2\sin 18^\circ$$

$$= \frac{\sqrt{5}-1}{2}$$

SECTION-5

SUBJECTIVE TYPE QUESTIONS

1. Applying $C_1 \to C_1 + C_2 + C_3$

$$z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0 = z\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & z+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & z+\omega-\omega^2 \end{vmatrix}$$

$\Rightarrow \quad z(z^2 - (\omega^2 - \omega)^2 - 3) = 0 \qquad \Rightarrow \; z^3 = 0$

2. $$\Delta = e^{\alpha+\beta+\gamma}\begin{vmatrix} 1 & e^{\alpha} & e^{2\alpha} - e^{-\alpha} \\ 1 & e^{\beta} & e^{2\beta} - e^{-\beta} \\ 1 & e^{\gamma} & e^{2\gamma} - e^{-\gamma} \end{vmatrix} = (e^{\beta} - e^{\alpha})(e^{\gamma} - e^{\alpha})\begin{vmatrix} 1 & e^{\alpha} & e^{2\alpha} - e^{-\alpha} \\ 0 & 1 & e^{\beta} + e^{\alpha} + e^{-\alpha}e^{-\beta} \\ 0 & 1 & e^{\gamma} + e^{\alpha} + e^{-\alpha}e^{-\gamma} \end{vmatrix}$$

$\Delta = (e^{\beta} - e^{\alpha})\,(e^{\gamma} - e^{\alpha})\,(e^{\gamma} - e^{\beta})\,(1 - e^{-(\alpha+\beta+\gamma)}) = 0$

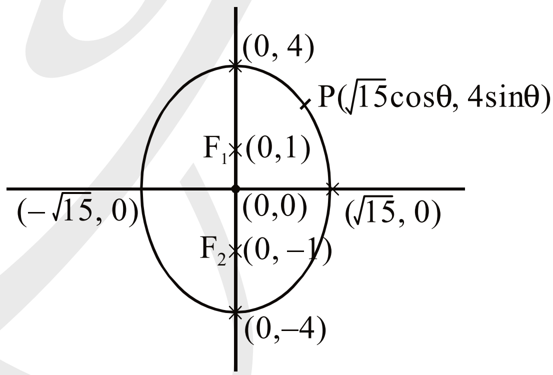
3. z lies on ellipse $\frac{x^2}{15} + \frac{y^2}{16} = 1$

$|z|^2 = 15\cos^2\theta + 16\sin^2\theta$

$= 15 + \sin^2\theta$

$m = \sqrt{15}$, $M = 4$

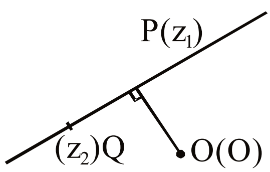
$\Rightarrow \quad m + M = 4 + \sqrt{15}$



4. $\frac{z_2 - z_1}{z_1}$ is purely imaginary

$\Rightarrow \quad \frac{z_2 - z_1}{z_1} + \frac{\overline{z}_2 - \overline{z}_1}{\overline{z}_1} = 0$

$\Rightarrow \quad \frac{z_2}{z_1} + \frac{\overline{z}_2}{\overline{z}_1} = 2$



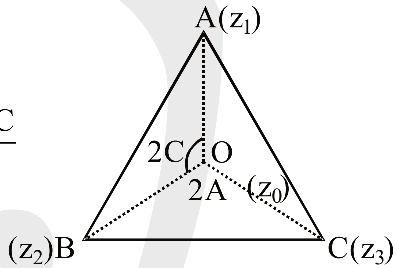
5. $z_1 = (a + b\omega + c\omega^2)^3$, $z_2 = (a + b\omega^2 + c\omega)^3$, $z_3 = (a\omega + b + c\omega^2)^3$

$z_4 = (a\omega + b\omega^2 + c)^3$, $z_5 = (a\omega^2 + b + c\omega)^3$, $z_6 = (a\omega^2 + b\omega + c)^3$

Clearly $z_1 = z_4 = z_5$ and $z_2 = z_3 = z_6$

6. $(z_0 - z_1)e^{i2C} = (z_0 - z_2)$

$(z_0 - z_3) = (z_0 - z_2)\, e^{i2A}$



$$\therefore \quad \left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$$

$$= \frac{(\cos 2C - i\sin 2C)\sin 2A + (\cos 2A + i\sin 2A)\sin 2C}{\sin 2B}$$

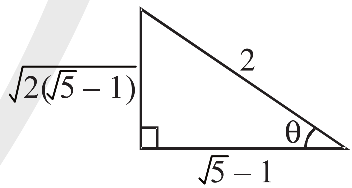
$$= \frac{\sin 2(A + C)}{\sin 2B} = -1$$

7. Let $\alpha = e^{i\theta}$, $z = ik$, $k, \theta \in R$

$\Rightarrow \quad -(\cos\theta + i\sin\theta)\, k^2 + ik + 1 = 0$

$\Rightarrow \quad 1 - k^2\cos\theta = 0$

$k(1 - k\sin\theta) = 0 \Rightarrow k\sin\theta = 1$



$\Rightarrow \quad \frac{1}{k^2} = \cos\theta = \sin^2\theta$

$\Rightarrow \quad \cos^2\theta + \cos\theta - 1 = 0$

$\Rightarrow \quad \cos\theta = \frac{-1 + \sqrt{5}}{2}$

$\Rightarrow \quad \tan^2\theta = \frac{2}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{2}$

8. $\cos 17\theta + i\sin 17\theta = (\cos\theta + i\sin\theta)^{17}$

$$\sin 17\theta = {}^{17}C_1\cos^{16}\theta\sin\theta - {}^{17}C_3\cos^{14}\theta\sin^3\theta + {}^{17}C_5\cos^{12}\theta\sin^5\theta$$

$$- {}^{17}C_7\cos^{10}\theta\sin^7\theta + \ldots - {}^{17}C_{15}\cos^2\theta\sin^{15}\theta + {}^{17}C_{17}\sin^{17}\theta$$

$$= \tan\theta\cos^{17}\theta({}^{17}C_1 - {}^{17}C_3\tan^2\theta + {}^{17}C_5\tan^4\theta - {}^{17}C_7\tan^6\theta + \ldots$$

$$- {}^{17}C_{15}\tan^{14}\theta + {}^{17}C_{17}\tan^{16}\theta)$$

If $\theta = \frac{r\pi}{17}$ $\quad r = 1, 2, ..., 8$

$\Rightarrow \quad 0 = {}^{17}C_1 - {}^{17}C_3\tan^2\theta + {}^{17}C_5\tan^4\theta - {}^{17}C_7\tan^6\theta + ... - {}^{17}C_{15}\tan^{14}\theta + {}^{17}C_{17}\tan^{16}\theta$

Put $\tan^2\theta = x$

$${}^{17}C_1 - {}^{17}C_3x + {}^{17}C_5x^2 - {}^{17}C_7x^3 + ... - {}^{17}C_{15}X^7 + {}^{17}C_{17}X^8 = 0$$

(roots: $\tan^2\frac{\pi}{17}$, $\tan^2\frac{2\pi}{17}$, ..., $\tan^2\frac{8\pi}{17}$)

$$\Rightarrow \quad \sum_{r=1}^{8}\tan^2\frac{r\pi}{17} = \frac{{}^{17}C_{15}}{{}^{17}C_{17}} = \frac{17\times16}{2}$$

$$\prod_{r=1}^{8}\tan^2\frac{r\pi}{17} = \frac{{}^{17}C_1}{{}^{17}C_{17}} = 17$$

$$\Rightarrow \quad \frac{a}{b} = 8$$

9. $$\left|\frac{75-27z_1\bar{z}_2}{z_1-z_2}\right| = \left|\frac{27\left(\frac{25}{9}-z_1\bar{z}_2\right)}{z_1-z_2}\right|$$

$$= \left|\frac{27(z_1\bar{z}_1 - z_1\bar{z}_2)}{(z_1-z_2)}\right| = \left|\frac{27z_1(\bar{z}_1-\bar{z}_2)}{(z_1-z_2)}\right|$$

$$= 27|z_1| = 27\times\frac{5}{3} = 45$$

10. Let $A(z_1)$, $B(z_2)$, $C(z_3)$, $D(z_4)$

$AB \perp CD \Rightarrow \frac{z_1-z_2}{z_4-z_3}$ is purely imaginary

$$\Rightarrow \quad \text{Re}\left(\frac{-5+9i}{3+i(k+3)}\right) = 0$$

$$\Rightarrow \quad \text{Re}((-5+9i)(3-i(k+3))) = 0$$

$$\Rightarrow \quad -15+9(k+3) = 0$$

$$\Rightarrow \quad k = -\frac{4}{3}$$

11. Let $\omega = y - 1 + yi,\ y \in R$

$$|z| \le |\omega|$$

$\Rightarrow$ $$(x-1)^2 + x^2 \le (y-1)^2 + y^2 \qquad \forall\, y \in R$$

$\Rightarrow$ $$(x-1)^2 + x^2 \le \min\{(y-1)^2 + y^2,\ y \in R\}$$

$\Rightarrow$ $$(x-1)^2 + x^2 \le \frac{1}{2}$$

$\Rightarrow$ $$(2x-1)^2 \le 0$$

$\Rightarrow$ $$x = \frac{1}{2}$$

$\Rightarrow$ $$z = \frac{1}{2} + \frac{i}{2}$$

12. $\text{Re}(z) > 1$

$\Rightarrow$ z lies in the right side of perpendicular bisector of points with complex representation 2 and 0.

$\Rightarrow$ $$|z-2| < |z|$$

$\Rightarrow$ $$\left|\frac{z-2}{2z}\right| < \frac{1}{2} \quad \Rightarrow \quad L = \frac{1}{2}$$

13. $|z_1 - z_2|\,|z_2 - z_3| + |z_2 - z_3|\,|z_3 - z_1| + |z_3 - z_1|\,|z_1 - z_2|$

$$\le |z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$$

$$= 2\sum |z_1|^2 - \sum z_1\bar{z}_2$$

$$= 3(|z_1|^2 + |z_2|^2 + |z_3|^2) - |z_1 + z_2 + z_3|^2$$

$$\le 3(3r^2) - 0 = 9r^2$$

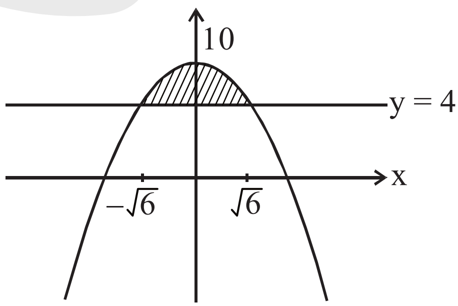
14. $y \ge 4$ and $x^2 + 4 + y - 4 \le 10$

$\Rightarrow$ $$y \le 10 - x^2$$

$$\text{Area} = 2\left(\int_0^{\sqrt{6}} (10 - x^2)dx - 4\sqrt{6}\right)$$

$$= 2\left(10\sqrt{6} - \frac{6\sqrt{6}}{3} - 4\sqrt{6}\right) T$$

$$= 8\sqrt{6}$$



15. $(z_1 + z_2 + z_3)(\overline{z}_1 + \overline{z}_2 + \overline{z}_3) + (-z_1 + z_2 + z_3)(-\overline{z}_1 + \overline{z}_2 + \overline{z}_3)$

$$+ (z_1 - z_2 + z_3)(\overline{z}_1 - \overline{z}_2 + \overline{z}_3) + (z_1 + z_2 - z_3) + (\overline{z}_1 + \overline{z}_2 - \overline{z}_3)$$

$$= 4(|z_1|^2 + |z_2|^2 + |z_3|^2) = 56$$

16. Let $z_1 = e^{i\theta_1}$, $z_2 = 2e^{i\theta_2}$, $z_3 = 3e^{i\theta_3}$

$\because \quad z_1 + z_2 + z_3 = 0$

$\Rightarrow \quad z_1^3 + z_2^3 + z_3^3 = 3z_1z_2z_3$

$\Rightarrow \quad e^{i3\theta_1} + 8e^{i3\theta_2} + 27e^{i3\theta_3} = 18e^{i(\theta_1 + \theta_2 + \theta_3)}$

$\Rightarrow \quad \sin 3\theta_1 + 8\sin 3\theta_2 + 27\sin 3\theta_3 = 18\sin(\theta_1 + \theta_2 + \theta_3)$.

17. Let $z = (\cos\alpha + \cos\beta) + i(\sin\alpha + \sin\beta)$

$\Rightarrow \quad z^7 + \overline{z}^7 = 2\mathrm{Re}(z^7)$

$$z^7 = \left(2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + i2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right)^7$$

$$= 2^7\cos^7\left(\frac{\alpha-\beta}{2}\right)\left(e^{i\frac{\alpha+\beta}{2}}\right)^7$$

$$= 2^7\cos^7\left(\frac{\alpha-\beta}{2}\right) e^{\frac{i7(\alpha+\beta)}{2}}$$

$$\mathrm{Re}(z^7) = 2^7\cos^7\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{7(\alpha+\beta)}{2}\right)$$

$\Rightarrow \quad m = 8,\ n = \frac{7}{2}$

18. $z^2 - \sqrt{3}z + 1 = 0 \quad \Rightarrow \quad z = \frac{\sqrt{3} \pm i}{2}$

$\Rightarrow \quad z = e^{i\pi/3}$ or $e^{-i\frac{\pi}{3}}$

$\therefore \quad z^r + \frac{1}{z^r} = 2\cos\frac{r\pi}{3}$

$$\sum_{r=1}^{5}\left(z^r+\frac{1}{z^r}\right)^2 = 4\sum_{r=1}^{5}\cos^2\left(\frac{r\pi}{3}\right) = 2\sum_{r=1}^{5}\left(1+\cos\frac{2r\pi}{3}\right)$$

$$= 2\left(5+\left(1-4\left(\frac{1}{2}\right)\right)\right) = 8$$

19. $\left|\text{Re}\left((1+i\sqrt{3})^n\right)\right| = \left|\text{Re}\left((-2\omega^2)^n\right)\right| = \begin{cases} 2^{n-1} & n \neq 3k \\ 2^n & n = 3k \end{cases} \quad k \in I$

$$\sum_{n=1}^{600}\log_2\left|f(1+i\sqrt{3})^n\right| = (1+2+3+4+\ldots\ldots+600) - \underbrace{(1+1+\ldots\ldots 1)}_{400\text{ times}}$$

$$= \frac{600\times 601}{2} - 400 = 179900$$

20. $\dfrac{z^4}{1+z^4}+\dfrac{z}{1+z}+\dfrac{z^2}{1+z^2}+\dfrac{z^4}{z+z^4} = \dfrac{z^5}{z+z^5}+\dfrac{z}{1+z}+\dfrac{z^2}{1+z^2}+\dfrac{z^5}{z^2+z^5}$

$$= \left(\frac{1}{1+z}+\frac{z}{1+z}\right)+\left(\frac{z^2}{1+z^2}+\frac{1}{z^2+1}\right)$$

$$= 2$$

21. $a^3 - 3ab^2 + i(3a^2b - 107 - b^3) = c$

$\Rightarrow$ $c = a^3 - 3ab^2$ and $(3a^2 - b^2)b = 107$

$\Rightarrow$ $b = 1$ and $3a^2 - b^2 = 107$

$\Rightarrow$ $a = 6$

$\Rightarrow$ $c = 6(36 - 3) = 198$

or $3a^2 - b^2 = 1$ and $b = 107$

$\Rightarrow$ $3a^2 = 11450$ which is not possible

Hence, $a + b + c = 6 + 1 + 198 = 205$

22. Let the line be

$$y = mx + 3$$

Put $z_k = x_k + iy_k$

$\Rightarrow$ $y_k = mx_k + 3$

$\Rightarrow$ $\displaystyle\sum_{k=1}^{5} y_k = m\sum_{k=1}^{5} x_k + 15$

$$\sum_{k=1}^{5} z_k = \sum_{k=1}^{5} \omega_k = 3 + 504i$$

$\Rightarrow$ $\sum y_k = 504, \quad \sum x_k = 3$

$\Rightarrow$ $504 = m(3) = 15$

$\Rightarrow$ $m = 163$

23. $z\omega = e^{i\frac{2k_1\pi}{18}} e^{i\frac{2k_2\pi}{48}} = e^{i\frac{2\pi}{144}(8k_1+3k_2)} = e^{i\frac{2r\pi}{144}}$

r can take atmost 144 distinct values and all values are attained using the periodicity of 2π.

For example $3 \times 47 + 8 = 149$

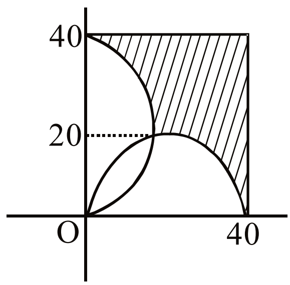
$$\frac{149 \times 2\pi}{144} - 2\pi = \frac{2\pi \times 5}{144}$$

Which is not attained for $8k_1 + 3k_2$ between 0 to 143.

24. $0 < x < 40, 0 < y < 40$ and $\frac{40}{\bar{z}} = \frac{40(x+iy)}{x^2+y^2}$

$\Rightarrow$ $0 < \frac{40x}{x^2+y^2} < 1$ $\Rightarrow$ $40x < x^2 + y^2$

and $0 < \frac{40y}{x^2+y^2} < 1$ $\Rightarrow$ $40y < x^2 + y^2$



$A = (40)^2 - [\pi(20)^2 - \{(20)^2 - 2((20)^2 - \frac{\pi}{4}(20)^2)\}]$

$\Rightarrow$ $A = (20)^2 \left(3 - \frac{\pi}{2}\right) \approx 571.68$

$\Rightarrow$ $[A] = 571$

25. $\left(13 - \frac{1}{z}\right)^{10} + 1 = 0 \Rightarrow 13 - \frac{1}{z} = e^{i\left(\frac{\pi+2k\pi}{10}\right)}$ $k = 0, 1, 2, ..., 9$

$\Rightarrow$ $\frac{1}{z} = 13 - \cos\frac{(2k+1)\pi}{10} - i\sin\frac{(2k+1)\pi}{10}$

$\frac{1}{z}\frac{1}{\bar{z}} = (13 - \cos\theta)2 + \sin2\theta = 170 - 26\cos\theta$

$$\sum_{r=1}^{5}\frac{1}{|z_r|^2} = 170 \times 5 - 26\left(\cos\frac{\pi}{10} + \cos\frac{3\pi}{10} + \cos\frac{5\pi}{10} + \cos\frac{7\pi}{10} + \cos\frac{9\pi}{10}\right)$$

$$= 850 - 26(0) = 850$$

26. $\left(z^3 + \frac{1}{z^3}\right) + \left(z + \frac{1}{z}\right) + 1 = 0$

$\Rightarrow \quad \left(z + \frac{1}{z}\right)^3 - 2\left(z + \frac{1}{z}\right) + 1 = 0$

$\Rightarrow \quad z + \frac{1}{z} = 1, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$

$z^2 - z + 1 = 0 \quad \Rightarrow \quad z = \frac{1 \pm i\sqrt{3}}{2}$

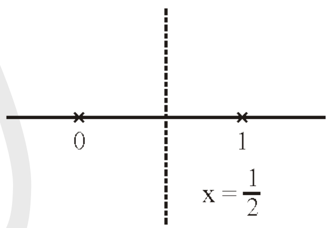
$z + \frac{1}{z} = \frac{\sqrt{5}-1}{2} = 2\cos 72^\circ \quad \Rightarrow \quad z = e^{\pm i(72^\circ)}$

$z + \frac{1}{z} = \frac{-1-\sqrt{5}}{2} = 2\cos 144^\circ \quad \Rightarrow \quad z = e^{\pm i(144^\circ)}$

$\Rightarrow \quad P = C^{i60^\circ} e^{i72^\circ} e^{i144^\circ} = e^{i276^\circ}$

27. $|f(z) - z| = |f(z)|$

$\Rightarrow \quad |(a + bi)z - z| = |(a + bi)z| \quad \forall\, z \in C$



$\Rightarrow \quad |(a - 1) + bi| = |a + bi|$

$\Rightarrow \quad a = \frac{1}{2}$

$|a + ib| = 8$

$\Rightarrow \quad \frac{1}{4} + b^2 = 64$

$\Rightarrow \quad \boxed{b^2 = \frac{255}{4}}$

28. $$F\left(F(z)\right)=\frac{\dfrac{z+i}{z-i}+i}{\dfrac{z+i}{z-i}-i}=\frac{(z+1)i}{(z-1)}$$

$$F\left(F(F(z))\right)=\frac{\left(\dfrac{z+1}{z-1}\right)i+i}{\left(\dfrac{z+1}{z-1}\right)i-i}=z$$

$$z_{2020}=F(z_{2019})=F(z_0)$$

$$=\frac{z_0+i}{z_0-i}=\frac{\dfrac{1}{137}+i+i}{\dfrac{1}{137}+i-i}=1+274i$$

29. $$\frac{\left(x^{18}-1\right)^2}{(x-1)^2}-x^{17}=\frac{\left(x^{36}-2x^{18}+1\right)-\left(x^{19}-2x^{18}+x^{17}\right)}{(x-1)^2}$$

$$\frac{x^{36}-x^{19}+1-x^{17}}{(x-1)^2}=\frac{\left(x^{17}-1\right)\left(x^{19}-1\right)}{(x-1)^2}$$

$P(x)=0 \quad \Rightarrow \quad x=e^{i\frac{2\pi k_1}{17}},\ e^{i\frac{2\pi k_2}{19}}$

$k_1 = 1, 2, ..., 16$ and $k_2 = 1, 2, ..., 18$

$\therefore \quad \sum_{r=1}^{5} a_r=\frac{1}{19}+\frac{1}{17}+\frac{2}{19}+\frac{2}{17}+\frac{3}{19}=\frac{159}{323}$

30. $1+\frac{n}{z}=-\frac{i}{4} \Rightarrow z=\frac{-n}{1+\frac{i}{4}}=\frac{-n\left(1-\frac{i}{4}\right)}{\frac{17}{16}}$

$$z=-\frac{16}{17}n\left(1-\frac{i}{4}\right)$$

$\therefore \quad 164=\frac{4n}{17}$

$\Rightarrow \quad n=697$

31. $\left(i\ e^{-it}\right)^n = i\ e^{-int}$

$\Rightarrow \quad i^n\ e^{-int} = i\ e^{-int}$

$\Rightarrow \quad i^n = i$

$\Rightarrow \quad n = 4k + 1,\ k \in I.$

32. $f(-\omega^2) = a\omega^{4036} - b\omega^{4034} + c\omega^{4032}$

$\Rightarrow \quad 2015 + 2019\sqrt{3}i = a\omega - b\omega^2 + c$

$\Rightarrow \quad 2015 + 2019\sqrt{3}i = \frac{b-a}{2} + c + i(a+b)\frac{\sqrt{3}}{2}$

$\Rightarrow \quad a + b = 4038$

$\Rightarrow \quad a = b = 2019$

and $\quad \frac{b-a}{2} + c = 2015 \Rightarrow c = 2015$

$\therefore \quad f(1) = a + b + c = 4038 + 2015 = 6053$

CHAPTER 8

LOGARITHM

SECTION-1

SINGLE CHOICE QUESTIONS

1. $\log_2 5 = 2a$, $\log_5 2 + \ln_5 3 = b$

$$\Rightarrow \quad \frac{1}{2a} + \log_5 3 = b \Rightarrow \log_5 3 = b - \frac{1}{2a} = \frac{2ab-1}{2a}$$

$$\log_3 2 = \frac{\log_5 2}{\log_5 3} = \frac{\left(\frac{1}{2a}\right)}{\left(\frac{2ab-1}{2a}\right)} = \frac{1}{2ab-1}$$

2. $\ln 5(\ln 5 + \ln(x+1)) = \ln 2(\ln 2 + \ln y)$

$\ln 2 \ln(x+1) = \ln y \ln 5$

$$\Rightarrow \quad \ln^2 5 + \ln 5 \ln(x+1) = \ln^2 2 + \frac{\ln^2 2 \ln(x+1)}{\ln 5}$$

$$\Rightarrow \quad \ln^2 5 - \ln^2 2 = \ln(x+1)\left(\frac{\ln^2 2 - \ln^2 5}{\ln 5}\right)$$

$$\Rightarrow \quad \ln(x+1) = -\ln 5$$

$$\Rightarrow \quad x + 1 = \frac{1}{5} \Rightarrow x = -\frac{4}{5}$$

3. Adding $\log_8(ab) + \log_4 a^2b^2 = 12$

$$\Rightarrow \quad \log_2(ab) + 3\log_2(ab) = 36$$

$$\Rightarrow \quad (ab)^4 = 2^{36}$$

$$\Rightarrow \quad ab = 2^9 = 512$$

4. $w = x^{24} \Rightarrow x = w^{\frac{1}{24}}$

$w = y^{40} \Rightarrow y = w^{\frac{1}{40}}$

$w = (xyz)^{12} \Rightarrow \quad z = w^{\frac{1}{12} - \frac{1}{24} - \frac{1}{40}} = w^{\frac{1}{60}}$

$\Rightarrow \quad \log_z w = \log_{w^{\frac{1}{60}}} w = 60$

5. $\log_2\left(\frac{1}{3}\log_2 x\right) = \frac{1}{3}\log_2(\log_2 x)$

$\Rightarrow \quad \left(\frac{1}{3}\log_2 x\right)^3 = \log_2 x$

$\Rightarrow \quad (\log_2 x)^2 = 27$

$\Rightarrow \quad \log_2 x = 3\sqrt{3}$

6. The number $2\log_{(2000)^6} 4 + 3\log_{(2000)^6} 5$

$= \log_{(2000)^6}(4^2 \times 5^3) = \frac{1}{6}$

7. $abc = b^3 = 6^6$

$\Rightarrow \quad b = 36$

Check a = 11, 20, 27, 32, 35

$\Rightarrow \quad a = 27, b = 36, c = 48$

8. $(24\cos x)^2 = (24\sin x)^3$

$\Rightarrow \quad 24\sin^3 x + \sin^2 x - 1 = 0$

$\Rightarrow \quad (3\sin x - 1)(8\sin^2 x + 3\sin x + 1) = 0$

$\Rightarrow \quad \sin x = \frac{1}{3}$

$\Rightarrow \quad \cot^2 x = \operatorname{cosec}^2 x - 1 = 9 - 1 = 8$

9. $n = b^2, 2b = n^2$

$$\log_{b^2}(2b) = \log_n(n^2) = 2$$

10. $\log_{10} x(3\log_{10} x - 2) = 1$

$\Rightarrow$ $$\log_{10} x = -\frac{1}{3} \text{ or } 1$$

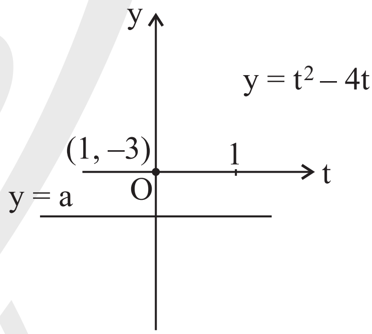
$$\log_{10} x = -\frac{1}{3} \text{ is rejected}$$

$\Rightarrow$ $$x = 10$$

SECTION-2

ONE OR MORE THAN ONE CORRECT QUESTIONS

1. Let $t = 3^{-|x-2|}$



$\Rightarrow$ $t^2 - 4t = a,\ t \in (0, 1]$

$\Rightarrow$ $a \in [-3, 0)$

$$t = 2 - \sqrt{4+a} \quad \because \quad t \le 1$$

$$3^{-|x-2|} = 2 - \sqrt{4+a}$$

$\Rightarrow$ $$x = 2 \pm \log_3(2 - \sqrt{4+a})$$

2. $(x + y) \ln x = n \ln y$ ... (1)

$(x + y) \ln y = 2n \ln x + n \ln y$ ... (2)

Divide (2) by (1)

$$\frac{\ln y}{\ln x} = 2\frac{\ln x}{\ln y} + 1 \qquad \Rightarrow \qquad \frac{\ln y}{\ln x} = 2, -1$$

$\frac{\ln y}{\ln x} = -1$ gives $(x, y) = (1, 1)$

Taking $\ln y = 2 \ln x \Rightarrow y = x^2$

$\Rightarrow \quad x^{x+x^2} = x^{2n}$

$\Rightarrow \quad x^2 + x - 2n = 0 \quad \Rightarrow x = \dfrac{\sqrt{1+8n}-1}{2}$

& $\quad y = x^2 = \dfrac{1+8n+1-2\sqrt{1+8n}}{4} = \dfrac{4n+1-\sqrt{1+8n}}{2}$

3. $(2x+y)^2 = x^2 + xy + 7y^2$

$\Rightarrow \quad x^2 + xy - 2y^2 = 0$

$\Rightarrow \quad (x-y)(x+2y) = 0$

Case I : If y = x, then

$(3x+y)^2 = 3x^2 + 4xy + ky^2$

$\Rightarrow \quad 16y^2 = 7y^2 + ky^2$

$\Rightarrow \quad k = 9$

Case II : If x = –2y

$\Rightarrow \quad (-5y)^2 = 12y^2 - 8y^2 + ky^2$

$\Rightarrow \quad k = 21$

4. $a^{128} + 128 = \log_a \log_a 2^{24}$

$a^{\left(a^{a^{128}} \cdot a^{128}\right)} = 2^{24}$

$\left(a^{a^{128}}\right)^{a^{a^{128}}} = 8^8$

$\Rightarrow \quad a^{a^{128}} = 8$

$\Rightarrow \quad \left(a^{a^{128}}\right)^{128} = 8^{128}$

$\Rightarrow \quad \left(a^{128}\right)^{a^{128}} = (64)^{64}$

$\Rightarrow \quad a^{128} = 64$

$\Rightarrow \qquad a = 2^{\frac{3}{64}}$

$\Rightarrow \qquad 2^{\frac{3}{64}\times 256} = \log_{2^{\frac{3}{64}}} b$

$\Rightarrow \qquad b = \left(2^{\frac{3}{64}}\right)^{2^{12}} = 2^{3\times 2^6} = 2^{192}$

5. If $k < 0$

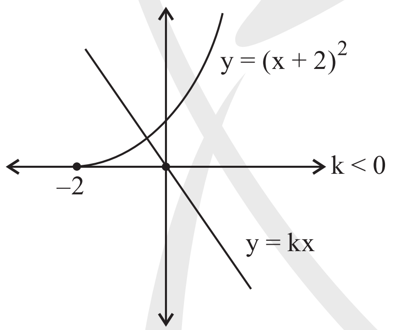
Equation has only one root.

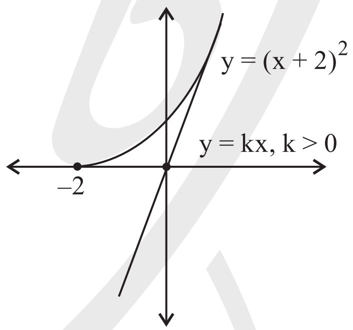
If $k > 0$, $\qquad kx = x^2 + 4x + 4$

$$x^2 + (4 - k)x + 4 = 0$$

$D = 0 \quad \Rightarrow \qquad (k - 4)^2 - 16 = 0$

$\Rightarrow \qquad k = 8$





6. $\log_{2^a}\left(\dfrac{1000}{b}\right) = 1$

$\Rightarrow \qquad \dfrac{1000}{b} = 2^a$

$\Rightarrow \qquad b = \dfrac{1000}{2^a}$

$(a, b) = (1, 500), (2, 250), (3, 125)$

7. $\log_{30} 8 = \frac{3(1-\log_{10} 5)}{(1+\log_{10} 3)} = \frac{3(1-a)}{1+b}$

$$\log_{243} 32 = \log_3 2 = \frac{1-\log_{10} 5}{\log_{10} 3} = \frac{1-a}{b}$$

$$\log_{40}(15) = \frac{\log_{10} 3 + \log_{10} 5}{1+2(1-\log_{10} 5)}$$

$$= \frac{1+b}{1+2(1-a)} = \frac{a+b}{3-2a}$$

SECTION-3

COMPREHENSION TYPE QUESTIONS

Q. 4 to Q. 6.

Adding $3xyz - 9 + \log_5 (xyz) = 32 + 81 + 256$

$\Rightarrow$ $3xyz + \log_5 (xyz) = 378$

$\because$ $f(x) = 3x + \log_5 x$ is strictly increasing $\forall x > 0$

$\Rightarrow$ $xyz = 125$

$\therefore$ $\log_5 x = 32 - 125 + 3 = -90$

$\log_5 y = -41$

$\log_5 z = 134$

Q. 7. to Q. 9

$$2\log_x (2y) = 2\log_{2x} 4z = \log_{2x^4} (8yz) = K$$

$$2y = x^{\frac{K}{2}}$$

$$4z = (2x)^{\frac{K}{2}}$$

$$8yz = (2x^4)^K$$

$$\Rightarrow \quad (2x)^{\frac{K}{2}} x^{\frac{K}{2}} = (2x^4)^K$$

$$\Rightarrow \quad 2^{-\frac{K}{2}} = x^{3K} \quad x = 2^{-\frac{1}{6}}$$

$$y = \frac{1}{2}\left(2^{-\frac{1}{6}}\right)^{\frac{K}{2}} = 2^{-1-\frac{K}{12}}$$

$$z = \frac{1}{4}\left(2^{\frac{5}{6}}\right)^{\frac{K}{2}} = 2^{\frac{5K}{12}-2}$$

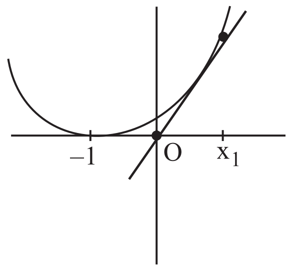
$$y^5 z = 2^{-5-2} = \frac{1}{128}$$

$$xy^5 z = \frac{1}{2^7 2^{\frac{1}{6}}} = \frac{1}{2^{\frac{43}{6}}}$$

SECTION-4

SUBJECTIVE TYPE QUESTIONS

1. $ax = (x + 1)^2$



For tangency $a = 2(x_1 + 1)$

$$ax_1 = (x_1+1)^2 = 2x_1(x_1+1)$$

$\Rightarrow x_1 = 1 \quad \Rightarrow \quad a = 4$

For eqn. to have exactly one root

$a \in (-\infty, 0) \cup \{4\}$

2. $$\left(\frac{3x+x^3}{1+3x^2}\right) = \ln\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right) = \ln\left(\frac{1+3x+3x^2+x^3}{1-3x+3x^2-x^3}\right)$$

$$= \ln\left(\frac{(1+x)^3}{(1-x)^3}\right) = 3f(x)$$

3. $10^{10^{10}} \le b \le 10^{10^{100}}$

$$N = 10^{10^{100}} - 10^{10^{10}} + 1 = \underbrace{999...9}_{(10^{100}-10^{10})\text{ times}}\underbrace{000..0}_{(10^{10}-1)\text{ times}}1$$

$$P = \underbrace{9+9+9+...9}_{(10^{100}-10^{10})\text{ times}} + 1 = 9(10^{100} - 10^{10}) + 1$$

$$= 9(\underbrace{99...9}_{90\text{ times}}\underbrace{000...0}_{10\text{ times}}) + 1$$

$$= 8\underbrace{999...9}_{89\text{ times}}1\underbrace{00...0}_{9\text{ times}}1$$

$$q = 8 + \underbrace{9+9+...9}_{89\text{ times}} + 2 = 10 + 89 \times 9 = 811$$

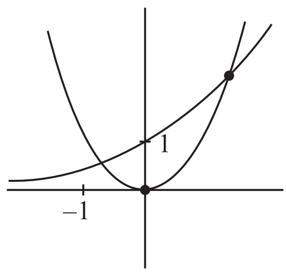
4. $x^2 + 2^x + \log_2(x^2 + 2^x) = 2^{x+1} + \log_2(2^{x+1})$

Let $f(x) = x + \ln_2 x$

$f(x)$ is strictly increasing $\forall n > 0$

$\Rightarrow \quad x^2 + 2^x = 2^{x+1}$

$\Rightarrow \quad x^2 = 2^x$



$x = 2, 4$ and one value of x lies in interval $(-1, 0)$.

$\therefore\ [x_1] + [x_2] + [x_3] = 2 + 4 + (-1) = 5.$

5. $8 \times 27(3^{3(\log_6 x - 1)} + 2^{3(\log_6 x - 1)} - 6^{3(\log_6 x - 1)})$

$$= 8 \times 27\left[\underbrace{(3^{3\log_6 x - 3} - 1)(1 - 2^{3\log_6 x - 3})}_{\le 0} + 1\right]$$

$\le 8 \times 27$

$\Rightarrow \quad S = 8 \times 27$

6. $\log_6 \dfrac{x^2+x}{x+4} > 1$ $\Rightarrow \dfrac{x^2+x}{x+4} > 6$

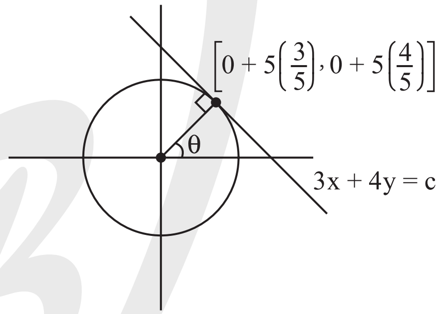
$\Rightarrow \dfrac{x^2-5x-24}{x+4} > 0$ $\Rightarrow x \in (-4,-3) \cup (8,\infty)$

7. $n \in [2^i, 2^{i+1}) \Rightarrow [\log_2 n] = i,\ \ i = 1,3,5,7,9$

$N = (2^2-2^1)+(2^4-2^3)+(2^6-2^5)+(2^8-2^7)+(2^{10}-2^9)$

$= 2+2^3+2^5+2^7+2^9 = \dfrac{2(2^{10}-1)}{4-1} = 681$

8. $\tan\theta = \dfrac{4}{3}$



max. value of 3x+4y = 3(3) + 4(4) = 25

9. If a, b, c are in G.P.

$\Rightarrow$ log a, log b, log c are in A.P.

Let log a = α, log b = α + d, log c = α + 2d

$\dfrac{\alpha+2d}{\alpha} + \dfrac{\alpha+d}{\alpha+2d} = \dfrac{2\alpha}{\alpha+d}$

$\Rightarrow \quad 4d^2 = -3\alpha^2 - 9\alpha d$

$7\log_c b - 2(\log_c b)^2 = 7\dfrac{(\alpha+d)}{(\alpha+2d)} - 2\dfrac{(\alpha+d)^2}{(\alpha+2d)^2}$

$= \dfrac{(5\alpha^2+12d^2+17\alpha d)}{\alpha^2+4d^2+4\alpha d}$

$= \dfrac{5\alpha^2-(9\alpha^2+27\alpha d)+17\alpha d}{\alpha^2+4d\alpha+(-3\alpha^2-9\alpha d)} = 2$

10. $2 + \log_{|x|}\left(\dfrac{|x|-\sqrt{3}}{\sqrt{3}|x|-2\sqrt{2}}\right) = \log_{|x|}\left(\dfrac{\sqrt{3}|x|+2\sqrt{2}}{|x|+\sqrt{3}}\right)$

$\Rightarrow |x|^2\left(\dfrac{|x|-\sqrt{3}}{\sqrt{3}|x|-2\sqrt{2}}\right) = \dfrac{\sqrt{3}|x|+2\sqrt{2}}{|x|+\sqrt{3}}$

$\Rightarrow \quad x^2(x^2-3) = 3x^2 - 8$

$\Rightarrow \quad x^4 - 6x^2 + 8 = 0$

$\Rightarrow \quad x^2 = 4, 2, \ x = \pm 2, \pm \sqrt{2}$

11. Let $\log_{10}x = \alpha$, $\log_{10}y = \beta$, $\log_{10}z = \gamma$

$\alpha + \beta + \gamma = 81, \ \alpha\beta + \beta\gamma + \alpha\gamma = 468$

$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$\Rightarrow \quad \alpha^2 + \beta^2 + \gamma^2 = 81^2 - 2(468) = 5625$

$\Rightarrow \quad |\vec{V}| = \frac{1}{25}\sqrt{\alpha^2 + \beta^2 + \gamma^2} = \frac{75}{25} = 3$

12. $\sum_{r=1}^{160} \frac{\log_3\left(\frac{r+2}{r+1}\right)}{\log_3(r+1)\log_3(r+2)} = \sum_{r=1}^{160}\left(\frac{1}{\log_3(r+1)} - \frac{1}{\log_3(r+2)}\right)$

$= \frac{1}{\log_3 2} - \frac{1}{\log_3 162} = \frac{4}{(\log_3 162)(\log_3 2)}$

13. $2^x = 3^y$, $2^x = 6.2^y$

$\Rightarrow \quad 3^y = 6.2^y \Rightarrow y = \log_{\frac{3}{2}} 6$

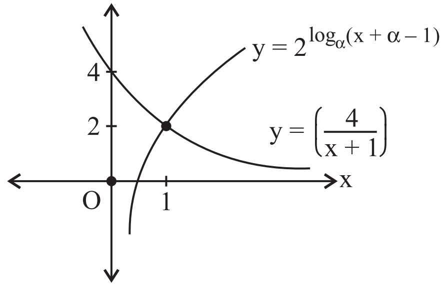
$\Rightarrow \quad 2^x = 3^{\log_{\frac{3}{2}} 6} \Rightarrow x = \log_2 3^{(\log_{3/2} 6)}$

$\Rightarrow \quad x = \left(\log_{\frac{3}{2}} 6\right)(\log_2 3)$

$\Rightarrow \quad ab = 2 \times \frac{3}{2} = 3$

14. $\log_\alpha\left(\frac{4}{x+1}\right) = \log_\alpha(2x + \alpha - 1)\log_\alpha 2 = \log_\alpha 2^{\log_\alpha(x+\alpha-1)}$

$\Rightarrow \quad \frac{4}{x+1} = 2^{\log_\alpha(x+\alpha-1)}$



15. $a = K_1^4, b = K_1^5, c = K_2^2, d = K_2^3$

$\Rightarrow \quad (K_2 - K_1^2)(K_2 + K_1^2) = 19$

$\Rightarrow \quad K_2 + K_1^2 = 19$ and $K_2 - K_1^2 = 1$

$\Rightarrow \quad K_2 = 10, K_1 = 3$

$\Rightarrow \quad d - b = (10)^3 - 3^5 = 1000 - 243 = 757$

17. $[\log_2 r] = m$ for $2^m \le r < 2^{m+1}$

No. of values of $r = 2^{m+1} - 2^m = 2^m$

$\therefore \quad \sum_{m=0}^{7} m2^m = 1538$

$\therefore \; 1538 + 8(n - 2^8 + 1) = 2018$

$\Rightarrow \quad n = 315$

18. $(\log_{2019} x)^2 + \frac{1}{2} = 2\log_{2019} x$

$\Rightarrow \; 2(\log_{2019} x)^2 - 4\log_{2019} x + 1 = 0$

$\Rightarrow \; \log_{2019} x_1 + \log_{2019} x_2 = \frac{4}{2} = 2$

$\Rightarrow \; x_1 x_2 = (2019)^2 = (2000 + 19)^2$

$= 1000K + 361$

19. $[\log_2 n] = 2K$

$\Rightarrow \quad 2^{2K} \le n < 2^{2K+1}$

$\Rightarrow \quad$ No. of values of $n = 2^{2K}$

$[\log_2 n] = 2$, no of 'n' = 4

$[\log_2 n] = 4$, no of 'n' = 16

$[\log_2 n] = 6$, no of 'n' = 64

$[\log_2 n] = 8$, no of 'n' = 256

Total no. of values of n = 340.

20. $(\log_a b)^2 - 5\log_a b + 6 = 0$

$\Rightarrow \quad \log_a b = 2 \text{ or } 3$

$\Rightarrow \quad b = a^2 \text{ or } a^3$

If $b = a^2 \Rightarrow a \le 44$

If $b = a^3 \Rightarrow a \le 12$

$\Rightarrow$ No. of ordered pairs (a, b) = 43 +11 = 54.

21. $a_1^{12} r^{66} = 8^{2006}$

$\Rightarrow \quad a_1^2 r^{11} = 2^{1003}$

Let $\quad a_1 = 2^x, r = 2^y$

$\Rightarrow \quad 2x + 11y = 1003$

$\Rightarrow \quad y = \dfrac{1003 - 2x}{11}$

$\Rightarrow \quad x = 11k + 1, x \le 496$

$\Rightarrow \quad$ No. of x = 46

22. $(\sqrt{x}\log_b x)^3 = b^{56}$ ... (1)

$(\log_b x)^{54} = x$ ... (2)

$\Rightarrow \quad (\sqrt{x}\, x^{\frac{1}{54}})^3 = b^{56}$

$\Rightarrow \quad x^{\frac{14}{9}} = b^{56} \quad \Rightarrow \quad x = b^{36}$

From (2), we get

$(36)^{54} = x = b^{36}$

$\Rightarrow \quad b = (36)^{3/2} = 216$